ADDITIONAL ENERGY DENSITY IN SIX-DIMENSIONAL SPACE-TIME

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ABSTRACT

The electrodynamical variables in symmetric, psudo-Euclidean six-dimensional space-time have been constructed in terms of covariant six-vector vector formulation. The EM field tensor, Maxwell equations and wave equation in six space explicitly inherit their dependence on temporal degrees of freedom. Representing six-fields in vector form, we have expressed six-Poynting vector and energy density associated with fields in six-space and it has been shown that presence of additional fields substantially enhances EM field energy density. The observation of a limit for high energy process beyond which time trajectory of the process changes, may reveal the presence of extra time dimensions.

Keywords: *Six-dimensional space-time, Maxwell equations, Energy density;*

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1. INTRODUCTION:

In order to explore natural phenomena of all possible events, various consistent theoretical models in unification programme, gravitation, superstring theories and extended relativity have been developed which essentially involve multi-dimensional structure of space-time. In extended relativity, the localization of superluminal events is possible only in time dimensions [1-5], and to incorporate sub- or superluminal phenomena, the theories with higher dimensional space-time have been constructed. The most symmetrical, with minimum number of space-time structure is Six-dimensional space-time [6-13] with three-space, three-time orthogonal dimensions. The temporal degrees of freedom incorporate new fields and these electromagnetic fields play important role in kinematics, classical and quantum field theoretical studies [14] in six-space. At high energies, temporal fluctuations become more prominent and considerably stable particle would become unstable [15]. Studying charge-field interaction through Cerenkov radiation in six-space, We [16] have shown that large amount of energy is required to turn the time trajectory of the radiation. Observationally, An event in six dimensional space-time, through space-time structural mappings [17] may be subjected to real, observational four-dimensional Minkowski world.

Keeping in view the role of temporal degrees of freedom in six-space, In the present paper, the space-time structure is expressed through six-vectors and covariant electrodynamics has been developed. The EM fields, Field tensor and wave equations have been identified. Expressing Maxwell set of field equations in vectorial representation, the energy flow, energy density associated with fields have been shown to carry contributions from temporal degrees of freedom.

2. BACKGROUND: STRUCTURE OF SIX-DIMENTIONAL SPACE-TIME

The six-dimensional space-time is represented by six-coordinates, comprised of a spatial vector *r* and a temporal vector ***t*****,** such that,

$$\{x^{\mu}\} \equiv (x_r\,, x_t\,), \quad (1)$$

$$\{x^{\mu}\} \equiv (\,x_1\,, x_2\,, x_3\,, x_4\,, x_5\,, x_6\,)\,. \quad (2)$$

These six-coordinates are orthogonal to each other, such that, a position vector$\{x^{\mu}\}$is specified by following six-component column vector;

$$\{x^{\mu}\} \equiv (\,x^1, x^2\,, x^3\,, x^4 = \,t^1, x^5 = \,t^2\,, x^6 = \,t^3\,)^{\mathrm{T}} \quad (3)$$

Where T denotes the transpose. The Greek indices μ,ν acquire values from 1 to 6, and sub- or superscripts μ,ν =1,2,3 represent spatial coordinates and μ,ν = 4,5,6 account for temporal coordinates. The space and time dimensions, composed of three-dimensional vector space $[R^3]$ and three-dimensional temporal vector $[T^3]$, are orthogonal to each other, such that,

$$[R^3] \perp [T^3] \quad . \tag{4}$$

In general, the velocity in six-space is a dyadic, which without loss of generality, may be expressed in six-component form [18]. We may define a unit time vector α, in the time field of the particle such that, the components of velocity vector $\vec{v}$ for a moving particle, are integrated with the components of the unit time vector $\vec{\alpha}$. The six-velocity vector in six-space is defined as;

$$\{ v^{\mu} \} = dx^{\mu}/ d\tau = \gamma(v)\ [v, \alpha]^{T} \tag{5}$$

With $\gamma(v) = dt / d\tau$. The time vector is directed tangentially to the time trajectory of the particle and infinitesimal increments dt and $d\tau$ are measured along the time curve of the particle in moving and instantaneous frames respectively. let us consider another frame of reference K'', with relative six-velocity $\{v_{\nu}\}$ with respect to frame K'. The velocity composition law may be expressed as,

$$\Lambda\{v_{\mu}\}.\ \Lambda\{v_{\nu}\} = \Lambda\{v_{\nu}\}.\ \Lambda\{v_{\mu}\} = \Lambda\{v_{\lambda}\} \tag{6}$$

