ANALYSIS OF BASIC SERIES INVERTER VIA THE APPLICATION OF ROHIT TRANSFORM

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Abstract

An electrical circuit that performs the function of converting DC voltage into AC voltage at the desired frequency is called an inverter. This paper presents the application of a new integral transform called Rohit transform for the analysis of a basic series inverter, which has been usually done via means of algebraic and analytic methods. In this paper, the basic series inverter is analyzed and its response is obtained via the application of the Rohit Transform.

Index Terms: *Rohit Transform, Basic series inverter, Response.*

INTRODUCTION

An inverter performs the function of converting DC voltage into AC voltage at the desired frequency. The classification of inverters as per the connections of semiconductor devices like thyristor (SCR) and commutating elements like a capacitor (C) and an inductor (Ł) is series inverters, parallel inverters and bridge inverters. In a series inverter, shown in figure 1, commutating elements Ł and C are connected permanently in series with the load resistor R [1, 2].

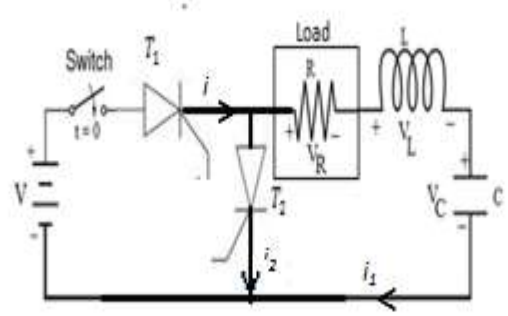

Figure 1: Basic Series Inverter

The thyristors T_1 and T_2 are alternately turned on. The values of commutating elements Ł and C in series with load resistor Ʀ are such that Ʀ < $\sqrt{\frac{4Ł}{C}}$ so that the Ł - C- Ʀ network forms an underdamped circuit. A series inverter can be a voltage fed inverter in which a voltage source of constant potential is connected to the Ł - C- Ʀ network or it can be a current fed inverter in which a current source providing a constant current is connected to the Ł - C- Ʀ network. An Inverter is used in the domestic installations and in the commercial installations as a source of stand by electric supply or uninterruptible power supply, in industrial installations for induction heating and for a variable speed AC drives [3, 4]. The basic series inverter is usually analyzed by calculus approach [1-3]. In this paper, the basic series inverter is analyzed via the application of the Rohit Transform. The Rohit Transform was proposed by the author Rohit Gupta in recent years and generally, it has been applied in different areas of science and engineering [5, 6 & 7]. The Rohit Transform [5, 6] of g(y), $y \geq 0$ is defined as

$\mathrm{R}\{g(y)\} = r^3 \int_0^\infty e^{-ry} g(y)dy = G(r)$, provided that the integral is convergent, where r may be a real or complex parameter.
The Rohit Transform (RT) of some derivatives [5, 6, 7] of g(y) are given by
$R\,\{g'(y)\} = rR\{g(y)\} - r^3 g(0)$
Or
$R\,\{g'(y)\} = rG(r) - r^3 g(0)$,

$R\{g''(y)\} = r^2 G(r) - r^4 g(0) - r^3 g'(0)$,

And so on.
METHODOLOGY
To derive the governing differential equation:
Considering a basic series inverter which consists of a series LCR network connected to a steady excitation voltage source of constant potential V through a thyristor T_1 and a thyristor T_2 is also connected in parallel to the series LCR network [1-4] as shown in figure 1.

In the first mode of operation of the basic series inverter [3, 4], since the thyristor T_2 is off and the thyristor T_1 is on, therefore, the equivalent circuit, in this case, is shown in figure 2.
When the switch is closed at t = 0, the potential drops across the network elements are given by $V_R\,(t) = R\,I(t)$, V_L (t) = Ł$[I'(t)]$ and $V_C\,(t) = \frac{q(t)}{C}$, where q (t) is the charge on the capacitor at any instant of time t.

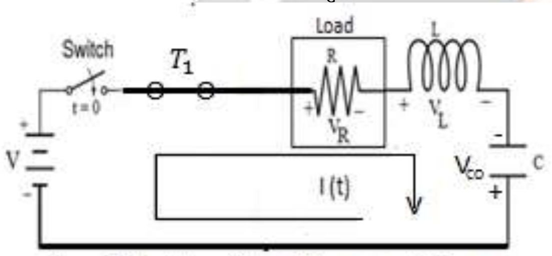

Figure 2: Equivalent Circuit (T_2 is OFF and T_1 is ON)

