ANALYSIS OF UNIFORM INFINITE FIN VIA MEANS OF ROHIT TRANSFORM

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Abstract

Heat transfers by virtue of temperature gradient and in practice, to improve the heat convection rate fins or spines are projected out from the conducting medium. Generally, the distribution of temperature and hence the amount of heat convected from the fin surfaces has been determined via the calculus approach. This paper presents the use of a new integral transform called Rohit Transform for the analysis of uniform infinite fin for obtaining the distribution of temperature and hence the rate of heat convected into the surroundings from an infinite uniform fin. This approach put forward the Rohit Transform as a new mathematical tool for the analysis of uniform infinite fin for obtaining the distribution of temperature and hence the rate of heat convected into the surroundings by uniform infinite fin.

Keywords: *Heat convected, Uniform Infinite Fin, Rohit Transform.*

INTRODUCTION

Fins or spines are the extended surfaces projected from heat-conducting surfaces to improve the heat dissipation into the surroundings [1, 2 & 3]. Fourier's law expressed as $\mathbf{H} = -k\mathcal{A}\frac{dt}{dy}$, is the basic law of conduction or dissipation of heat, where k is the thermal conductivity of the medium, $\mathcal{A}$ is the area of the cross-section of the medium, $\mathbf{H}$ is the rate of heat dissipated, $\frac{dt}{dy}$ is the temperature gradient and the negative sign indicates that the heat is transferring in the direction of decreasing temperature. Generally, the temperature distribution and hence the rate of heat convected from the infinite fin surface have been determined via the calculus approach [1-4]. This paper presents a new integral transform called Rohit Transform for the analysis of uniform infinite fin to obtain the temperature distribution and hence the rate of heat convected into the surroundings by uniform infinite fin. The Rohit Transform was proposed by the author Rohit Gupta in recent years and generally, it has been applied in different areas of science and engineering [5, 6, 7 & 8]. The Rohit Transform [6, 7] of g(y), y ≥ 0 is defined as $R\{g(y)\} = r^3\int_0^\infty e^{-ry}\, g(y)dy = G(r)$, provided that the integral is convergent, where r may be a real or complex parameter.
The Rohit Transform (RT) of some derivatives [5-8] of g(y) are given by

$R\,\{g'(y)\} = rR\{g(y)\} - r^3g(0)$

Or

$R\,\{g'(y)\} = rG(r) - r^3g(0)$,

$R\{g''(y)\} = r^2G(r) - r^4g(0) - r^3g'(0)$,

And so on.

MATERIAL AND METHOD

The differential equation which describes the heat dissipated from a uniform infinite fin is given by [9-12]

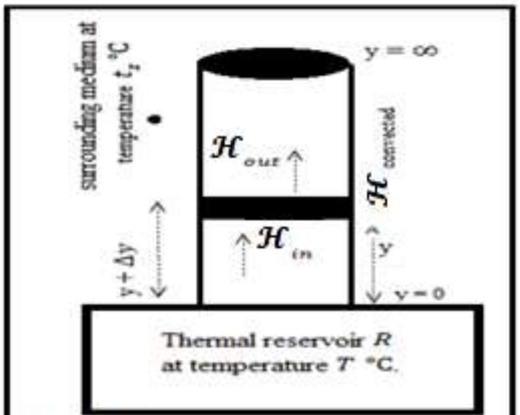

Figure 1: conduction of heat through a infinite fin.

$t''(y)$ - $\frac{\sigma P}{\mathcal{K}\mathcal{A}}$ [$t(y) - t_s$] = 0 ……… (1), where we assumed that the one end of the fin is connected to a heat source at y = 0 and the other end at y = infinity is free for losing heat into the surroundings. The source of heat is maintained at fixed temperature 'T' and t_s is the temperature of the surroundings of the infinite fin and is kept constant.

For convenience, let $(\frac{\sigma P}{\mathcal{K}\mathcal{A}})^{\frac{1}{2}} = \beta$……. (2)

And $t(y) - t_s$ = $\tau(y)$ ... ... (3) known as the excess temperature at the length 'y' of the infinite fin.

Then equation (1) can be rewritten as

$\tau''(y)$ - $\beta^2\, \tau(y)$ = 0 ……….. (4)

Equations (1) and (4) are the general form of energy equations for one-dimensional heat dissipation from the surface of the infinite fin. In equation (2), β is a constant provided that σ is constant over the entire surface the infinite fin and k is constant within the range of temperature considered.

The necessary initial conditions are [11, 12]

(i) $t(0)$ = T. In terms of excess temperature, at y = 0, $t - t_s$ = T- t_s or $\tau(0)$ = τ_0… (8)

(ii) $t(\infty) = t_s$. In terms of excess temperature, at y = ∞, $\tau(\infty)$ = 0

Taking Rohit Transform [6, 7 & 13] of equation (4), we get

$q^2\bar{\tau}$(q) – $q^4\tau(0)$ -$q^3\tau'(0)$- $\beta^2\bar{\tau}$(q) = 0... (5)

Applying boundary condition: $\tau(0)$ = τ_0, equation (5) becomes

$q^2\bar{\tau}$(q) – $q^4\tau_0 - q^3\tau'(0)$- $\beta^2\bar{\tau}$(q) = 0

Or

$q^2\bar{\tau}(q) - \beta^2\bar{\tau}(q) = q^3\tau'(0) + q^4\tau_0$….. (6)

In this equation, $\tau'(0)$ is some constant.

