1D DB-FDTD formulation using flux density for study of electromagnetic wave propagation

RANDRIAMAROSON Rivo Mahandrisoa [1], RAKOTOMIRAHO Soloniaina[2], RATSIMBA Mamy Nirina[3]

[1] *PhD, SE-I-MSDE, ED-STII, Antananarivo, Madagascar*
[2] *Professor, SE-I-MSDE, ED-STII, Antananarivo, Madagascar*
[3] *PhD, SE-I-MSDE, ED-STII, Antananarivo, Madagascar*

ABSTRACT

In order to implement simultaneously in the same FDTD simulation an ABC (Absorbing Boundary Condition) formed by loss layers and a dielectric medium within the FDTD grid, the DB-FDTD formulation for the 1D case is studied in this paper. The DB-FDTD formulation uses the electric and magnetic flux densities in Maxwell's equations. The DB-FDTD formulation allows to separate the update equations in the program into two categories. One equation group will be used for the implementation of loss layers, and another group for the definition of simulation space using the space-dependent electromagnetic parameters. Simulations for the 1D case are implemented with sources introduced by TFSF (Total Field Scattered Field) limit. In order to compare the classic FDTD formulation with the DB-FDTD formulation, the execution times of the programs are calculated.

Keyword: *FDTD 1D, ABC, Loss layer, Dielectric medium, DB-FDTD, Flux density*

1. INTRODUCTION

Termination of an FDTD grid using loss layers properly absorbs waves exiting the grid. In order to implement such terminations, electrical and magnetic (virtual) conductivities are brought into the updates equations [1]. This can be confusing when modeling an anisotropic conductive medium, because in doing so the values of electrical and magnetic conductivities are formed so that they are spatially dependent. Thus the classical FDTD implementation of an anisotropic medium requires more attention to its implementation with grid terminations with loss layers.

So that the implementation of FDTD grid terminations with loss layers does not interfere with the modeling of the anisotropic medium within the grid, this paper will discuss the FDTD formulation (DB-FDTD) using electric flux densities (D) and magnetic flux densities (B).

2. REFORMULATION USING FLUX DENSITIES

A more general form of Maxwell's equations uses electric flux density (D) and magnetic flux density (B) in addition to electric and magnetic fields. This general form, presented in Eq.1, introduces the material equations linking the flux densities and the electric and magnetic fields in frequency domain (Eq.1.b and Eq.2.d). Eq.1.a defined relationship between the magnetic field and the electric flux density, and Eq.1.c defined relationship between the electric field and the magnetic flux density [2]. The polarization functions are defined with electrical and magnetic conductivities (σ_{ep} and σ_{mp}) which are fictitious parameters allowing the implementation of the grid termination by absorbing layer.

$$\nabla \times \vec{H} = \frac{\partial \vec{D}}{\partial t} + \frac{\sigma_{ep}}{\varepsilon} \vec{D} \tag{1.a}$$

$$\widehat{D}(\omega) = \hat{\varepsilon}(\omega).\widehat{E}(\omega) \tag{1.b}$$

$$\nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t} - \frac{\sigma_{mp}}{\mu}\vec{B} \tag{1.c}$$

$$\widehat{B}(\omega) = \hat{\mu}(\omega).\widehat{H}(\omega) \tag{1.d}$$

2.1. Electrical and magnetic flux densities update equations

The Ampère (Eq.1.a) and Faraday (Eq.1.c) equations lead to formulations with finite difference of flux densities [3]. These formulations lead to the updating equations for the electric flux density (Eq.2) and those for the magnetic flux density (Eq.3). Eq.2 and Eq.3 relate to the updates of the fields for the 1D case where there are field's variations only for the propagation in the x direction.