Where,

$$\{v_{\lambda}\} = [\{v_{\nu}\} + \{v_{\mu}\}] / [\ 1 + \{v_{\nu}\}^{T}.\{v_{\mu}\}\ ] \tag{7}$$

In general, the matrices $\Lambda\{v_{\mu}\}$ and $\Lambda\{v_{\nu}\}$ do not commute and hence Thomas precession will be predicted. Any inertial observer, say K_o, in six space-time, may always choose the axes, in such a way that under a transcendent Lorentz transformation is represented by ;

$$x \rightarrow t_x;\ y \rightarrow t_y;\ z \rightarrow t_z \tag{8}$$

and

$$t_x \rightarrow x\ ;\ t_y \rightarrow y;\ t_z \rightarrow z \tag{9}$$

the reference metric for $D(3 \oplus 3)$ formulation is given by;

$$g_{\mu\nu} = (\ 1,1,1,-1,-1,-1). \tag{10}$$

The quadratic invariance between six-dimensional reference frames is represented as:

$$|ds|^2 = -|d\vec{r}|^2 + \left|d\vec{t}\right|^2 = g_{\mu\nu}x^{\mu}x^{\nu} \tag{11}$$

An inertial observer in six-space has equal freedom to choose any of the axes, as the formal expression of LT's is independent of any space-direction. The Lorentz transformations between a non-prime and prime frame in six-space, say K and K', may be defined as,

$$\{x^{\mu}\} \rightarrow \{x^{\mu}\}' = \Lambda\{x^{\mu}\} \tag{12}$$

where, Λ is 6x6 Lorentz transformation matrix [19].

3. EM FIELDS IN SIX-SPACE:

The covariance requirement of the six- space suggests that six potential may be represented by two vectors, having components-

$$\{A_{\mu}\} = \{\ A^1, A^2, A^3, A^4, A^5, A^6\}^T \equiv \{\ A, \phi\}^T \tag{13}$$

Where T denotes the transpose and { } represents six-vectors in six-dimensional space. The six- differential operator ∂_{μ} may be constructed as two differential constituents;

$$\{\ \partial_{\mu}\} = \{\ \partial_r\ , \partial_t\}^T \equiv (\ \partial/\partial x_{\mu}\ )\ , \tag{14}$$

Acting in spatial and temporal sub vector spaces, $[R]^3$ and $[T]^3$, respectively. The tensor transformation properties of all these six-vectors will be governed by metric tensor defined by (10). Using definitions of six-potential and six-derivatives, the skew symmetric EM field tensor in six-vector notation, may be defined as ;

$$F_{\mu\nu} = \partial_{\nu}A_{\mu} - \partial_{\mu}A_{\nu} \equiv \frac{\partial A_{\mu}}{\partial x_{\nu}} - \frac{\partial A_{\nu}}{\partial x_{\mu}} \tag{15}$$

Which represents the six-dimensional curl of the six-potential. This 6x6 EM field tensor has thirty-six components in total, which may be arranged in the following matrix form;

$$F_{\mu\nu} = \begin{vmatrix} F_{ij} & F_{im} \\ F_{mi} & F_{mn} \end{vmatrix}, \tag{16}$$

Where i, j = 1,2,3; m = i+3, n = j+3 and F_{ij}, F_{il}, F_{mj} and F_{ml} are 3x3 sub-matrices. The EM field tensor, equation (15), yields following components of six-dimensional fields;

$$\partial_r \times A = H_r \tag{17}$$

$$\partial_t \times \phi = H_t \quad (18)$$
$$\partial_{x_i}\phi_m - \partial_{t_m} A_i = E_{im} \quad (19)$$

