ANALYSIS OF CORONAVIRUS DISEASE MODEL BY DIFFERENTIAL TRANSFORM METHOD (DTM)

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ABSTRACT

This study investigates the analysis of Coronavirus disease model by Differential Transformation Method (DTM). The model is transformed using DTM operational properties, furthermore, the power series of the model system is generated and also an approximate solution of the model was established. The accuracy of DTM is demonstrated against Runge-Kutta method of order four (RKM) numerical solution and it demonstrated high accuracy of the results. Plotted DTM solution is found to be in good agreement with the popular Runge-Kutta solution.

Keyword: - Coronavirus, Differential Transform Method, Bounded, Runge-Kutta Method.

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1. 1NTRODUCTION

The outbreak of the novel coronavirus diseases caused by SARS-CoV-2 is causing great challenges to the global health. Non pharmaceutical interventions are being deployed due to the unavailability of certified effective drugs or vaccine for the virus [8,10]. The novel coronavirus (SARS-CoV-2) is a new strain of the virus that has not been previously identified in humans. Severe acute respiratory syndrome coronavirus disease is the virus that produce the coronavirus disease (COVID-19). Based on daily reports, six months after the first case of COVID-19 was reported in Wuhan, China, more than 12,000,000 people has been infected with corona virus, claimed the life of 570,288, with about 7, 814,689 recovered from the infection [12,13]. In view of this, the Nigeria Centre for Disease Control (NCDC) through the National Emergency Operations Centre (EOC) has continued to lead the national public health response in Nigeria with oversight of the Presidential Task Force on COVID-19 (PTF-COVID-19). The NCDC is working closely with all states of the Federation to support their response activities to the pandemic. Several measures have been instituted by the Federal Government of Nigeria through the PTF-COVID-19 together with the Federal Ministry of Health to curtail the spread of the disease and protect the health of Nigerians, all resources, guidelines and real-time updates on COVID-19 in Nigeria were on [2].
The concept of differential transform method was first introduced in 1986 by Zhou to solve linear and nonlinear initial value problems in electric circuit analysis. Since then, several researches have been conducted in applying differential transform method to different types of equations. These researches confirm the fact that this method is reliable, efficient as well as having a wider applicability, see [2, 3, 4].Currently, the application of DTM is practice in the mathematical Biology and mathematical epidemiology research.
Presently, this method has been extended to solve SIR (susceptible-infected recovered) epidemic models [5, 11], influenza epidemic model [6], Lassa fever epidemic model [9].
In this work we aim to present the application of differential transform method to the proposed deterministic mathematical model of coronavirus disease suggested by [8].

2. MODEL FORMULATIONS

The following assumptions are considered to formulate the coronavirus disease [8] model.
i. Allowing recruitment of immigrant with coronavirus infection.

ii. Using combine incidence rates of the form $\lambda = \dfrac{\beta(I_D + I_U + I_S)}{1 + c(I_D + I_U + I_S)}$

iii. Transmission occurred through direct contact with infectious with COVID-19.

iv. All identified individual with COVID-19 infections are quarantined

Using mentioned assumptions, the equations below establish the interaction between different populations:

$$\left.\begin{aligned}
\frac{dS}{dt} &= (1-\pi)\rho - \frac{\beta(I_D + I_U + I_S)}{1 + c(I_D + I_U + I_S)}S - \mu S + \phi R \\
\frac{dL}{dt} &= \frac{\beta(I_D + I_U + I_S)}{1 + c(I_D + I_U + I_S)}S - (\kappa + \sigma_1 + \mu)L \\
\frac{dI_D}{dt} &= \pi\rho + \omega\kappa L - (\sigma_2 + \delta_1 + \mu)I_D \\
\frac{dI_U}{dt} &= (1-\omega)\kappa L - (\gamma_1 + \delta_2 + \mu)I_U \\
\frac{dI_S}{dt} &= \sigma_1 L + \sigma_2 I_D - (\gamma_2 + \delta_3 + \mu)I_S \\
\frac{dD}{dt} &= \delta_1 I_D + \delta_2 I_U + \delta_3 I_S - \mu D \\
\frac{dR}{dt} &= \gamma_1 I_U + \gamma_2 I_S - (\phi + \mu)R
\end{aligned}\right\} \qquad (1)$$

