ANALYSIS ON FUZZY STRONGLY g^* AND δg^* SUPER CLOSED SETS IN FUZZY TOPOLOGICAL SPACES

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ABSTRACT

In this paper we investigate the concept of fuzzy generalized closed sets and the properties of fuzzy strong g^* and δg^* super closed sets in a fuzzy topological spaces. And explore some of its characterization along with theorems.

Keyword: *Fuzzy strong g^* super closed sets,fuzzy generalized super closed set, fuzzy topological space, fuzzy super closure and interior.*

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1 INTRODUCTION

It 1965, Zadeh introduced the concept of fuzzy sets. As a generalization of topological spaces change introduced the concept of fuzzy topological space in 1968. g^*-closed sets were introduced and studied by Veerakumar for general topology. Recently Parimelazhagan and Subramonia pillai introduced strongly g^*-closed sets in topological space . In the present paper, we introduce fuzzy strongly g^*-closed sets in fuzzy topological space and investigate certain basic properties of these fuzzy sets.

A family τ of fuzzy sets of X is called a fuzzy topology on X if $0,1$ belongs to τ and τ is super cosed with respect to arbitrary union and finite intersection. Then members of τ are called fuzzy set A of X the closure of A (denoted by cl(A)) is the intersection of all the fuzzy super closed super sets of A and the interior of A (denoted by int(A)) is the union of all fuzzy super open subsets of A.

1.1 Definition

A subset B of a fuzzy topological space (X, τ) is called

1. Fuzzy super closure scl(B)={x∈X:cl(U)∩B≠ ∅}
2. Fuzzy super interior sint(B)={x∈X:cl(U)≤B≠ ∅}
3. Fuzzy super closed if scl(B) ≤B.
4. Fuzzy super open if 1-B is fuzzy super closed sint(B)=B.

1.2 Definition

Fuzzy generalized super closed set if cl≤U whenever B≤U and U is fuzzy super open in (X, τ).

1.3 Definition

Let (X,τ) be a topological space and B be its subset, then B is fuzzy strongly g^*-super closed set if cl(int(B))≤U whenever A ≤U and U is fuzzy g^*-super open.

1.4 Definition

Let (X,τ) be a fuzzy topological space. A fuzzy set A of (X,τ) is called fuzzy strongly g^*-closed if Cl(Int(A)) ≤ H whenever A≤ H and H is fg-open in X.

1.5 Definition

Let (X,τ) be a fuzzy topological space .A fuzzy set A of (X,Γ) is called fuzzy strongly g^*- super closed if cl(int(A)) ≤H , whenever A≤H and H is fg-super open in X.

1.6 Theorem Every super closed set is fuzzy strongly g^*super closed set.

Proof:

suppose B is fuzzy g^*super closed set in X. let G be a fuzzy super open set containing in X.

Then G contains cl(B).Now G≥ cl(B)≥ cl(int(B)).

Thus B is fuzzy strongly g^*-super closed in X.

1.7 Theorem If B is a subset of a fuzzy topological space X is super open and fuzzy strongly g^*-super closed then it is fuzzy super closed.

Proof:

suppose a subset B of X is both fuzzy super open and fuzzy strongly g^*-super closed.

Now B≥ cl(int(B)≥ cl(B).Therefore cl(B)≥B. we have B≥cl(B). Thus B is fuzzy super closed in X.

1.8 Theorem A set B is fuzzy strongly g^*-super closed iff cl(int(B))-B contains no non empty fuzzy super closed set.

Proof:

Necessary: Suppose that H is non empty fuzzy super closed subset of cl(int(B)).Now H≤cl(int(B))-Aimplies H≤cl(int(B))∩ B^C,since cl(int(B))-B=cl(int(B))∩ A^C.Thus H≤cl(int(B)).Now H≤ B^C implies B≤ H^C. Hence H^C is fuzzy g^*-super open and A is fuzzy strongly g^*-super closed,We have cl(int(B))_ H^C. Thus H≤cl(int(B)))^C. Hence H≤ (cl(int(B)))∩(cl(int(B)))^C=ϕ. Therefore H=≤(cl(int(A))_ H^Ccontains no non empty fuzzy super closed sets.