The Eq. (18), mathematically analogous to equation (17) shows possible curl of temporal part of six-potential in temporal dimensions, therefore, may be named as temporal magnetic field. The fields represented by equations (17) and (18) are generated in six-space as rotations of spatial and temporal vector potential in respective spatial and temporal dimensions i.e. $[R]^3$ and $[T]^3$, respectively. The electric field components constitute a 3x3 dyadic in six-space. We may equally choose row or column generalization to construct three component vector in respective subspaces of the formalism. The row generalization leads to components like (E_{14} E_{15} , E_{16}), (E_{24} E_{25} , E_{26}) and (E_{34} , E_{35} , E_{36}) and column generalization leads to choice of electric field components as (E_{14} , E_{24} , E_{34}), (E_{15} , E_{25} , E_{35}) and (E_{16} , E_{26} , E_{36}). The first set corresponds to time multiplicity and second set corresponds to space multiplicity, respectively. The EM field tensor in terms of six-fields, equations (17-19) may also be represented as ;

$$F_{\mu\nu} = \begin{bmatrix} H_r & E_{im} \\ E_{mi} & H_t \end{bmatrix} \quad (20)$$

Such that H_r and H_t represent antisymmetric 3x3 component matrix with three non-zero magnetic field components in respective subspaces. In general, the 6x6 antisymmetric EM field tensor has fifteen independent components- explicitly- three spatial magnetic field, three temporal magnetic field and nine electric field components.

The six-current in six-space, may be defined as;

$$\{J_\mu\} \equiv \{ J, \rho\}^T . \quad (21)$$

The space current density J, (or three spatial current), can be derived from charge source density vector (or three temporal current), through the following relationship;

$$J_i = (v_{ij})\rho^j \qquad (i,j = 1,2,3) \quad (22)$$

Following the dyadic generalization of the dyadic velocity such that;

$$v == (v_{ij}).\, \delta_{ij} \quad (23)$$

And assuming the vector charge source density to be equivalent and fundamental one in all temporal planes of the theory, i.e. $\rho^1 = \rho^2 = \rho^3$,

the equation (23) may take the form,

$$J^i = \rho\ (v_{ij}).\, \delta_{ij} \quad (24)$$

or more generally,

$$J = \rho.v \qquad \text{or,} \quad (25)$$
$$\{J_\mu\} \equiv \{ v, I\}^T.\rho \quad (26)$$

Where *I* being of the same order of matrix *v*. The six-current satisfies the continuity equation as;

$$\partial_\mu J^\mu = 0. \quad (27)$$

In six-space, the Maxwell field equations in terms of six-EM field tensor and six-current may be written as;

$$F_{\mu\nu}{}^{,\nu} = J^\mu \quad (28)$$
$$\tilde{F}_{\mu\nu\rho\sigma}{}^{,\sigma} = 0 \quad (29)$$

Where $\tilde{F}$ represents dual of EM field tensor, defined as ;

$$\tilde{F}_{\mu\nu\rho\sigma}{}^{,\sigma} = \epsilon_{\mu\nu\rho\sigma\zeta\eta}\ F^{\zeta\eta} \quad (30)$$

The tensor $\epsilon_{\mu\nu\rho\sigma\zeta\eta}$ is completely anti-symmetric tensor of rank six, such that,

$$\epsilon_{\mu\nu\rho\sigma\zeta\eta} = \begin{cases} +1 \text{ for even pernutation} \\ -1 \text{ for odd permutation} \\ 0 \text{ if indices are repeated.} \end{cases} \quad (31)$$

The dual tensor in six-space has higher rank than that of its original tensor. Explicitly, we have following field equations in six-space;

$$(\partial_r \times H_r)_p = \partial_{x_m}(E_{pi}) + J_p \quad (32)$$
$$(\partial_t \times H_t)_p = \partial_{x_i}(E_{ip}) + \rho_p \quad (33)$$
$$\partial_{x_m}(H_r) = -\epsilon_{pmn}\ \partial_{x_j}(E_{nj}) \quad (34)$$
$$\partial_{x_j}(H_t) = -\epsilon_{pmn}\ \partial_{x_m}(E_{jn}) \quad (35)$$
$$\partial_r .\ H_r = 0 \quad (36)$$