$$D_z^{n+1}(i) = C_{dd}(i)D_z^n(i) + C_{dh}(i)\left(\frac{H_y^{n+\frac{1}{2}}(i)-H_y^{n+\frac{1}{2}}(i-1)}{\Delta x}\right) \tag{2.a}$$

$$C_{dd}(i) = \frac{1-\frac{\sigma_{ep}(i)\Delta t}{2\varepsilon(i)}}{1+\frac{\sigma_{ep}(i)\Delta t}{2\varepsilon(i)}} \qquad ; \qquad C_{dh}(i) = \frac{\Delta t}{1+\frac{\sigma_{ep}(i)\Delta t}{2\varepsilon(i)}} \tag{2.b}$$

$$B_y^{n+\frac{1}{2}}(i) = C_{bb}(i)B_y^{n-\frac{1}{2}}(i) + C_{be}(i)\left(\frac{E_z^n(i+1)-E_z^n(i)}{\Delta x}\right) \tag{3.a}$$

$$C_{bb}(i) = \frac{1-\frac{\sigma_{mp}(i)\Delta t}{2\mu(i)}}{1+\frac{\sigma_{mp}(i)\Delta t}{2\mu(i)}} \qquad ; \qquad C_{be}(i) = \frac{\Delta t}{1+\frac{\sigma_{mp}(i)\Delta t}{2\mu(i)}} \tag{3.b}$$

The implementation of these equations in a time walking loop is done in the same way as when implementing the field update equations in a usual formulation of the FDTD.

Since the parameters σ_{ep} and σ_{mp} belong to fictitious sources in the medium, they do not come into play for the definition of the constituent materials inside the grid. However, to be able to terminate the FDTD grid with paired loss layers with the medium, these terms are used for absorption of incident waves at the edges of the grid.

2.2. Electric and magnetic fields update equations

Equations 1.b and 1.d must be formulated as a time domain difference for an FDTD implementation. It is therefore necessary to pass these equations from the frequency domain to the time domain. For this, the medium is considered as a medium with electrical and magnetic losses, the material parameters of which are given in Eq.4 [2] [4].

$$\hat{\varepsilon}(\omega) = \varepsilon_0\hat{\varepsilon}_r(\omega) \tag{4.a}$$

$$\hat{\varepsilon}_r(\omega) = \varepsilon_r + \frac{\sigma_e}{j\omega\varepsilon_0} \tag{4.b}$$

$$\hat{\mu}(\omega) = \mu_0\hat{\mu}_r(\omega) \tag{4.c}$$

$$\hat{\mu}_r(\omega) = \mu_r + \frac{\sigma_m}{j\omega\mu_0} \tag{4.d}$$

Using Eq.4.b in Eq.1.b, and Eq.4.d in Eq.1.d, the equations relating flux densities to fields are obtained in Eq.5.

$$\widehat{D}(\omega) = \varepsilon_0\varepsilon_r\widehat{E}(\omega) + \frac{\sigma_e}{j\omega}\widehat{E}(\omega) \tag{5.a}$$

$$\widehat{B}(\omega) = \mu_0\mu_r\widehat{H}(\omega) + \frac{\sigma_m}{j\omega}\widehat{H}(\omega) \tag{5.b}$$

Applying the inverse Fourier transform to Eq.5.a provides the electric flux density as a function of time (Eq.6). In Eq.6, the integral is approximated by a sum over the time step Δt, thus giving the time-sampled form of the electric flux density at Eq.prg.7.

$$D(t) = TF^{-1}\{\widehat{D}(\omega)\} = \varepsilon E(t) + \sigma_e \int_0^t E(\tau)d\tau \tag{6}$$

$$D^n = \varepsilon E^n + \sigma_e \Delta t \sum_{i=0}^n E^i \tag{7}$$

In Eq. 7, the resolution of the field at time step n depend on the current value of the flux density (D^n) as well as on the current value of the field (E^n). This is an inconsistency, because the current value of a field should depend only on the current and / or passed value of another field and / or the passed value of the field to be calculated. In order to correct the formulation of Eq.7, the term E^n is removed from the summation (Eq.8).

$$D^n = \varepsilon E^n + \sigma_e \Delta t E^n + \sigma_e \Delta t \sum_{i=0}^{n-1} E^i \tag{8}$$

