# 2D DB-FDTD formulation using flux density for study of plane wave propagation in TM mode

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## ABSTRACT

*In order to implement simultaneously in the same two-dimensional FDTD simulation an ABC (Absorbing Boundary Condition) formed by loss layers as well as a dielectric medium within the FDTD gate, the DB-FDTD formulation for the case 2D is studied in this paper. The DB-FDTD formulation uses the electric and magnetic flux densities in Maxwell's equations. The DB-FDTD formulation allows to separate the update equations in the program into two categories. One equation group will be used for the implementation of loss layers, and another group for the definition of simulation space using the space-dependent electromagnetic parameters. Simulations for the 2D case are implemented with sources introduced by TFSF (Total Field Scattered Field) limit, thus introducing a plane wave into the simulation space. The simulation of the plane wave propagation is made for vacuum and for a medium where a dielectric cylinder is present in the vacuum.*

**Keyword**: *FDTD 2D, ABC, Loss layer, Dielectric Cylinder, DB-FDTD, Plane Wave*

## 1. INTRODUCTION

Termination of an FDTD grid using loss layers properly absorbs waves exiting the grid. In order to implement such terminations, electrical and magnetic (virtual) conductivities are involve [1]. This can cause confusion when modeling an anisotropic conductive medium, for to do the values of electric and magnetic conductivities are formed so that they are dependent on space. Thus the classical FDTD implementation of an anisotropic medium requires more attention on its implementation with grid terminations with loss layers.

For the implementation of FDTD grid terminations with loss of layers does not interfere with the modeling of the anisotropic medium in the grid, this work will discuss the FDTD formulation (DB-FDTD) using electric flux density (D) and magnetic flux density (B). The simulation of the propagation of a plane wave in TM (Transverse Magnetic) mode within a 2D FDTD grid will be the subject of particular attention in this paper.

## 2. REFORMULATION A L'AIDE DES DENSITES DE FLUX

A more generalized form of Maxwell's equations uses electric flux density ($D$) and magnetic flux density ($B$) in addition to electric and magnetic fields. This general form, presented in Eq.1, introduces the material equations linking the flux densities and the electric and magnetic fields in frequency domain (Eq.1.b and Eq.2.d). Eq.1.a defined relationship between the magnetic field and the electric flux density, and Eq.1.c defined relationship between the electric field and the magnetic flux density [2]. The polarization functions are defined with electrical

and magnetic conductivities ($\sigma_{ep}$ and $\sigma_{mp}$) which are fictitious parameters allowing the implementation of the grid termination by absorbing layer.

$$\nabla \times \vec{H} = \frac{\partial \vec{D}}{\partial t} + \frac{\sigma_{ep}}{\varepsilon} \vec{D} \qquad (1.a)$$

$$\hat{D}(\omega) = \hat{\varepsilon}(\omega).\hat{E}(\omega) \qquad (1.b)$$

$$\nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t} - \frac{\sigma_{mp}}{\mu} \vec{B} \qquad (1.c)$$

$$\hat{B}(\omega) = \hat{\mu}(\omega).\hat{H}(\omega) \qquad (1.d)$$

**2.1. Electrical and magnetic flux densities update equations**

The Ampère (Eq.1.a) and Faraday (Eq.1.c) equations lead to formulations with finite difference of flux densities [3]. These formulations lead to the updating equations for the electric flux density (Eq.2) and for the magnetic flux density (Eq.3). Eq.2 and Eq.3 relate to the updates of the fields for the 2D case where there are variations only for the propagation in $x$ and $y$ directions in mode TM, there is no variation in the $z$ direction. The multiplication coefficients of the densities and fields are expressed using loss factors to facilitate the implementation of the termination of the grid with loss layers [1].