$$\partial_t . \mathrm{H_t} = 0 \quad (37)$$

Where, *i,j = 1,2,3 p,m,n=4,5,6;* ϵ_{pmn} is the skew symmetric permutation symbol with components, $\epsilon_{123}= \epsilon_{231}= \epsilon_{312}= -\epsilon_{132}= -\epsilon_{321}= -\epsilon_{213}= 1$. The Lorentz gauge condition for potentials may be modified in six-space, so that extended condition is given by,

$$\partial_r A + \partial_t \phi = 0 \quad (38)$$

The wave equation in six-space, with Lorentz gauge equation (38), may assume the form;

$$\partial_t{}^2 A - \partial_r{}^2 A = J \quad (39)$$

$$\partial_t{}^2 \phi - \partial_r{}^2 \phi = \rho \, . \quad (40)$$

The wave equations, may also be represented in abbreviated form, as;

$$\hexagon\, A_\mu = J_p \quad (41)$$

$$\hexagon = \partial_r{}^2 - \partial_t{}^2 \quad (42)$$

$\hexagon$ is the D'Alembertian operator and $\partial_r{}^2, \partial_t{}^2$ represent the norm of the differential operators in spatial and temporal planes respectively.

4. ENERGY DENSITY ASSOCIATED WITH FIELDS IN SIX-SPACE:

The electric field dyadic represented by equation (19), may also be represented in two vectors by choosing space-time indices as per selective spatial or temporal subspaces of six-space, as;

$$E_{\odot j} = -\partial_{t_j} A - \partial_r \phi_j \equiv E_r \left(E_{1j}, E_{2j}, E_{3j} \right), \forall \odot = 1,2,3 \quad (43)$$

$$E_{j\odot} = -\partial_t . A_j - \partial_{r_j} \phi \equiv E_t \left(E_{j1}, E_{j2}, E_{j3} \right)^T \quad (44)$$

In other words, E_r represents the vector electric fields, with fixed time trajectory in $[R]^4$ sub-space, whereas E_t selects space trajectory to represent electric field vector in $[T]^4$ sub-space, respectively.

In terms of these electric field vectors, the Maxwell equations are given by;

$$\partial_r \times H_r = (\partial_t)^j E_{\odot j} + J \quad (45)$$

$$\partial_t \times H_t = (\partial_x)^i E_{i\odot} - \rho \quad (46)$$

$$\partial_r \times E_{\odot j} = -(\partial_t)^i H_r \quad (47)$$

$$\partial_r \times E_{i\odot} = -(\partial_x)^j H_t \quad (48)$$

$$\partial_r . H_r = 0 \quad (49)$$

$$\partial_t . H_t = 0 \quad (50)$$

The $\odot$ in these equations takes multiple values for a fixed indices *i ,j* and *r, t* denote respective sub-spaces. These field equations lead to the following expressions for the flow of field energy in spatial and temporal subspaces, respectively,

$$E_{\odot j}\, (\partial_t)^m E_{\odot k} + (1/2)\, (\partial_t)^j |H_r|^2 = E_{\odot j} . J - div_r . S_j^r \quad (51)$$

$$E_{i\odot} . (\partial_x)^i E_{i\odot} + (1/2)\, (\partial_x)^i |H_t|^2 = -E_{i\odot} . \rho - div_t . S_i^t \quad (52)$$

The first term in equation (51) may be expended to,

$$E_{\odot j}\, (\partial_t{}^1 E_{r1} + \partial_t{}^2 E_{r2} + \partial_t{}^3 E_{r3}) = (1/2)\, \partial_t{}^j \left|E_{\odot j}\right|^2 \quad (53)$$

where, we have used orthogonality of the six-dimensions, which eliminates the terms, following time-trajectory other than that of $E_{\odot j}$. Similarly, the RHS of equation (52) reduces to,

$$\text{RHS} = (1/2)\, \partial_x{}^i \left|E_{i\odot}\right|^2 \quad (54)$$

Using equations (53) and (54) equations (51,52) may be written as;

$$(1/2)\, \partial_t{}^j \left(\left|E_{\odot j}\right|^2 + |H_r|^2\right) = -E_{\odot j} . J - div_r . S_j^r \quad (55)$$

$$(1/2)\, \partial_r{}^i \left(\left|E_{i\odot}\right|^2 + |H_t|^2\right) = -E_{i\odot} . \rho - div_t . S_i^t \quad (56)$$