Using Eq. 8, the current value of the field can be calculated from the current value of the flux density (D^n) and the previous values of the field ($E^i\big|_{i\in[0,n-1]}$). The equations for updating the electric and magnetic fields are thus obtained in Eq. 9.

$$E^n = \frac{D^n - \sigma_e \Delta t \sum_{i=0}^{n-1} E^i}{\varepsilon + \sigma_e \Delta t} \tag{9.a}$$

$$H^n = \frac{B^n - \sigma_m \Delta t \sum_{i=0}^{n-1} H^i}{\mu + \sigma_m \Delta t} \tag{9.b}$$

The summation terms are defined by the auxiliary terms I_e for Eq.9.a and I_m for Eq.9.b. These auxiliary terms are given in Eq. 10.

$$I_e^{n-1} = \sigma_e \Delta t \sum_{i=0}^{n-1} E^i \tag{10.a}$$

$$I_m^{n-1} = \sigma_m \Delta t \sum_{i=0}^{n-1} H^i \tag{10.b}$$

In the end, the equations for updating the fields associated with the equations for updating the auxiliary terms are given in Eq.10 for the electric field and in Eq.11 for the magnetic field.

$$E^n = \frac{D^n - I_e^{n-1}}{\varepsilon + \sigma_e \Delta t} \tag{10.a}$$

$$I_e^n = I_e^{n-1} + \sigma_e \Delta t E^n \tag{10.b}$$

$$H^n = \frac{B^n - I_m^{n-1}}{\mu + \sigma_m \Delta t} \tag{11.a}$$

$$I_m^n = I_m^{n-1} + \sigma_m \Delta t E^n \tag{11.b}$$

All the information about the media is contained in Eq.10 and Eq.11. The medium can be defined using the coefficients of these equations, using the electromagnetic parameters.

3. DB-FDTD ALGORITHM USING FLUX DENSITIES

3.1. Grid termination with absorbent layers

In order for the grid to behave like an infinite space, its boundaries will be made up with absorbent layers whose absorption factors are gradually increased as the layers progress. The application of the grid termination by absorbent layers, only involves the flux density updating equations (Eq.2 and Eq.3). Equation 12 represents the calculation of loss factors.

$$pe(i) = \frac{\sigma_{ep}(i)\Delta t}{2\varepsilon(i)} = pm(i) = \frac{\sigma_{mp}(i)\Delta t}{2\mu(i)} = 0.333 \left(\frac{i}{taille_{perte}}\right)^3 ;$$

$$i = [1, taille_{perte}], \ et \ i = [taille_{grille} - taille_{perte}, taille_{grille}] \tag{prg.12}$$

The loss factors have zero values for the entire space constituting the interior of the grid. As the loss factors Do not intervene in Eq.10 and Eq.11, the definition of loss layers as ABC of the FDTD grid does not in any way affect the implementation of the definition of the materials constituting the medium of simulation.

3.2. Calculation algorithm

The steps for calculating fields using the DB-FDTD formulation involving flux densities are similar to the steps for calculating using the conventional FDTD formulation. These calculation steps are:

1. Updating the electric flux density D at time $n + 1$ (Eq.2)

2. Injection from the source to the chosen source node

3. Updating the electric field E at time $n + 1$ (Eq.10.a)

4. Update of the electrical auxiliary term I_e at time $n + 1$ (Eq.10.b)

5. Updating the magnetic flux density setting B at time $n + \frac{1}{2}$ (Eq.3)

6. Updating the magnetic field H at time $n + \frac{1}{2}$ (Eq.11.a)

7. Update of the magnetic auxiliary term I_m at time $n + \frac{1}{2}$ (Eq.11.b)

8. Repeat steps 1 to 7 for each time step

3.3. TFSF formulation

The injection of the source can be done by TFSF formulation by applying the corrections to the flux densities. The corrections are made in a similar way as for the conventional FDTD formulation [5]. For the 1D case the TFSF formulation, for a wave propagating in the direction of x positive, is described in Eq. 13.