Subject to the initial conditions $S(0) = S_0, L(0) = L_0, I_D(0) = I_{D0}, I_U(0) = I_{U0}, I_S(0) = I_{S0}, D(0) = D_0, R(0) = R_0$

Table 1.Description of variables and parameters

Variable	**Description**		
S	Susceptible individual		
E	Expose individual		
I_D	Infectious detected individual		
I_U	Infectious undetected individual		
I_S	Infectious isolated individual		
D	Death individual		
R	Recovered individual		
Parameter	**Description**	**Value**	**Source**

ρ	Immigration rate	600	Estimated
π	Proportion of persons coming from a high-risk area	0.47	Estimated
$\delta_1 = \delta_2 = \delta_3$	Disease induced death rate	0.015	Estimated
ω	Proportion of individual with symptoms	0.5	[9]
κ	Progression rate	0.09	Estimated
γ_1	Infectious undetected individual recovery rate	0.0714	[12]
γ_2	Infectious isolated individual recovery rate	0.05	[12]
μ	Natural death rate	0.1	[12]
σ_1	Isolation rate of exposed individual	0.07143	Estimated
σ_2	Isolation rate of infectious detected	0.04762	[8]
β	Contact rate	0.2	[8]
ϕ	Loss of immunity from recovered individual	0.35	Estimated
c	Effect of lockdown	0.2	Estimated

2.1 MODEL ANALYSIS

The model system (1) describes human population, all the solutions of state variable with positive initial conditions are non-negative for all $t > 0$ and they are bounded in the feasible region

$$\Gamma = \left\{ \left(S, L, I_D, I_U, I_S, D, R\right) \in R_+^7 : 0 \le N = S + L + I_D + I_U + I_S + D + R \le \frac{\rho}{\mu} \right\}$$

2.2 CONCEPT OF THE DIFFERENTIAL TRANSFORMS METHOD

If $p(t)$ is the given function in Taylor series about the point $t=0$, such that $p(t)=\sum_{i=0}^{k}\frac{t^k}{k!}\left[\frac{d^k b}{dt^k}\right]_{t=0}$, then the differential transform of $p(t)$ is given as $P(t)=\frac{1}{k!}\left[\frac{d^k b}{dt^k}\right]_{t=0}$ and the inverse differential transform is $p(t)=\sum_{k=0}^{\infty}t^k P(k)$.

2.3 OPERATIONAL PROPERTIES OF DIFFERENTIAL TRANSFORMS METHOD

Given the functions $u(t)$ and $v(t)$ with time t, then $U(k)$ and $V(k)$ are the transformed functions of $u(t)$ and $v(t)$, respectively. Then, the following properties hold:

1. If $p(t)=u(t)\pm v(t)$, then $P(t)=U(k)\pm V(k)$
2. If $p(t)=\alpha u(t)$, then $P(t)=\alpha U(k)$
3. If $p(t)=u'(t)$, then $B(k)=(k+1)U(k+1)$
4. If $p(t)=u''(t)$, then $B(k)=(k+1)(k+2)U(k+2)$
5. If $p(t)=u^m(t)$ then $B(k)=(k+1)(k+2)\cdots(k+m)U(k+m)$
6. If $p(t)=u(t)v(t)$, then $P(t)=\sum_{l=0}^{k}V(l)U(k-l)$
7. If $p(t)=1$, then $P(k)=\delta(k)$
8. If $p(t)=t$, then $P(k)=\delta(k-1)$
9. If $p(t)=t^m$, then $P(k)=\delta(k-m)$, $\delta(k-m)=\begin{cases}1 & if \quad k=0\\ 0 & if \quad k\neq m\end{cases}$
10. If $p(t)=e^{\lambda t}$, then $B(k)=\frac{\lambda^k}{k!}$
11. $p(t)=(1+t)^m$, then $B(k)=\frac{m(m-1)\cdots(m-k+1)}{k!}$

2.4 SOLUTION OF CORONAVIRUS DISEASE MODEL USING DIFFERENTIAL TRANSFORM METHOD

Using operational properties (1), (2), (3), (6) and (7) of DTM in subsection (1) and applying it into the model system (1), the first equation is transformed into

$$(k+1)S(k+1)=(1-\pi)\rho-\beta\sum_{l=0}^{k}S(l)\left(\frac{I_D(k-l)+I_U(k-l)+I_S(k-l)}{1+c[I_D(k-l)+I_U(k-l)+I_S(k-l)]}\right)-\mu S(k)+\phi R(k)$$