where,

$$S_j^r = (E_{\odot j} \times H_r) \quad (57)$$

$$S_i^t = (E_{i\odot} \times H_t) \quad (58)$$

May be visualized as spatial and temporal parts of the Poynting vector. The net flow of EM field energy, therefore, in six-vector form may be expressed as;

$$\{ S^d \} = \{S_j^r ,\ S_i^t \}^T , \quad (59)$$

Where d represents the spatial or temporal constituent of the Poynting vector. To calculate total energy associated with EM fields, we specify six-volume given by-

$$\mathrm{dV} = \mathrm{d}V_r\ \mathrm{d}V_t \equiv \mathrm{d}x_1.\mathrm{d}x_2.\mathrm{d}x_3.\mathrm{d}t_1.\mathrm{d}t_2.\mathrm{d}t_3 \quad . \tag{60}$$

Integrating sum of equations (55) and (56) in six-volume, we get;

$$\int_{\mathrm{v}} (1/2)[\ \partial_t{}^j\ (|E_{\odot j}|^2 + |H_r|^2) + \ \partial_r{}^i\ (|E_{i\odot}|^2 + |H_t|^2)]\ \mathrm{dV}$$
$$= -\int_{\mathrm{v}} (E_{\odot j}.J +\ E_{i\odot}.\rho)\ \mathrm{dV} - \oint (S^d)^T\ (n^d).df^d \tag{61}$$

Where, n^d is unit column vector, defined by,

$$\{\ n^d\} = \{n^r\ , n^t\ \}^T, \tag{62}$$

Such that, $n^r\ , n^t$ are the unit vectors in the direction positive normal to the spatial and temporal surfaces respectively, over which the surface integral equation (61) extends. The LHS of the equation (61) leads to the following expression for the energy density of the EM fields in six-space;

$$\chi = (|E_{\odot j}|^2 + |H_r|^2 + \ |E_{i\odot}|^2 + |H_t|^2)/2 \tag{63}$$

For closed systems, the volume integral extends over entire six-space and then surface integral on RHS of equation (61) vanishes as field is zero at infinity and reduces to,

$$\int_{\mathrm{v}} (1/2)[\ \partial_t{}^j\chi^r + \partial_r{}^i\chi^t + (E_{\odot j}.J +\ E_{i\odot}.\rho)]\ \mathrm{dV} = 0 \tag{64}$$

where,

$$\chi^r = (1/2)(|E_{\odot j}|^2 + |H_r|^2) \tag{65}$$

$$\chi^t = (1/2)(\ |E_{i\odot}|^2 + |H_t|^2) \tag{66}$$

The second bracket in equation (64) is equivalent to the rate of change of particle's kinetic energy, hence equation may be written as;

$$\int_{\mathrm{v}} (1/2)[\ \partial_t{}^j\chi^r + \partial_r{}^i\chi^t + \partial^d.\mathcal{E}_{kin}\ ]\ \mathrm{dV} = 0 \tag{67}$$

Here, ∂^d represents total six-derivative and,

$$\partial^d.\mathcal{E}_{kin} = (E_{\odot j}.J +\ E_{i\odot}.\rho) \tag{68}$$

The integral in equation represents the total energy, including kinetic energy of the particles, carried by the six EM fields. However, if we integrate equation over any finite volume then the surface integrals on RHS does not vanish in general and hence for open system, we have,

$$\int (1/2)[\ \partial_t{}^j\chi^r + \partial_r{}^i\chi^t + \partial^d.\mathcal{E}_{kin}\ ]\ \mathrm{dV} = \oint (S^d)^T\ (n^d).df^d \tag{69}$$

Since the LHS of equation is the total energy of the EM fields, its RHS is the flux of the field energy across the surface bounding the volume. Thus for open systems, the flow of field energy is directed by the six- Poynting vector given by equation (69).