$$D_z^{n+1}(i_{src}) = D_z^{n+1}(i_{src}) + \frac{C_{dh}}{\eta} E_{zinc}\left(-\frac{1}{2}, n + \frac{1}{2}\right) \tag{13.a}$$

$$B_y^{n+\frac{1}{2}}\left(i_{src} - \frac{1}{2}\right) = B_y^{n+\frac{1}{2}}\left(i_{src} - \frac{1}{2}\right) - C_{be} E_{zinc}(0, n) \tag{13.b}$$

4. SIMULATION DB-FDTD 1D

4.1. Wave propagation in vacuum

Fig. 1 gives snapshots of an EM wave traveling through a vacuum. The source, which is a Ricker wavelet of frequency $f = 500 \ THz$, is introduced by TFSF limit at node 30, and the grid (of $150 \ nodes$ size) is terminated by loss layers ($taille_{perte} = 15$). For all the nodes of the grid the permittivity and the permeability are uniform ($\varepsilon_r = 1; \mu_r = 1$), and the conductivities are zero ($\sigma_e = \sigma_m = 0$). Fig. 2, which is a cascading view of the snapshots, shows the absorption by the loss layers.
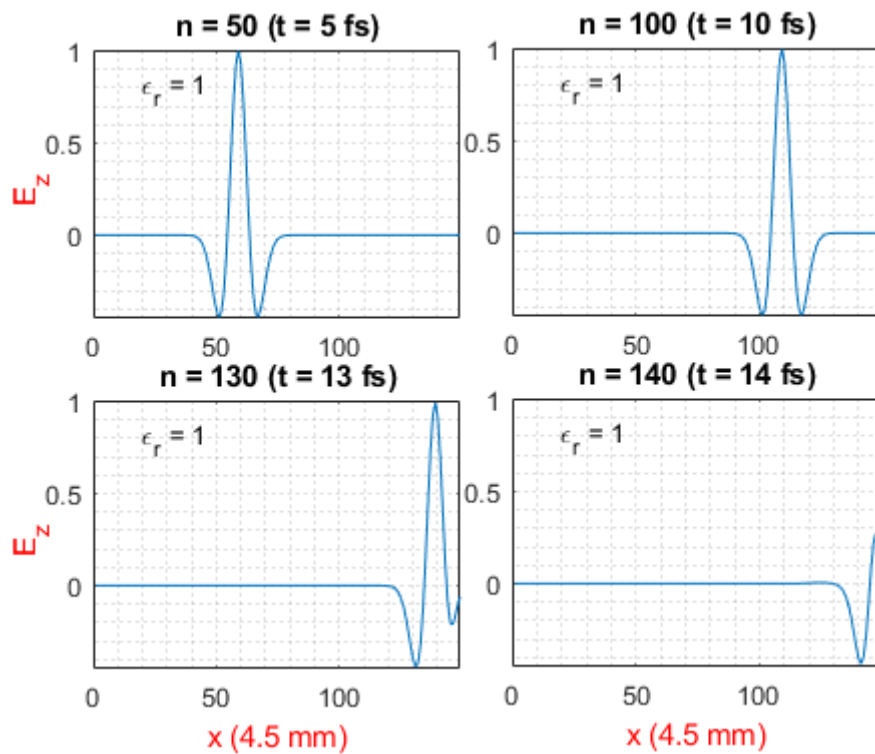
Fig.1: Snapshots of wave propagation in vacuum using the DB-FDTD formulation
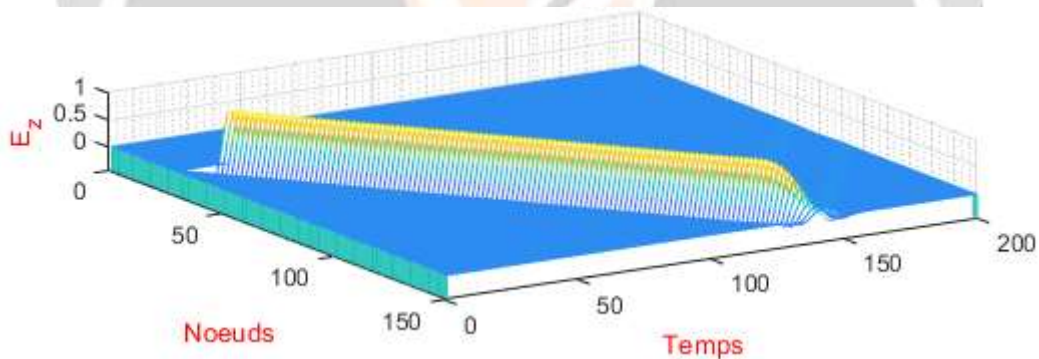