In this mode of operation, the capacitor is assumed to be initially charged to a potential V_{CO}, with the upper plate having negative polarity and the lower plate having positive polarity. Therefore, the application of Kirchhoff's loop law to the loop shown in figure 2 provides
$V_R(t) + V_Ł\,(t) + V_C\,(t) = V + V_{CO}$
Or
R I (t) + Ł$[I'(t)]$ + $\frac{q(t)}{C}$ = V + V_{CO} ….. (1)
Differentiate equation (1), we get a linear homogeneous differential equation of order 2 as given below:
R$[I'(t)]$ + Ł$[I''(t)]$ + $\frac{1}{C}I(t)$ = 0
Or
Ł$[I''(t)]$ + R$[I'(t)]$ + $\frac{1}{C}I(t)$ = 0
Or
$[I''(t)] + \frac{R}{Ł}[I'(t)] + \frac{1}{ŁC}\,I(t) = 0$ …. (2), where $[q'(t)]$ is the instantaneous electric current flowing in the series Ł - C- R network circuit
To solve equation (2), we first write the relevant boundary conditions as follows:
Since the current through the inductor and the electric potential across the capacitor cannot be changed instantaneously [8, 9, 10], therefore, as the switch is closed at the instant t = 0, then I (0) = 0.
Since at the instant t = 0, I (0) = 0, therefore, equation (1) provides Ł$[I'(0)] = V + V_{CO}$ or $[I'(0)] = \frac{V + V_{CO}}{Ł}$.
The Rohit Transform [5, 6 & 7] of equation (2) provides $q^2\bar{I}(q) - q^4 I(0) - q^3[I'(0)] + \frac{R}{Ł}\{q\,\bar{I}(q) - q^3 I(0)\} + \frac{1}{ŁC}\,\bar{I}(q)$ = 0..................... (3)
Applying boundary conditions: $I(0) = 0$ and $[I'(0)] = \frac{V + V_{CO}}{Ł}$, equation (3) becomes,
$q^2\bar{I}(q) - q^3\frac{V + V_{CO}}{Ł} + \frac{R}{Ł}q\,\bar{I}(q) + \frac{1}{ŁC}\,\bar{I}(q) = 0$

Or

$\bar{I}(q)\ [q^2 + \frac{R}{Ł}q + \frac{1}{ŁC}] = \frac{V + V_{CO}}{Ł} q^3$

Or

$\bar{I}(q) = \frac{V + V_{CO}}{Ł}\left[\frac{q^3}{q^2 + \frac{R}{Ł}q + \frac{1}{ŁC}}\right]$

Or

$\bar{I}(q) = \frac{V + V_{CO}}{Ł}\left[\frac{q^3}{q^2 + 2\,q\,\frac{R}{2Ł} + \left(\frac{R}{2Ł}\right)^2 - \left(\frac{R}{2Ł}\right)^2 + \frac{1}{ŁC}}\right]$

Or

$\bar{I}(q) = \frac{V + V_{CO}}{Ł}\left[\frac{q^3}{\left(q + \frac{R}{2Ł}\right)^2 - \left(\frac{R}{2Ł}\right)^2 + \frac{1}{ŁC}}\right]$

Or

$\bar{I}(q) = \frac{V + V_{CO}}{Ł}\left[\frac{q^3}{\left(q + \frac{R}{2Ł}\right)^2 + [\frac{1}{ŁC} - \left(\frac{R}{2Ł}\right)^2]}\right]$ ………… (4)

According to the condition [11, 12, 13] for the circuit to be underdamped, $\frac{1}{ŁC} > \left(\frac{R}{2Ł}\right)^2$ or $\frac{1}{ŁC} - \left(\frac{R}{2Ł}\right)^2 > 0$, therefore on putting $\frac{1}{ŁC} - \left(\frac{R}{2Ł}\right)^2 = \omega^2$ or $\omega = \sqrt{\frac{1}{ŁC} - \left(\frac{R}{2Ł}\right)^2}$ in equation (4), we can rewrite equation (4) as

$\bar{I}(q) = \frac{V + V_{CO}}{Ł}\left[\frac{q^3}{\left(q + \frac{R}{2Ł}\right)^2 + \omega^2}\right]$

Or

$\bar{I}(q) = \frac{V + V_{CO}}{Ł}\left[\frac{q^3}{\left(q + \frac{R}{2Ł}\right)^2 - (i\omega)^2}\right]$

Or

$\bar{I}(q) = \frac{V + V_{CO}}{Ł}\left[\frac{q^3}{\left(q + \frac{R}{2Ł} - i\omega\right)\left(q + \frac{R}{2Ł} + i\omega\right)}\right]$

Or

$\bar{I}(q) = \frac{V + V_{CO}}{2i\omega Ł}\left[\frac{q^3}{\left(q + \frac{R}{2Ł} - i\omega\right)} - \frac{q^3}{\left(q + \frac{R}{2Ł} + i\omega\right)}\right]$