Dividing both sides by $(k+1)$ in order to make $S(k+1)$ as subject of the expression, then

$$S(k+1)=\frac{1}{k+1}\left\{(1-\pi)\rho-\beta\sum_{l=0}^{k}S(l)\left(\frac{I_D(k-l)+I_U(k-l)+I_S(k-l)}{1+c[I_D(k-l)+I_U(k-l)+I_S(k-l)]}\right)-\mu S(k)+\phi R(k)\right\}$$

Secondly, the equation of model system (1) is transformed into

$$(k+1)L(k+1)=\beta\sum_{l=0}^{k}S(l)\left(\frac{I_D(k-l)+I_U(k-l)+I_S(k-l)}{1+c\left[I_D(k-l)+I_U(k-l)+I_S(k-l)\right]}\right)-P_1L(k)$$

This implies

$$L(k+1)=\frac{1}{k+1}\left\{\beta\sum_{l=0}^{k}S(l)\left(\frac{I_D(k-l)+I_U(k-l)+I_S(k-l)}{1+c\left[I_D(k-l)+I_U(k-l)+I_S(k-l)\right]}\right)-P_1L(k)\right\}$$

Thirdly, the equation of model system (1) is transformed into

$$(k+1)I_D(k+1)=\pi\rho+\omega\kappa L(k)-P_2I_D(k)$$

This implies

$$I_D(k+1)=\frac{1}{k+1}\left\{\pi\rho+\omega\kappa L(k)-P_2I_D(k)\right\}$$

The fourth equation of model system (1) is transformed into

$$(k+1)I_U(k+1)=(1-\omega)\kappa L(k)-P_3I_U(k)$$

Similarly

$$I_U(k+1)=\frac{1}{k+1}\left\{(1-\omega)\kappa L(k)-P_3I_U(k)\right\}$$

Also, the fifth, sixth and seventh equation of the model system (1) yields

$$I_S(k+1)=\frac{1}{k+1}\left\{\sigma_1L(k)+\sigma_2I_D(k)-P_4I_S(k)\right\}$$

$$D(k+1)=\frac{1}{k+1}\left\{\delta_1I_D(k)+\delta_2I_U(k)+\delta_3I_S(k)-\mu D(k)\right\}$$

$$R(k+1)=\frac{1}{k+1}\left\{\gamma_1I_U(k)+\gamma_2I_S(k)-P_5R(k)\right\}$$

Thus, the system of equations governing the COVID-19 model in (1) is transformed into differential transformation methods of the form:

$$S(k+1)=\frac{1}{k+1}\left\{(1-\pi)\rho-\beta\sum_{l=0}^{k}S(l)\left(\frac{I_D(k-l)+I_U(k-l)+I_S(k-l)}{1+c\left[I_D(k-l)+I_U(k-l)+I_S(k-l)\right]}\right)-\mu S(k)+\phi R(k)\right\}$$

$$L(k+1)=\frac{1}{k+1}\left\{\beta\sum_{l=0}^{k}S(l)\left(\frac{I_D(k-l)+I_U(k-l)+I_S(k-l)}{1+c\left[I_D(k-l)+I_U(k-l)+I_S(k-l)\right]}\right)-P_1L(k)\right\}$$

$$I_D(k+1)=\frac{1}{k+1}\left\{\pi\rho+\omega\kappa L(k)-P_2I_D(k)\right\}$$

$$I_U(k+1) = \frac{1}{k+1}\{(1-\omega)\kappa L(k) - P_3 I_U(k)\}$$

$$I_S(k+1) = \frac{1}{k+1}\{\sigma_1 L(k) + \sigma_2 I_D(k) - P_4 I_S(k)\}$$

$$D(k+1) = \frac{1}{k+1}\{\delta_1 I_D(k) + \delta_2 I_U(k) + \delta_3 I_S(k) - \mu D(k)\}$$

$$R(k+1) = \frac{1}{k+1}\{\gamma_1 I_U(k) + \gamma_2 I_S(k) - P_5 R(k)\}$$

Subject to initial conditions from [8] and which reduces to $S(0)=750$, $L(0)=290$, $I_U(0)=25$, $I_D(0)=65$, $I_S(0)=220$, $D(0)=18$, $R(0)=192$. Besides, the parameter values were taken table1 to solve $S(k+1)$, $L(k+1)$, $I_D(k+1)$, $I_U(k+1)$, $I_S(k+1)$, $D(k+1)$ and $R(k+1)$ respectively.