$$D_z^{n+1}(i,j) = C_{dd}(i,j)D_z^n(i,j) + C_{dh}(i,j)\left\{\left(\frac{H_y^{n+\frac{1}{2}}(i,j)-H_y^{n+\frac{1}{2}}(i-1,j)}{\Delta x}\right) - \left(\frac{H_x^{n+\frac{1}{2}}(i,j)-H_x^{n+\frac{1}{2}}(i,j-1)}{\Delta y}\right)\right\} \qquad (2.a)$$

$$C_{dd}(i,j) = \frac{1-\frac{\sigma_{ep}(i,j)\Delta t}{2\varepsilon(i,j)}}{1+\frac{\sigma_{ep}(i,j)\Delta t}{2\varepsilon(i,j)}} = \frac{1-pez(i,j)}{1+pez(i,j)} \quad ; \quad C_{dh}(i,j,k) = \frac{\Delta t}{1+\frac{\sigma_{ep}(i,j)\Delta t}{2\varepsilon(i,j)}} = \frac{\Delta t}{1+pez(i,j)} \qquad (2.b)$$

$$B_x^{n+\frac{1}{2}}(i,j) = C_{bb}(i,j)B_x^{n-\frac{1}{2}}(i,j) + C_{be}(i,j)\left(\frac{E_z^n(i,j+1)-E_z^n(i,j)}{\Delta y}\right) \qquad (3.a)$$

$$C_{bxbx}(i,j) = \frac{1-\frac{\sigma_{mp}(i,j)\Delta t}{2\mu(i,j)}}{1+\frac{\sigma_{mp}(i,j)\Delta t}{2\mu(i,j)}} = \frac{1-pmx(i,j)}{1+pmx(i,j)} \quad ; \quad C_{bxez}(i,j) = \frac{-\Delta t}{1+\frac{\sigma_{mp}(i,j)\Delta t}{2\mu(i,j)}} = \frac{-\Delta t}{1+pmx(i,j)} \qquad (3.b)$$

$$B_y^{n+\frac{1}{2}}(i,j) = C_{bb}(i,j)B_y^{n-\frac{1}{2}}(i,j) + C_{be}(i,j)\left(\frac{E_z^n(i+1,j)-E_z^n(i,j)}{\Delta x}\right) \qquad (3.c)$$

$$C_{byby}(i,j) = \frac{1-\frac{\sigma_{mp}(i,j)\Delta t}{2\mu(i,j)}}{1+\frac{\sigma_{mp}(i,j)\Delta t}{2\mu(i,j)}} = \frac{1-pmy(i,j)}{1+pmy(i,j)} \quad ; \quad C_{byez}(i,j) = \frac{\Delta t}{1+\frac{\sigma_{mp}(i,j)\Delta t}{2\mu(i,j)}} = \frac{\Delta t}{1+pmy(i,j)} \qquad (3.d)$$

The implementation of these equations in a time walking loop is done in the same way as when implementing the field update equations in a usual formulation of the 2D FDTD.

Since the parameters $\sigma_{ep}$ and $\sigma_{mp}$ belong to fictitious sources in the medium, they do not come into play for the definition of the constituent materials inside the grid. However, to be able to terminate the FDTD grid with paired loss layers with the medium, these terms are used for absorption of incident waves at the edges of the grid.

### 2.2. Electric and magnetic fields update equations

Equations 1.b and 1.d must be formulated as a time domain difference for an FDTD implementation. It is therefore necessary to pass these equations from the frequency domain to the time domain. For this, the medium is considered as a medium with electrical and magnetic losses, the material parameters of which are given in Eq.4 [2] [4].

$$\hat{\varepsilon}(\omega) = \varepsilon_0 \hat{\varepsilon}_r(\omega) \tag{4.a}$$

$$\hat{\varepsilon}_r(\omega) = \varepsilon_r + \frac{\sigma_e}{j\omega\varepsilon_0} \tag{4.b}$$

$$\hat{\mu}(\omega) = \mu_0 \hat{\mu}_r(\omega) \tag{4.c}$$

$$\hat{\mu}_r(\omega) = \mu_r + \frac{\sigma_m}{j\omega\mu_0} \tag{4.d}$$

Using Eq.4.b in Eq.1.b, and Eq.4.d in Eq.1.d, the equations relating flux densities to fields are obtained in Eq.5. Applying the inverse Fourier transform to Eq.5.a provides the electric flux density as a function of time (Eq.6). In Eq.6, the integral is approximated by a sum over the time step $\Delta t$, thus giving the time-sampled form of the electric flux density at Eq.prg.7.