Taking inverse Rohit Transform [5], we get

$\bar{I}(q) = \frac{V + V_{CO}}{2i\omega Ł}\left[e^{-(\frac{R}{2Ł}t - i\omega)} - e^{-(\frac{R}{2Ł}t + i\omega)}\right]$

Or

$I(t) = \frac{V + V_{CO}}{Ł}\frac{e^{-\frac{R}{2Ł}t}}{\omega}\sin\omega t$

Or

$I(t) = \frac{V + V_{CO}}{\omega Ł} e^{-\frac{R}{2Ł}t}\sin\omega t$ ……. (5)

This equation (5) confirms that the current $I(t)$ is sinusoidal in nature with exponentially decreasing amplitude. At $\omega t = \pi\ or\ t = \frac{\pi}{\omega}$, $I(\frac{\pi}{\omega}) = 0$ i.e. at the instant $t = \frac{\pi}{\omega}$, the current in the circuit becomes zero.

To find the voltage drop across the inductor, differentiating equation (5) w.r.t. t, we get

$[I'(t)] = \frac{V + V_{CO}}{\omega Ł} e^{-\frac{R}{2Ł}t}[\omega\cos\omega t - \frac{R}{2Ł}\sin\omega t]$ …. (6)

For simplifying equation (6), let us put $\frac{R}{2Ł} = b$, where b is known as damping constant, and $\frac{1}{ŁC} = \omega_o^2$, where ω_o is known as resonant frequency such that $\omega = \sqrt{\omega_o^2 - b^2}$ or $\omega_o = \sqrt{\omega^2 + b^2}$, then we can rewrite equation (6) as

$[I'(t)] = \frac{V + V_{CO}}{\omega Ł} e^{-bt}[\omega\cos\omega t - b\sin\omega t]$

Or

$[I'(t)] = \frac{V + V_{CO}}{\omega Ł} e^{-bt}\,\omega_o[\frac{\omega}{\omega_o}\cos\omega t - \frac{b}{\omega_o}\sin\omega t]$ …… (7)

Put $\frac{\omega}{\omega_o} = \cos\varphi$ and $\frac{\mathrm{b}}{\omega_o} = \sin\varphi$ such that $\varphi = \tan^{-1}\frac{\mathrm{b}}{\omega}$, equation (7) becomes

$[I'(t)] = \frac{\mathrm{V} + \mathrm{V_{CO}}}{\omega Ł} e^{-\mathrm{b}t}\, \omega_o [\cos\varphi \cos\omega t - \sin\varphi \sin\omega t]$

Or

$[I'(t)] = \frac{\mathrm{V} + \mathrm{V_{CO}}}{\omega Ł} e^{-\mathrm{b}t}\, \omega_o \cos(\omega t + \varphi)$ ………. (8)

The voltage drop across the inductor [4, 14] is given by

$\mathrm{V}_Ł(\mathrm{t}) = Ł[I'(t)] = \frac{\mathrm{V} + \mathrm{V_{CO}}}{\omega} e^{-\mathrm{b}t}\, \omega_o \cos(\omega t + \varphi)$ … … … … (9)

We can determine the voltage across capacitor as

$\mathrm{V_R}(\mathrm{t}) + \mathrm{V}_Ł(\mathrm{t}) + \mathrm{V_C}(\mathrm{t}) = \mathrm{V}$

Or

$\mathrm{V_C}(\mathrm{t}) = \mathrm{V} - \mathrm{V_R}(\mathrm{t}) - \mathrm{V}_Ł(\mathrm{t})$

Or

$\mathrm{V_C}(\mathrm{t}) = \mathrm{V} - \mathrm{R}\, I(t) - \mathrm{V}_Ł(\mathrm{t})$ …… (10)

Using equations (5) and (9) in equation (6) and simplifying, we get

$\mathrm{V_C}(\mathrm{t}) = \mathrm{V} - \frac{\mathrm{V} + \mathrm{V_{CO}}}{\omega} e^{-\mathrm{b}t}\, \omega_o \cos(\omega t - \varphi)$ .. (11)

At $\omega t = \pi \; or \; t = \frac{\pi}{\omega}$, $\mathrm{V_C}\left(\frac{\pi}{\omega}\right) = \mathrm{V} - \frac{\mathrm{V} + \mathrm{V_{CO}}}{\omega} e^{-\mathrm{b}\frac{\pi}{\omega}}\, \omega_o \cos(\pi - \varphi)$