The iteration of the model is performed, and the result of the DTM of coronavirus disease model obtained as follow

$S(1) = -409.6142415$, $L(1) = 662.2805381$, $I_D(1) = 284.4797$, $I_U(1) = 8.39$, $I_S(1) = -12.49$
$D(1) = 2.85$, $R(1) = -73.615$

$S(2) = -0.275504825$, $L(2) = 80.30359136$, $I_D(2) = 132.7702677$, $I_U(2) = 14.11936411$,
$I_S(2) = 24.88354032$, $D(2) = 1.96034775$, $R(2) = 16.550648$

$S(3) = -0.752459061$, $L(3) = 305.0838865$, $I_D(3) = 88.00752023$, $I_U(3) = 0.327270713$,
$I_S(3) = 2.650940509$, $D(3) = 0.793520935$, $R(3) = -1.731830662$

$S(4) = 1.39424382$, $L(4) = 58.03351234$, $I_D(4) = 70.35424799$, $I_U(4) = 3.416942908$,
$I_S(4) = 6.386413736$, $D(4) = 0.321358469$, $R(4) = 0.233809488$

$S(5) = -0.021944884$, $L(5) = 60.57608645$, $I_D(5) = 54.63410005$, $I_U(5) = 0.394917979$,
$I_S(5) = 1.288368962$, $D(5) = 0.240472813$, $R(5) = 0.091615228$

$S(6) = 2.399909064$, $L(6) = 47.96639985$, $I_D(6) = 45.97355442$, $I_U(6) = 0.442051863$,
$I_S(6) = 1.119340803$ $D(6) = 0.136785587$, $R(6) = 0.0085647898667$

Therefore, the series solution of the transformed expressions when $k=6$ for $S(t)$, $L(t)$, $I_D(t)$, $I_U(t)$, $I_S(t)$ $D(t)$ and $R(t)$ are obtained as

$$S(t) = 750 - 409.6142415t - 0.275504825t^2 - 0.752459061t^3 + 1.39424382t^4 - 0.021944884t^5 + 2.399909064t^6 \cdots$$

$$L(t) = 290 + 662.2805381t + 80.30359136t^2 + 305.0838865t^3 + 58.03351234t^4 + 60.57608645t^5 + 47.96639985t^6 \cdots$$

$$I_D(t) = 65 + 284.4797t + 132.7702677t^2 + 88.00752023t^3 + 70.35424799t^4 + 54.63410005t^5 + 45.97355442t^6 \cdots$$

$$I_U(t) = 25 + 8.39t + 14.11936411t^2 + 0.327270713t^3 + 3.416942908t^4 + 0.394917979t^5 \\ +0.442051863t^6 \cdots$$

$$I_S(t) = 220 - 12.49t + 24.88354032t^2 + 2.650940509t^3 + 6.386413736t^4 + 1.288368962t^5 \\ +1.119340803t^6 \cdots$$

$$D(t) = 18 + 2.85t + 1.960347775t^2 + 0.793520935t^3 + 0.321358469t^4 + 0.240472813t^5 \\ +0.136785587t^6 \cdots$$

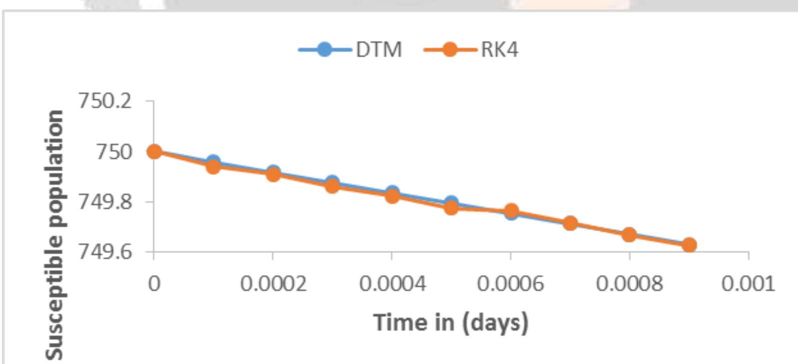
$$R(t) = 192 - 73.615t + 16.550648t^2 - 1.731830662t^3 + 0.233809488t^4 + 0.091615228t^5 \\ +0.0085647898667t^6 \cdots$$