Fig.2: Cascading view of wave propagation in vacuum using the DB-FDTD formulation

4.2. Wave propagation striking a dielectric medium

The hardware parameters of the medium can be changed using $\varepsilon(i)$ and $\sigma(i)$. Fig.3 shows the propagation of a wave passing through a dielectric layer ($\varepsilon_r = 4$) from node 71 to node 101. The conductivity of the dielectric is that of an isolating material ($\sigma_e = 10^{-17}\ S.m^{-1}$), a conductive material ($\sigma_e = 5,8 \times 10^7\ S.m^{-1}$) would reflect the incident wave to the medium made of this material (Fig.4). The observed amplitudes of the wave correspond to the reflection and transmission coefficients calculated for the left limit (Eq.14.a) and right limit (Eq.14.b) of the dielectric.

$$\Gamma_G = -\frac{1}{3} \text{ et } T_G = \frac{2}{3} \tag{14.a}$$

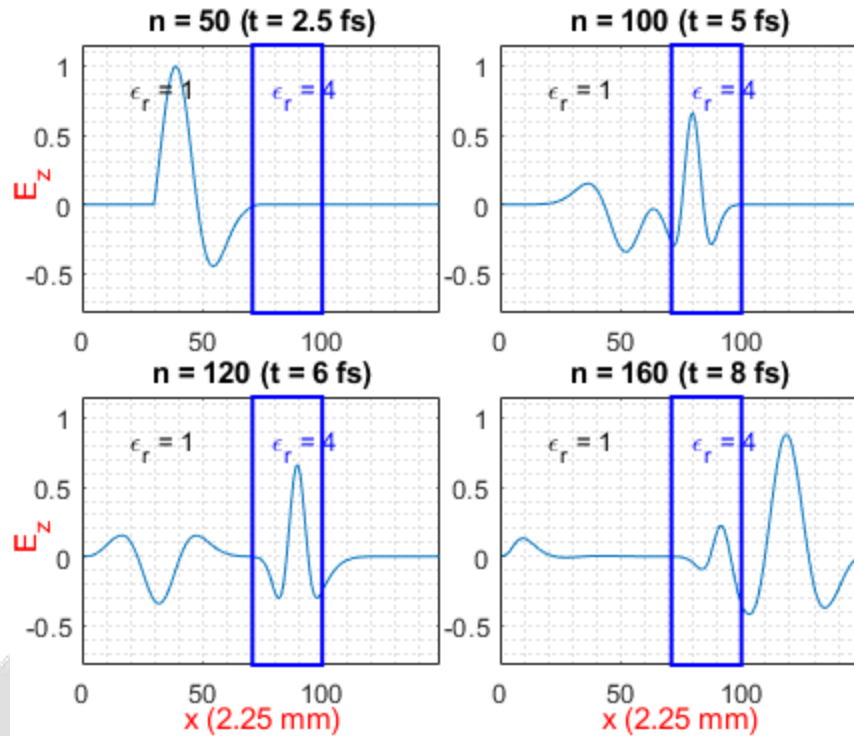$$\Gamma_D = \frac{1}{3} \text{ et } T_D = \frac{4}{3} \tag{14.b}$$