5. DISCUSSION:

The covariant formulation for orthogonal, symmetric space-time dimensions, specified by equation (3) has been developed with reference metric $g_{\mu\nu}$ given by equation (10). The skew symmetric 6x6 EM field tensor, equation (15), may be perceived as four 3x3 sub-matrices, equations (16,20), which clearly exhibit space-time rotational possibilities in six-space. The six covariant field equations expressed by equations (28,29) and in explicit form represented by equations (32-37), incorporate effect of the temporal vector space. The symmetry between spatial and temporal magnetic fields can be observed explicitly in six-dimensional Maxwell field equations. The Continuity equation (27), Lorentz gauge condition (38) and the wave equation (39) have been expressed in terms of spatial temporal potentials and currents. The Maxwell equations may also be represented in terms of extended D'Alembertian operator, equation (42), in six-space.

To derive the Poynting vector in six-space, the general dyadic electric field represented by equation (19), may be characterized by two electric field vectors, through selecting either time or space trajectory of event. However, this selection does not restrict the mixed components of the electric field dyadic. In general, any possible mixing of space and time coordinates may also be specified in terms of these two electric field vectors, equations (43,44). Following these vector forms of the electric field, the Maxwell set of equations (32-37) is written in vectorial form of fields, equation (45-50). The net flow of electromagnetic field energy is expressed by six- Poynting vector, equation (59), which has explicit spatial and temporal parts given by equations (65,66) in respective subspaces. Defining a unit six- volume equation (60), and integrating sum of equations (55,56),

the energy density of the six dimensional electromagnetic fields has been worked out for closed systems, equation (63). However, in case of open system the surface integral in equation (61) does not vanish and the rate of change of energy density is equal to the rate of change of particles kinetic energy which is exactly equal to the work done by the six dimensional electromagnetic fields on the particle, equation (69).
In the presence of large temporal deviations, the temporal part of the Poynting vector equation (58) gives rise to significant amount of energy flow supplied by the temporal magnetic field. Comparing equation (63) with that of conventional energy density we find that the energy density is increased largely due to the temporal contributions. It suggest that the ground state energy of the particles will increase in six-space configuration and consequently the particle would became more unstable, as pointed out by Dorling and Stranad [10]. The total field energy carried by the six dimensional fields is enhanced largely and reversing the argument we may infer that in conventional, i.e. four dimensional observational process, if large amount of energy is supplied the particles trajectory may change accordingly. In other words the observation of a limit for high energy process beyond which time trajectory of the process changes, may reveal the presence of extra time dimensions.

REFERENCES:

[1] Mignani R & Recami E., Lett Nuovo Cim **16** 449 (1976)
[2] Rajput B S and Chandola H C, Ind. J. Pure Appl. Phys **21** 651 (1983); **24**, 58 (1986).
[3] Recami E and Mignani R, Riv Nuovo Cimento **9** 11 (1986)
[4] Antippa A F, Phys. Rev D **11** 724 (1975)
[5] Gonzalez-Gascon F, Phys. Lett. A **56** 249 (1976)
[6] Demers P., Can. J. Phys., **53**, 1687 (1975).
[7] Ziino G, Lett. Nuovo Cimento, **24**, 171 (1979).
[8] Pappas P. T., Lett. Nuovo Cimento, **22**, 601 (1978).
[9] Cole E. A. B, Phys. Lett., **75** 29 (1979); Nuovo Cim B, **85**, 105 (1985).
[10] Pavsic M, J. Phys. A, **14**, 3217 (1981).
[11] Dattoli G and Mignani R, Lett. Nuovo. Cim. **22** 65 (1978);
[12] Vysin V, Lett. Nuovo. Cim **22** 76 (1978)
[13] Chaudhary P and Rajput B S, Indian J Phys. **85** 1843 (2011)
[14] Negi P S, Chandola H C & Rajput B S, Prog Theor. Phys. **80** 1 (1988).
[15] Dorling J, Am. J. Phys. **38** 539 (1970); Strand J, Lett. Nuovo. Cim. **26** 535 (1979)
[16] Chaudhary P, Chandola H C and Rajput B S, Nuovo. Cim. A**100** 2976 (1988)
[17] Chaudhary P and Rajput B S, Ind. J. Pure Appl. Phys. **29** 153 (1991)
[18] Cole E A B and Bhuchanun S, J Phys A **15** 1255 (1982)
[19] Chaudhary Prakash, *IJARIIE*, **8(1)** 1065 (2022), DUI: 16.0415/IJARIIE-15992