3. NUMERICAL SIMULATIONS AND DISCUSSION

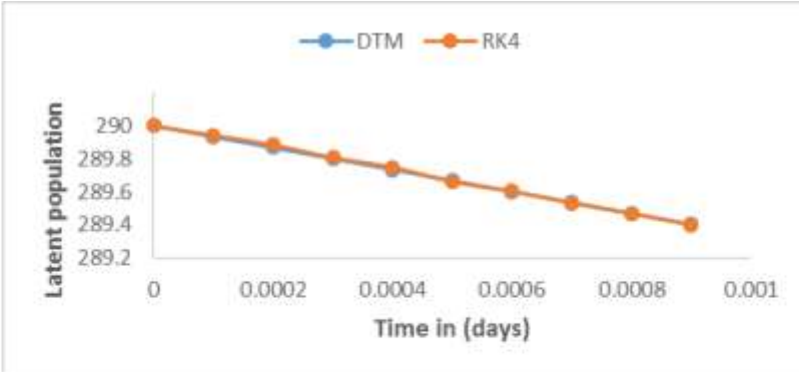
In this section, numerical simulation of model system (1) was presented using a set of parameter values given in Table (1) with initial conditions and the result where $S(k)$, $L(k)$, $I_D(k)$, $I_U(k)$, $I_S(k)$, $D(k)$ and $R(k)$ are the differential transforms of the corresponding functions $S(t)$, $L(t)$, $I_D(t)$, $I_U(t)$, $I_S(t)$, $D(t)$ and $R(t)$ respectively with initial conditions given as

$$S(0) = 750,\ L(0) = 290,\ I_U(0) = 25,\ I_D(0) = 65,\ I_S(0) = 220,\ D(0) = 18,\ R(0) = 192.$$

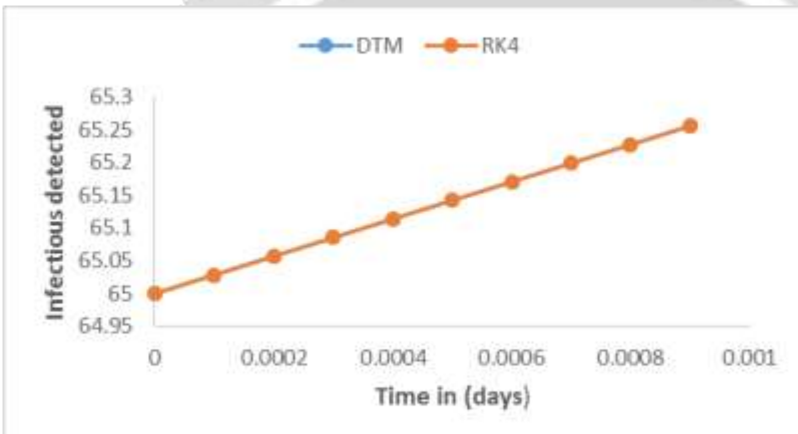
The DTM is demonstrated against maple built-in fourth order Runge-Kutta method for the solutions of coronavirus disease model, both methods converges.



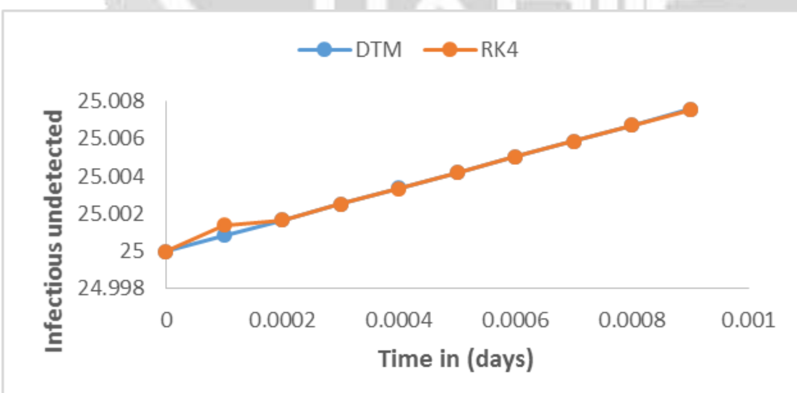
(a).