Fig.3: Snapshots of a wave passing through an isolating medium with relative permittivity $\varepsilon_r = 4$
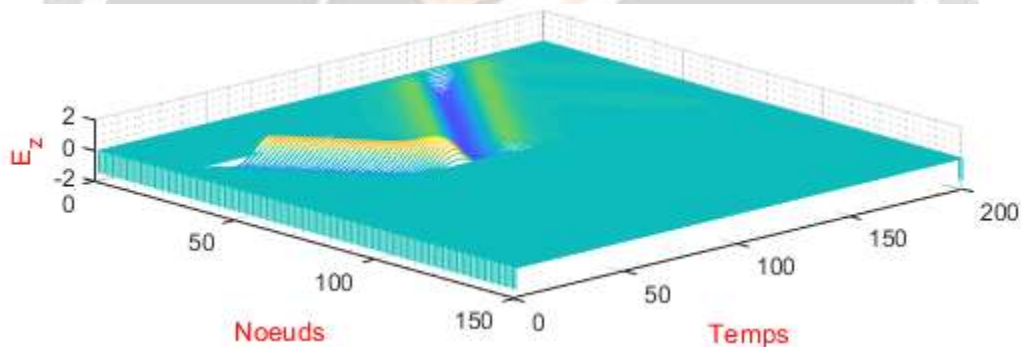

Fig.4: Cascading view of a wave hitting a conductive medium

4.3. Comparison of the classic FDTD formulation and the DB-FDTD formulation

Tab. 1 gives the execution time of the FDTD and DB-FDTD programs for various simulation scenarios. The start of the execution time is defined before the definition of the various variables and the end by the end of the time loop traversal. The execution time therefore concerns the time necessary for the resolution of the propagation of a wave and does not include the time necessary for post-processing.

The execution times for a scenario using an isolating medium are longer than others. This is due to the size taken by the variable $\sigma_e = 10^{-17}\ S.m^{-1}$. For the scenario with a conductive medium, $\sigma_e = 5.8 \times 10^7\ S.m^{-1}$, the execution time is less important since the size of the variable is less voluminous compared to the previous scenario variable size. The scenario involving vacuum propagation has the smallest execution time because $\sigma_e = 0$. However, for all scenarios the execution time for a DB-FDTD formulation is greater than classic FDTD formulation execution time.

Tab.1: Execution time of FDTD and DB-FDTD 1D programs

	Program execution time (in ms)		
	Propagation in vacuum	Insertion of an isolating medium in vacuum	Insertion of a conductive medium in vacuum
FDTD	40.9	70.8	45.4
DB-FDTD	54.6	74.3	57.6

5. CONCLUSION

The results of the DB-FDTD formulation, using flux densities, are as accurate as for the classical FDTD formulation using electric and magnetic fields. The implementation of loss layers like ABC of the grid, as well as the implementation of TFSF limit in the DB-FDTD formulation generate the same results as for those of the classic FDTD formulation. The DB-FDTD formulation is more complex and introduces additional calculations for the FDTD program, which results in increased program execution time. But the strong point of this technique is the differentiation of the equations implementing the ABC and those implementing the parameters of the medium. The definition of dielectric materials in the simulation medium is thus facilitated.

6. REFERENCES

[1]. R.M. Randriamaroson, « Terminating FDTD Grid with Electrical and Magnetically loss layers», *IJARIIE*, vol. 6, Issue-5, 2020, pp. 1869-1875.

[2]. D. M. Sullivan ; "Electromagnetic Simulation using the FDTD method" ; Second Edition ; IEEE Press ; 2013

[3]. K. S. Yee, « Numerical solution of initial boundary value problems involving Maxwell's equations in isotropic media », *IEEE Trans. Antenn. Propag.*, vol. 17, 1966, pp. 585–589.

[4]. A. Taflove, « Computational electrodynamics – The Finite-Difference Time-Domain Method », 3rg Ed, Artech House, 2005.

[5]. R.M. Randriamaroson, « TFSF boundary implementation in the FDTD Algorithm for the Study of the Propagation of Electromagnetic Waves in Vacuum», *IJARIIE*, vol. 5, Issue-5, 2019, pp. 452-461.