$$\hat{D}(\omega) = \varepsilon_0 \varepsilon_r \hat{E}(\omega) + \frac{\sigma_e}{j\omega} \hat{E}(\omega) \tag{5.a}$$

$$\hat{B}(\omega) = \mu_0 \mu_r \hat{H}(\omega) + \frac{\sigma_m}{j\omega} \hat{H}(\omega) \tag{5.b}$$

$$D(t) = TF^{-1}\{\hat{D}(\omega)\} = \varepsilon E(t) + \sigma_e \int_0^t E(\tau)d\tau \tag{6}$$

$$D^n = \varepsilon E^n + \sigma_e \Delta t \sum_{i=0}^n E^i \tag{7}$$

In Eq. 7, the resolution of the field at time step $n$ depend on the current value of the flux density $(D^n)$ as well as on the current value of the field $(E^n)$. This is an inconsistency, because the current value of a field should depend only on the current and / or passed value of another field and / or the passed value of the field to be calculated. In order to correct the formulation of Eq.7, the term $E^n$ is removed from the summation (Eq.8).

$$D^n = \varepsilon E^n + \sigma_e \Delta t E^n + \sigma_e \Delta t \sum_{i=0}^{n-1} E^i \tag{8}$$

Using Eq. 8, the current value of the field can be calculated from the current value of the flux density $(D^n)$ and the previous values of the field $(E^i\big|_{i\in[0,n-1]})$. The equations for updating the electric and magnetic fields are thus obtained in Eq. 9.

$$E^n = \frac{D^n - \sigma_e \Delta t \sum_{i=0}^{n-1} E^i}{\varepsilon + \sigma_e \Delta t} \tag{9.a}$$

$$H^n = \frac{B^n - \sigma_m \Delta t \sum_{i=0}^{n-1} H^i}{\mu + \sigma_m \Delta t} \tag{9.b}$$

The summation terms are defined by the auxiliary terms $I_e$ for Eq.9.a and $I_m$ for Eq.9.b. These auxiliary terms are given in Eq. 10.

$$I_e^{n-1} = \sigma_e \Delta t \sum_{i=0}^{n-1} E^i \tag{10.a}$$

$$I_m^{n-1} = \sigma_m \Delta t \sum_{i=0}^{n-1} H^i \tag{10.b}$$

In the end, the equations for updating the fields associated with the equations for updating the auxiliary terms are given in Eq.10 for the electric field and in Eq.11 for the magnetic field.

$$E^n = \frac{D^n - I_e^{n-1}}{\varepsilon + \sigma_e \Delta t} \tag{10.a}$$

$$I_e^n = I_e^{n-1} + \sigma_e \Delta t E^n \tag{10.b}$$

$$H^n = \frac{B^n - I_m^{n-1}}{\mu + \sigma_m \Delta t} \tag{11.a}$$

$$I_m^n = I_m^{n-1} + \sigma_m \Delta t E^n \tag{11.b}$$

All the information about the media is contained in Eq.10 and Eq.11. The medium can be defined using the coefficients of these equations, using the electromagnetic parameters.

## 3. DB-FDTD ALGORITHM USING FLUX DENSITIES FOR PLANE WAVE PROPAGATION

### 3.1. Grid termination with loss layers

In order for the grid to behave like an infinite space, its boundaries will be made up with absorbent layers whose absorption factors are gradually increased as the layers progress. The application of the grid termination by absorbent layers, only involves the flux density updating equations (Eq.2 and Eq.3). Equation 12 represents the calculation of loss factors [1].

$$i \epsilon \left[1, taille_{perte}\right] \tag{12.a}$$

$$perte(i) = 0.333 \left(\frac{i}{taille_{perte}}\right)^3 \tag{12.b}$$

$$pez(i: taillex - i + 1, i) = perte(taille_{perte} - i + 1) \tag{12.c}$$

$$pez(i: taillex - i + 1, tailley - i + 1) = perte(taille_{perte} - i + 1) \tag{12.d}$$

$$pez(i, i: tailley - i + 1) = perte(taille_{perte} - i + 1) \tag{12.e}$$

$$pez(taillex - i + 1, i: tailley - i + 1) = perte(taille_{perte} - i + 1) \tag{12.f}$$

$$pez = pmx = pmy \tag{12.g}$$

The loss factors have zero values for the entire space constituting the interior of the grid. As the loss factors Do not intervene in Eq.10 and Eq.11, the definition of loss layers as ABC of the FDTD grid does not in any way affect the implementation of the definition of the materials constituting the medium of simulation.