Or

$\mathrm{V_C}\left(\frac{\pi}{\omega}\right) = \mathrm{V} + \frac{\mathrm{V} + \mathrm{V_{CO}}}{\omega} e^{-\mathrm{b}\frac{\pi}{\omega}}\, \omega_o \cos(\varphi)$…. (12)

As $\frac{\omega}{\omega_o} = \cos\varphi$, equation (12) becomes

Or

$\mathrm{V_C}\left(\frac{\pi}{\omega}\right) = \mathrm{V} + (\mathrm{V} + \mathrm{V_{CO}})e^{-b\frac{\pi}{\omega}}$

For convenience, let us write $\mathrm{V_C}\left(\frac{\pi}{\omega}\right) = \mathrm{V_{C1}}$, then

$\mathrm{V_{C1}} = \mathrm{V_C}\left(\frac{\pi}{\omega}\right) = \mathrm{V} + (\mathrm{V} + \mathrm{V_{CO}})e^{-b\frac{\pi}{\omega}}$

Or

$\mathrm{V_{C1}} = \mathrm{V_C}\left(\frac{\pi}{\omega}\right) = \mathrm{V} + (\mathrm{V} + \mathrm{V_{CO}})e^{-\frac{\mathrm{R}\,\pi}{2Ł\omega}}$ ……. (13)

This equation (13) provides the voltage across the capacitor at the instant $t = \frac{\pi}{\omega}$.

In the second mode of operation of the basic series inverter [1, 2], as both the thyristors T_1 and T_2 are in the off state as shown in figure 3, therefore, in this mode of operation, $I(t) = 0, \mathrm{V_C}(\mathrm{t}) = \mathrm{V_{C1}}$ and $\mathrm{V}_Ł(\mathrm{t}) = 0$.

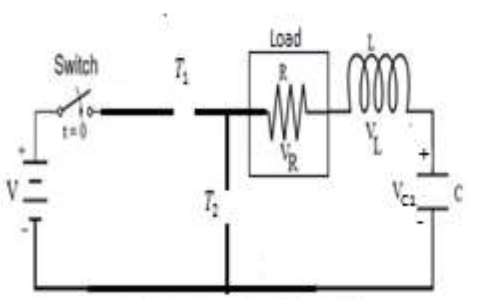

Figure 3: Equivalent Circuit (Both T_1 and T_2 are *OFF)*

In the third mode of operation of the basic series inverter [3, 4], since the thyristor T_1 is off and the thyristor T_2 is on, therefore, the equivalent circuit, in this case, is shown in figure 4.

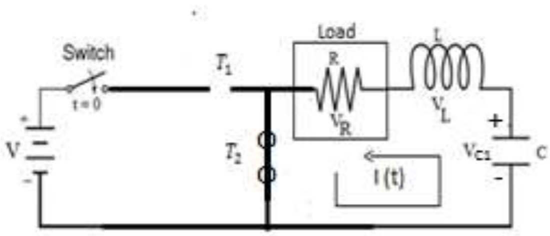

Figure 4: Equivalent Circuit (T_1 is OFF and T_2 is ON)

In this case, the capacitor is initially charged to a potential V_{C1} with positive polarity on the upper plate and negative polarity on the lower plate. The direction of flow of current on this case is in opposite direction to that of current that flows in the first mode of operation of the basic series inverter [4, 8].

The application of Kirchhoff's loop law to the loop shown in figure 4 provides

$V_R(t) + V_{\text{Ł}}(t) + V_C(t) = V_{C1}$

Or

R $I(t)$ + Ł$[I'(t)]$ + $\frac{q(t)}{C}$ = V_{C1} …… (14)

Equation (18) is similar to the equation (1). Here, on the right-hand side of equation (14), the term V_{C1} appears instead of term V + V_{CO} which appeared on the right-hand side of equation (1). Hence the solution of equation (18) can be obtained in a similar manner by the convolution method as that of equation (1) and is given by

$I(t)= \frac{V_{C1}}{\omega \text{Ł}} e^{-bt} \sin \omega t$

Or

$I(t)= \frac{V_{C1}}{\omega \text{Ł}} e^{-\frac{R}{2\text{Ł}}t} \sin \omega t$ ………. (15)

This equation (15) confirms that the current $I(t)$ is sinusoidal in nature with exponentially decreasing amplitude. It is clear from the equations (5) and (15) that the amplitude of current in the first mode of operation will be equal to the amplitude of current in the third mode of operation only if $V_{C1} = V + V_{CO}$.

CONCLUSION AND RESULT

In this paper, an attempt has been made to exemplify the application of a new integral transform called Rohit Transform for the analysis of basic series inverter. The results obtained are the same as obtained with other approaches or methods [1-4, 14]. The approach brought up the Rohit Transform as a powerful tool for analyzing and determining the responses of power electronic circuits.

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