Let us substitute $\tau'(0) = \varepsilon$,

Equation (6) becomes

$$q^2\bar{\tau}(q) - \beta^2\bar{\tau}(q) = q^3\varepsilon + q^4$$

Or

$\bar{\tau}$(q) = $\frac{q^3\varepsilon}{(q^2 - \beta^2)}$ + $\frac{q^4\tau_0}{(q^2 - \beta^2)}$ ……. (7)

Taking inverse Rohit Transform [5, 13] of above equation, we get

τ(y) = $\frac{\varepsilon}{\beta} sinhby$ + $\tau_0 \cos h\beta y$

Or

τ(y) = $\frac{\varepsilon}{2\beta}\left[e^{\beta y} - e^{-\beta y}\right]$ +$\tau_o \left[\frac{e^{\beta y} + e^{-\beta y}}{2}\right]$… (12)

Determination of the constant ε:

Applying initial condition: $\tau(\infty)$ = 0, we can write

$$\frac{\varepsilon}{2\beta}\left[\, e^{\beta(\infty)} - \, e^{-\beta(\infty)}\right] + \tau_o\left[\frac{e^{\beta(\infty)} + \, e^{-\beta(\infty)}}{2}\right] = 0$$

Or

$$\frac{\varepsilon}{2\beta}\left[\, e^{\beta(\infty)} - 0\right] + \tau_o\left[\frac{e^{\beta(\infty)} + 0}{2}\right] = 0$$

Or

$$\left[\frac{\varepsilon}{2\beta} + \frac{\tau_o}{2}\right] e^{\beta(\infty)} = 0$$

As $e^{\beta(\infty)} \neq 0$, therefore,

$$\left[\frac{\varepsilon}{2\beta} + \frac{\tau_o}{2}\right] = 0$$

Or

$\varepsilon = -\beta\tau_o$ …….. (8)

Put the value of ε from equation (8) in equation (7), we get

$\tau(\text{y}) = \frac{-\beta\tau_o}{2\beta}\left[\, e^{\beta y} - \, e^{-\beta y}\right] + \tau_o\left[\frac{e^{\beta y} + e^{-\beta y}}{2}\right]$

Or

$\tau(\text{y}) = \frac{-\tau_o}{2}\left[\, e^{\beta y} - \, e^{-\beta y}\right] + \tau_o\left[\frac{e^{\beta y} + e^{-\beta y}}{2}\right]$

Or

$\tau(\text{y}) = \frac{\tau_o}{2}\left[\, e^{\beta y} + \, e^{-\beta y} - \, e^{\beta y} + \, e^{-\beta y}\right]$

Or

$\tau(\text{y}) = \tau_o e^{-\beta y}$ …….. (9)

Equation (9) provides the distribution of temperature along the length of the infinite fin and confirms that the temperature of the infinite fin decreases along its length with the increase in distance from the heat source maintained at the temperature T.

The amount of heat convected from the surface of the infinite fin can be obtained by using the equation [10-12]

$\mathbf{H}_f = -k\mathcal{A}\,[D_y t(y)]_{\,y=0}$

Or

$\mathbf{H}_f = -k\mathcal{A}\,[D_y \tau(y)]_{\,y=0}$ …… (10)

Now since $\tau'(y) = -\beta\tau_o e^{-\beta y}$,

Therefore,

$[\tau'(y)]_{y=0} = -\beta\tau_o$….. (11)

Using (16) in (15), we get

$\boldsymbol{\mathcal{H}}_f = \mathcal{K}\mathcal{A}\beta\tau_o$

Or

$\boldsymbol{\mathcal{H}}_f = \mathcal{K}\mathcal{A}\beta(\text{T} - t_s)$….. (12)

Put the value of β from equation (2) in equation (12), we get

$$\boldsymbol{\mathcal{H}}_f = \mathcal{K}\mathcal{A}\left(\frac{\sigma\text{P}}{\mathcal{K}\mathcal{A}}\right)^{\frac{1}{2}}(\text{T} - t_s)$$

Or

$\boldsymbol{\mathcal{H}}_f = (\mathcal{K}\mathcal{A}\sigma\text{P})^{\frac{1}{2}}(\text{T} - t_s)$ .... (13)

This equation (13) provides the rate of heat convected from the surface of the infinite fin into its surroundings and confirms that the rate of convection of heat can be increased by increasing the surface area of the fin.

CONCLUSION

In this paper, a new integral transform called Rohit Transform is exemplified for the analysis of uniform infinite fin for determining the temperature distribution along the infinite fin and hence the amount of heat convected from its surface into the surroundings. This approach brought up the Rohit Transform as a new tool for the analysis of uniform infinite fin. It is concluded that the temperature of the infinite fin decreases with the increase in its length from the heat source, and the rate of heat convected from the infinite fin surface into the surroundings can be improved by increasing the surface area of the infinite fin. The results obtained by the application of Rohit Transform to the uniform infinite fin are the same as obtained with other methods or approaches [1, 2, 11 & 12].

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