### 3.2. TFSF formulation for introducing plane wave

The TFSF formulation is done by applying the corrections to the flux densities. The corrections are made in a similar way as for the conventional 2D FDTD formulation. For the simulation of a plane wave propagating in the direction of $x$ positive, the correction equations are applied to the electric flux density (Eq.13) and to the components of the magnetic flux density (Eq.14, Eq.15 ). The TFSF limit is defined by the nodes of coordinates $(i_d, j_d)$ and $(i_f, j_f)$ defining a rectangular limit [5].

$$j \in \left[j_d, j_f\right] \tag{13.a}$$

$$D_z^{n+1}(i_d, j) = c_{dzdz}D_z^n(i_d, j) - c_{dzby}H_{yinc}^{n+\frac{1}{2}}\left(i_d - \frac{1}{2}\right)/eps(i,j) \tag{13.b}$$

$$D_z^{n+1}(i_f, j) = c_{dzdz}D_z^n(i_f, j) + c_{dzby}H_{yinc}^{n+\frac{1}{2}}\left(i_f + \frac{1}{2}\right)/eps(i,j) \tag{13.c}$$

$$i \in \left[i_d, i_f\right] \tag{14.a}$$

$$B_x^{n+\frac{1}{2}}\left(i, j_d - \frac{1}{2}\right) = c_{bxbx}B_x^{n+\frac{1}{2}}\left(i, j_d - \frac{1}{2}\right) - c_{bxez}E_{zinc}^n(i) \tag{14.b}$$

$$B_x^{n+\frac{1}{2}}\left(i, j_f + \frac{1}{2}\right) = c_{bxbx}B_x^{n+\frac{1}{2}}\left(i, j_f + \frac{1}{2}\right) + c_{bxez}E_{zinc}^n(i) \tag{14.c}$$

$$j \in \left[j_d, j_f\right] \tag{15.a}$$

$$B_y^{n+\frac{1}{2}}\left(i_d - \frac{1}{2}, j\right) = c_{byby}B_y^{n-\frac{1}{2}}\left(i_d - \frac{1}{2}, j\right) - c_{byez}E_{zinc}^n(i_d) \tag{15.b}$$

$$B_y^{n+\frac{1}{2}}\left(i_f + \frac{1}{2}, j\right) = c_{byby}B_y^{n-\frac{1}{2}}\left(i_f + \frac{1}{2}, j\right) + c_{byez}E_{zinc}^n(i_f) \tag{15.c}$$

### 3.3. 2D DB-FDTD algorithm for plane wave propagation

The steps for calculating fields using the 2D DB-FDTD formulation involving flux densities are similar to the steps for calculating using the conventional 2D FDTD formulation. For the simulation of a plane wave, these calculation steps are:

1. Updating the electric flux density $D$ at time $n + 1$ (Eq.2)

2. Correction of electric flux density $D$ at time $n + 1$ (Eq.13)

3. Updating the electric field $E$ at time $n + 1$ (Eq.10.a)

4. Update of the electrical auxiliary term $I_e$ at time $n + 1$ (Eq.10.b)

5. Updating the magnetic flux density $B$ at time $n + \frac{1}{2}$ (Eq.3)

6. Correction of magnetic flux density $B$ at time $n + \frac{1}{2}$ (Eq.14, Eq.15)

7. Updating the magnetic field $H$ at time $n + \frac{1}{2}$ (Eq.11.a)

8. Update of the magnetic auxiliary term $I_m$ at time $n + \frac{1}{2}$ (Eq.11.b)

9. Repeat steps 1 to 8 for each time step

**4. DB-FDTD SIMULATION OF A PLANE WAVE MODE TM**

**4.1. Propagation of a plane wave in vacuum**

Fig. 1 presents the snapshots of a Ricker wavelet introduced by TFSF formulation of a 2D grid of dimension (150 × 100). In order to satisfy the stability criterion, the number of currents is set by $S_c = 0.7071$. The medium is considered to be not ferromagnetic ($\mu_r = 1, \sigma_m = 0$). Tab. 1 presents the parameters calculated for 2D simulations of the propagation of a plane wave traveling in a vacuum.