Sufficient: Let B ≤G,G is fuzzy g -super open. Suppose that cl(int(B)) is not contained in G then (cl(int((B)))^C is a non empty fuzzy super closed set of cl(int(B))-B which is a contradiction. Therefore cl(int(B)) ≤ G and hence B is fuzzy strongly g^*-super closed.

1.9 Theorem Suppose that B≤A≤X, B is fuzzy strongly g^*-super closed set relative to A and that both fuzzy super open and fuzzy strongly g^*-super closed subset of X then B is fuzzy strongly g^*-super closed set relative to X.

Proof:

Let B ≤G and G be a fuzzy super open set in X. But given that B ≤A ≤X, therefore B ≤A and B ≤G.This implies B ≤A∩G.

since B is fuzzy strongly g^*-super closed relative to A, cl(int((B)) ≤A∩G.that is A∩cl(int(B)) ≤A∩G.This implies A∩(cl(int(B))) ≤A∩G. This implies A∩(cl(int(B))) ≤G.

Thus (A∩(cl(int(B))) ∪(cl(int(B)))^c ≤G∪(cl(int(B)))^c

implies A∪(cl(int(int(B)))^c ≤G((cl(int(B)))^c.

Since A is fuzzy strongly g^*-super closed in X, we have (cl(int(A))) ≤G∪(cl(int(B)))^c. Also B≤A⇒cl(int(B)) ≤cl(int(A)).Thus cl(int(B)) ≤cl(int(A)) ≤G≤(cl(int(B)))^c.Therefore B is fuzzy strongly g^*-super closed set relative to X.

1.10 Theorem Let B be fuzzy strongly g^*-super closed and suppose that h is fuzzy super closed then B∩H is fuzzy strongly g^*-super closed set.

Proof:

To show that B∩H is fuzzy strongly g^*-super closed,

we have to sow cl(int$(B|H)$) ≤G whenever B∩H≤G and G is fuzzy g-super open. B∩H is fuzzy super closed in B so it is fuzzy strongly g^*-super closed in B.

since by the theorem We have B∩H is fuzzy strongly g^*-super closed in X.

Since B∩H ≤**B** ≤X.

1.11 Theorem If A is fuzzy strongly g^*-super closed and A≤**B** ≤cl(int(A)) then B is fuzzy strongly g^*-super closed.

Proof:

Given that B≤cl(int(A)) then cl(int(B))≤cl(int(A)),cl(int(B))-B≤cl(int(A))-A.

since A≤B. As A is fuzzy strongly g^*-super closed then we have cl(int(A))-A contains no non empty fuzzy super closed set, cl(int(B))-B contains no empty fuzzy super closed set.

Then we have B is fuzzy strongly g^*-super closed set.

1.12 Theorem Let A≤Y≤X and suppose that A is fuzzy strongly g^*-super closed in X then A is fuzzy strongly g^*-super closed relative to Y.

Proof:

Given that A≤Y≤X and A is fuzzy strongly g^*-super closed in X.

To show that A is fuzzy strongly g^*-super closed relative to Y, Let A≤Y∩G, where G is fuzzy g-super open in X.

Since A is strongly g^*-super closed relative to X, A≤G implies cl(int(A)) ≤G. That is Y∩cl(int(A) ≤Y∩G, where Y ∩cl(int(A) is closure of interior of A in Y.

Thus A is fuzzy strongly g^*-super closed relative to Y.

1.13 Theorem If A is a fuzzy strongly g^*-closed set in(X,τ) and A≤B≤cl(int(A)), then B is fuzzy strongly g^*-closed in (X, τ).

Proof :

Let A be a fuzzy strongly g^*-closed set in (X, τ).

Let B≤H Where H is fuzzy g-open set in X.

Then A≤H.since A is fuzzy strongly g^*-closed set, it follows that cl(int(A))≤H.

Now B≤cl(int(A)) implies cl(int(B)) ≤cl(int(cl(int(A)))=cl(int(A))

We get cl(int(B)) ≤H. Hence B is fuzzy strongly g^*-closed set in (X, τ).

1.14 Theorem Let a be a fuzzy strongly g^*-open in X and int(cl(A)) ≤B≤A then B is fuzzy strongly g^*-open in X.

Proof:

Suppose that a is fuzzy strongly g^*-open in X and int(cl(A)) ≤B≤A. then 1-A is fuzzy strongly g^*-closed in X and 1-A≤1-B≤cl(int(1-A)).