DTM
RK4
Latent population
290
289.8
289.6
289.4
289.2
0
0.0002
0.0004
0.0006
0.0008
0.001
Time in (days)

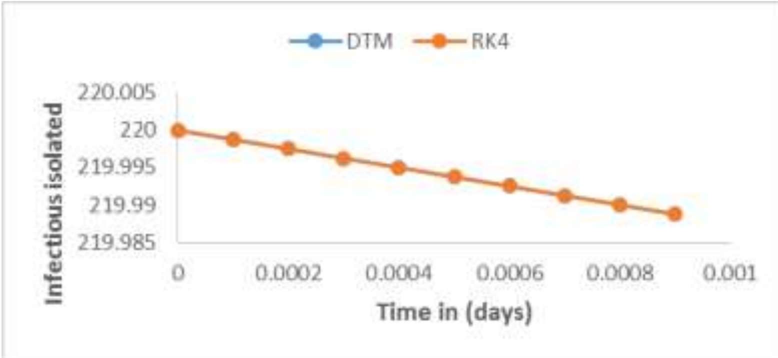
(b).


DTM
RK4
Infectious detected
65.3
65.25
65.2
65.15
65.1
65.05
65
64.95
0
0.0002
0.0004
0.0006
0.0008
0.001
Time in (days)

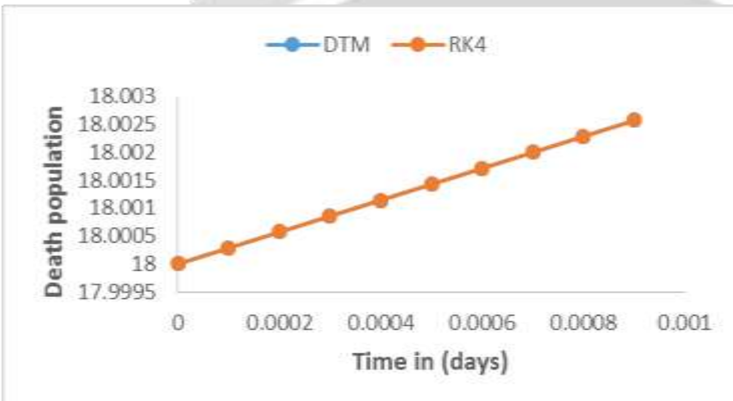
(c).


DTM
RK4
Infectious undetected
25.008
25.006
25.004
25.002
25
24.998
0
0.0002
0.0004
0.0006
0.0008
0.001
Time in (days)

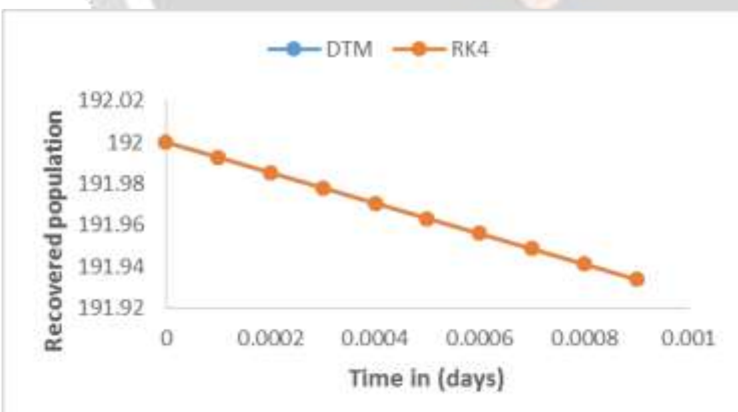
(d).



(e).



(f).



(g).

4. CONCLUSION

In this work, solution of coronavirus disease model using differential transform method has been successfully presented and analyzed rigorously. The study achieved the following:

In figure a, b, e & g susceptible population, latent population infectious isolated and recovery population decreasing as the time progressing, also in figure c, d & increasing as the time progressing. In view to this, it is concluded that mathematical modeling of epidemiology can be solved using differential transformation method because of it is efficiency, reliability and fast convergence rate. Also, it can be applied to solve problem in linear or non-linear ordinary differential equations.

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