**Tab.1:** 2D simulations parameters in vacuum

| Frequency | Wavelength | Spatial resolution | Spatial step | Time step |
|---|---|---|---|---|
| $f_{max} = 500\ THz$ | $\lambda_{min} = 600\ nm$ | $N_\lambda = 20$ | $\Delta x = 29.97\ nm$ <br><br> $\Delta y = 29.97\ nm$ | $\Delta t = 0.07\ fs$ |

In Fig. 1, the Ricker wavelet is introduced by TFSF limit, and therefore the propagation of a plane wave is seen in this figure. The wave propagates in the direction of the $x$ positive, and is absorbed at the right edge of the grid. At $t = 14.14\ fs$, half of the pulse has been absorbed by the paired loss layer in the middle, and there is no wave reflection at the edges of the grid.
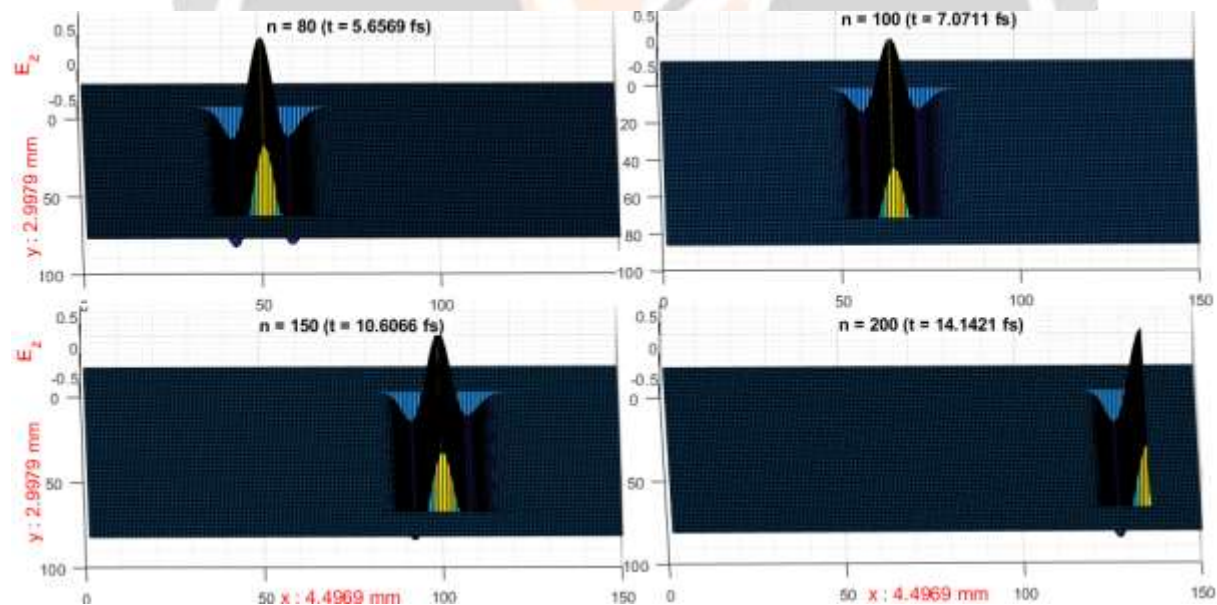


**Fig.1:** Snapshots of a plane wave propagating in vacuum using the DB-FDTD formulation

**4.2. Plane wave propagation striking a dielectric medium**

To simulate a plane wave interacting with an object, the object must be specified according to its electromagnetic properties: the dielectric constant and the conductivity. For example, the simulation concerning the propagation of a plane wave striking a dielectric cylinder of radius $r = 200\ nm \cong 20\ \Delta x$, which has a dielectric constant specified by the parameter $\varepsilon$ and a conductivity specified by the parameter $\sigma_e$, described in the Eq.10. The parameters

calculated for a simulation with a dielectric medium of relative permittivity $\varepsilon_r = 9$, are given in Tab. 2. In order to define a circle $C$ with radius $R$, and center coordinate $(i_c, j_c)$, the points belonging to the circle are defined in Eq.16.