Then we have 1-B is fuzzy strongly g^*-closed in X and hence B is fuzzy strongly g^*-open in X.

1.15 Theorem Every fuzzy g^*-super closed set is a fuzzy strongly g^*-super closed set in (X,Γ).
Proof:

suppose that A is fg^*-super closed in X. Let H be a fg- super open set in X such that A≤H. then cl(A) ≤H, since A is fg^*-super closed.

Now ,cl(int(A)) ≤cl(A) ≤H, hence A is fuzzy strongly g^*-closed set in X.

1.16 Theorem If A is fuzzy strongly g^*-super closed set in (X, Γ) and A≤B≤cl(int(A)), then B is fuzzy strongly g^*-super closed in (X, Γ).

Proof:

Let A be afuzzy strongly g^*-super closed set in (X, Γ) and B≤H where H is a fuzzy g-open set in X.Then A≤H.

Since A is a fuzzy strongly g^*-super closed set, it follows that cl(int(A)) ≤H. Now B≤cl(int(A)) implies cl(int(B)) ≤cl(int(cl(int(A))))=cl(int(A)).We get, cl(int(B)) ≤H.

Hence B is fuzzy strongly g^*-super closed set in (X, Γ).

1.17 Theorem If a fuzzy set A of a fuzzy topological space X is both fuzzy super open and fuzzy strongly g^*-super closed , then it is fuzzy super closed.

Proof:

Suppose that a fuzzy set A of X is both fuzzy super open and fuzzy strongly g^*-super closed. Now A≥cl(int(A)) ≥cl(A).That is A≥cl(A), since A≤cl(A).

So we get A=cl(A).

Hence A is fuzzy super closed inX.

1.18 Theorem If a fuzzy set A of a fuzzy topological space X is both fuzzy strongly g^*-super closed and fuzzy semi super open, then it is fg^*-super closed.

Proof:

Suppose a fuzzy set A of X is both fuzzy strongly g^*-super closed and fuzzy semi open in X. Let H be a fg-open set such that A≤H.

since A is fuzzy strongly g^* −super closed ,therefore cl(int(A)) ≤H.

Also since A is fs-super open,A≤cl(int(A)).

We have cl(A) ≤cl(int(A)) ≤H.

Hence A is fg^*-super closed in X.

1.19 Theorem If A is a fuzzy $\delta g^* -$ closed set in (X,τ) and $A \leq B \leq fcl\delta(A)$,then B is a fuzzy δg^* −super closed set in (X,Γ).

Proof:

Let A be a fuzzy $\delta g^* -$ closed set in (X,τ). Given $A \leq B \leq fcl\delta(A)$.

Suppose $B \leq H$ where H is fuzzy g −super open set. Since $A \leq B \leq H$ and A is a fuzzy δg^* −super closed set, we get $fcl\delta(A) \leq H$.

As $B \leq fcl\delta(A)$, $fcl\delta(B) \leq fcl\delta\big(fcl\delta(A)\big) = fcl\delta(A)$ we get $fcl\delta(B) \leq H$.

Hence B is a fuzzy δg^* −super closed set in (X,Γ).

1.20 Theorem If A is a fuzzy $\delta g^* -$ closed set in (X,τ) and $fint\delta(A) \leq B \leq A$, then B is a $\delta g^* -$ super open set in (X,τ).

Proof:

Let A be a fuzzy δg^* −open set and B be any fuzzy set in X .

such that $fint\delta(A) \leq B \leq A$. then $1 - A$ is a fuzzy $\delta g^* -$ closed set and

$1 - A \leq 1 - B \leq fcl\delta(A)$, as $1 - fint\delta(A) = fcl\delta(1 - A)$.

Therefore $1 - B$ is a fuzzy $\delta g^* -$ super closed.

Hence B is a fuzzy δg^* −super open.

1.21 Theorem Every fuzzy closed set is a fuzzy strongly g^* −super closed set in the fuzzy topological space (X,Γ).

Proof:

Let A be fuzzy super closed set in X and X be a $fg -$ super open set in X such that $A \leq H$. Since A is fuzzy super closed, $cl(A) = A$. Therefore $cl(A) \leq H$.

Now, $cl(int(A)) \leq cl(A) \leq H$.

Hence A is fuzzy strongly g^* −super closed set in X.

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