$$point(i, j) \in C : \sqrt{(i_c - i)^2 + (j_c - j)^2} \leq R \qquad (16)$$

**Tab.2**: Parameters of 2D simulations with a dielectric cylinder

| Frequency | Wavelength | Spatial resolution | Spatial step | Time step |
|-----------|------------|---------------------|--------------|-----------|
| $f_{max} = 500\ THz$ | $\lambda_{min} = 200\ nm$ | $N_\lambda = 40$ | $\Delta x = 9.99\ nm$  $\Delta x = 9.97\ nm$ | $\Delta t = 0.02\ fs$ |

Fig. 2 presents the snapshots of a plane wave striking an isolating material with relative permittivity $\varepsilon_r = 9$ and conductivity $\sigma_e = 10^{-17}\ S.m^{-1}$. On reaching the perimeter of the circle, part of the wave is reflected while another part continues to propagate in the circle but with a speed reduced to a third of that in vacuum ($c = c_0/\sqrt{\varepsilon_r}$). The fields reflected by the dielectric can go outside the TFSF limit and end up being absorbed by the absorbent layers terminating the grid.
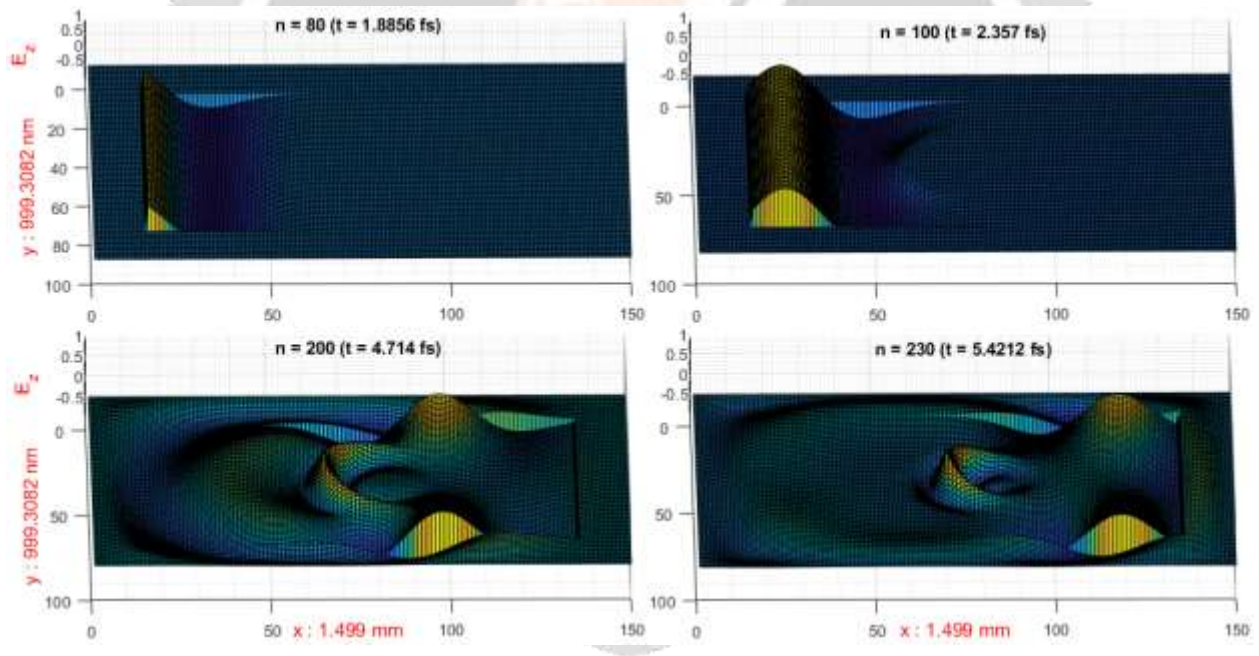


**Fig.2**: Illustration of a plane wave hitting an isolating dielectric medium

Fig. 3 illustrates the propagation of a plane wave in vacuum where a conductive material with relative permittivity $\varepsilon_r = 9$ and conductivity $\sigma_z = 5,8 \times 10^7\ S.m^{-1}$ is placed at the center of the FDTD grid. The material being conductive, the incident waves to the circle are completely reflected then absorbed by the terminations of the grid when leaving the TFSF limit.
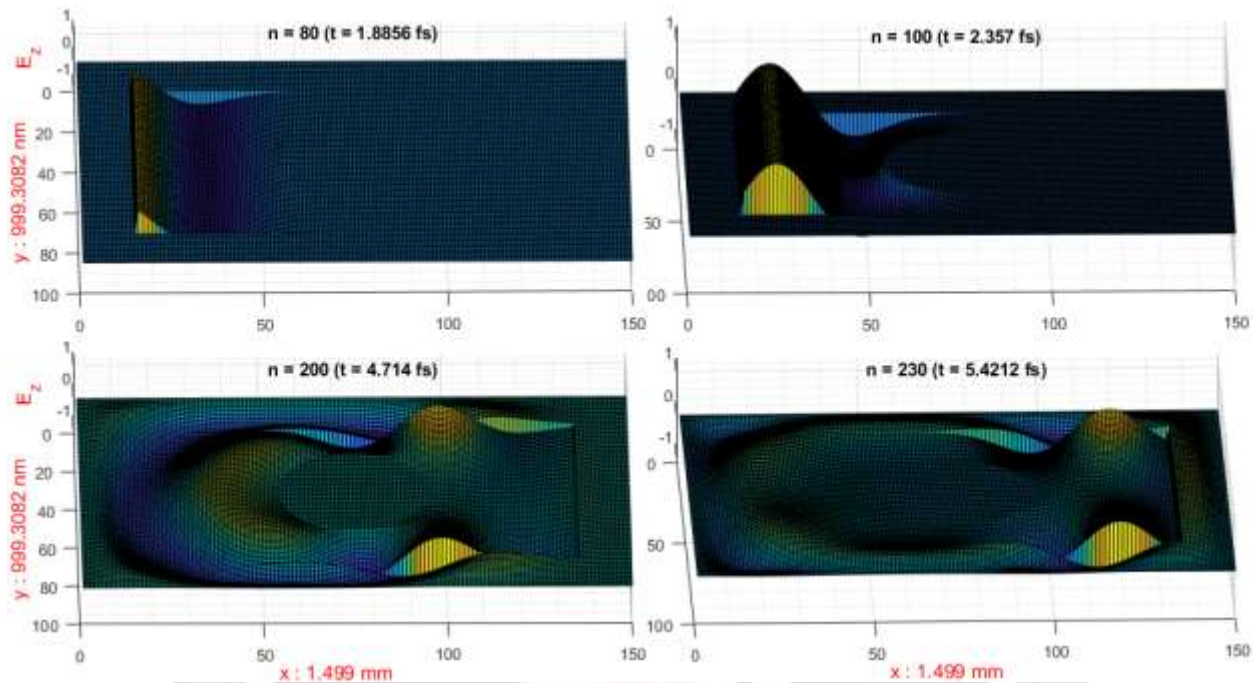
**Fig.3**: Illustration of a plane wave striking a conducting dielectric medium

## 5. CONCLUSION

The results of the 2D DB-FDTD formulation, using flux densities, are as accurate as for the classical 2D FDTD formulation using electric and magnetic fields. The implementation of the source as a plane wave, using a rectangular TFSF boundary, uses the same principles as for the implementation in the conventional 2D FDTD formulation. The only change is the application of the corrections for the flux densities for the DB-FDTD formulation instead of applying them to the fields as in the case of the classic FDTD formulation. The ABC consisting of the loss layer fulfills its role well by absorbing the fields leaving the TFSF limit and incident at the limits of the 2D FDTD grid.

The use of the DB-FDTD formulation allows to distinguish the implementation of ABC and the implementation of anisotropic medium. The simulations using the DB-FDTD formulation for various plane wave propagation scenarios in TM mode, namely in vacuum as well as in vacuum and striking a isolating then a conductive dielectric, allows to account for the precision of the formulation equaling the precision of the classic 2D FDTD formulation.

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