AN ANALYSIS ON DERIVATION IN NEAR-RINGS AND ITS GENERALIZATIONS

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Abstract

We present a past description of the study of derivatives, generalized derivatives, n-derivatives, generalized n-derivatives, and other kinds of derivatives in close circles, based on the works of several authors. In addition, recent results on semigroup ideals and generalized n-derivatives on these topics are discussed in detail. Examples of different concepts were also included.

Keywords: Derivation, Left Near-Rings, Right Near-Rings, Generalization

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Introduction

This paper is an attempt to discuss an up-to-date account of work on derivations and its various invariants in the setting of near-rings. The research has been described in a manner suitable for everybody who have some basic knowledge in near-ring theory. In order to make the treatment as self-contained as possible, and to bring together all the relevant material in a single paper, we have included several references. Sometimes, many results have been unified in a single theorem. Proper references of almost all the results are given. Let N be non-empty set, equipped with two binary operations say '+' and '.'. N is called a left near-ring if following condition is true

(i) (N , +) is a group (not necessarily abelian)
(ii) (N , .) is a semigroup and
(iii) $x.(y + z) = x.y + x.z$ for all $x, y, z \in N$.

Similarly a right near-ring can also be defined. A left near-ring N is called zero- symmetric if $0.x = 0$ for all $x \in N$ (recall that in a left near ring $x.0 = 0$ for all $x \in N$). Similar remarks hold for a right near-ring also. For a natural example of a near-ring, let (G, +) be a group (not necessarily abelian). Consider S, the set of all mappings from G to G. Then S is a zero- symmetric right near-ring with regard to the operations '+' and ' . ' defined as below:

where $f, g \in EndG$. It is to be noted that it is not a left near-ring.

Derivations in Near-Rings

The idea of derivation in rings is an ancient one that is important in many areas of mathematics. When Posner [16] established two extremely remarkable discoveries on derivations in prime rings in 1957, it marked a significant advancement. Additionally, there has been a lot of interest in researching the commutativity of rings, particularly prime and semi-prime rings that allow adequate restricted derivations. Numerous algebraists have investigated derivations in prime rings and semi-prime rings from diverse angles. E. Bell et al. [24] developed the idea of derivation in near-rings as a result of the concept of derivation in rings.

Definition. A derivation ′d ′ on N is defined to be an additive mapping $d : N \to N$ satisfying the product rule $d(xy) = xd(y) + d(x)y$ for all $x, y \in N$.

Example: Let $N = N1 \oplus N2$, where N1 is a zero symmetric left near-ring and N2 is a ring having derivation δ. Then $d : N \longrightarrow N$ defined by $d(x, y) = (0, \delta(y))$ for all $x, y \in N$ is a nonzero derivation of N , where N is a zero-symmetric left near- ring.

For an example of a derivation on noncommutative near-ring one can consider the following:

Example : Let us consider $(C, +, \ast)$ where ′∗ ′ is defined as $x \ast y = |x|y$ for all $x, y \in C$, then it can be easily seen that $(C, +, \ast)$ is a zero-symmetric left near-ring which is not a right near -ring. Assume N =

$$\left\{\begin{pmatrix} \boldsymbol{a} & \boldsymbol{b} \\ \mathbf{0} & \mathbf{0} \end{pmatrix}\right\} \mid a, \boldsymbol{b} \in \}$$

then N is a zero-symmetric left near-ring which is not a right near-ring. Define $d : N \longrightarrow N$ as d

$$\left\{\begin{pmatrix} \boldsymbol{a} & \boldsymbol{b} \\ \mathbf{0} & \mathbf{0} \end{pmatrix}\right\} = \left\{\begin{pmatrix} \mathbf{0} & \boldsymbol{a} \\ \mathbf{0} & \mathbf{0} \end{pmatrix}\right\}$$

Then d is a non-zero derivation on N . In a left near-ring, right distributive property does not hold in general, the following lemmas play a vital role in further study. For any $a, b, c \in N$ expanding $d(a(bc))$ and $d((ab)c)$ and comparing the relations so obtained we get the following (for reference see ([24], Lemma)).

Lemma Let d be an arbitrary derivation on a near-ring N . Then N satisfies the following partial distributive law:

$(ad(b) + d(a)b) = ad\,(b) + d(a)bc \; for \; all \; a, b, c \in$ N.

The study of derivation was initiated by E. Bell et al. [24], pertaining to the 3-prime near-rings and semiprime near-rings. Some basic properties of 3-prime near-rings are given below which are helpful in the study of derivations in 3-prime near-rings:

- If $z \in Z\setminus\{0\}$, then z is not a zero divisor.

- If Z contains a nonzero element z for which $z + z \in Z$, then (N, +) is abelian.

- Let d be a nonzero derivation on N. Then $xd(N) = \{0\}$ implies $x = 0$ and $d(N)x = \{0\}$ implies $x = 0$.

- If N is 2-torsion free and d is a derivation on N such that d 2 = 0, then d = 0.

In the year 1984 X.K.Wang ([41], Proposition 1) gave an equivalent definition of derivation on a near-ring N as below and also obtained partial commutativity of addition and partial distributive law in the near-ring N .

Definition. Let d be an arbitrary additive endomorphism of N . Then d is a derivation on Nif $d(xy) = d(x)y + xd(y)$ for all $x, y \in N$.
Lemma. Let d be a derivation on N .Then N satisfies the following partial distributive law: $(d(x)y + xd(y))z = d(x)yz + xd(y)z$ for all $x, y, z \in N$
Lemma. Let N be a near-ring with center Z, and let d be a derivation on N . Then $d(Z) \subseteq Z$.

Major study in this area was carried out by E. Bell et al. [24], Beidar, Fong and Wang [16]etc. which consists of commutativity of addition and multiplication of 3-prime near-ring and semiprime near-ring with constrained derivations. It has been also studied that under suitable constrained derivations, 3-prime near-rings behave like rings.

Now we list several commutativity theorems, obtained by above authors for 3-prime near-rings,admitting suitable constrained derivations as below.

Results given below have been proved by E. Bell et al. [24]. **Theorem**
If a 3-prime near-ring N , admits a non trivial derivation satisfying either of the following properties

(i) d(N) ⊆ Z,

(ii) [d(x), d(y)] = 0 for all x, y ∈N , then (N , +) is abelian and if N is 2-torsion free aswell, then N is a commutative ring.

Following results concerning commutativity of near-ring have been proved by Beidar, Fongand Wang [16]

Theorem. Let N be 3-prime near-ring which admits derivations d1 and d2. Suppose Nsatisfies any one of the following properties:

$$(1) d_1^2 \neq 0 \neq d_2^2 \, and \; d_1(x)d_2(y)d_2(y)d_2(y)d_1(x) \, for \;\; all \; x, y \in n.$$

$$(11)\; 2n \neq 0, d_1 \neq 0, d_2 \neq 0 and \; d_1 0 \; and \; d_1(x)d_2(y) = d_2(y)d_1(x) for \;\; all \; x, y \in n.$$

$$then \; n \; is \; a \; comminutative \;\; ring.$$

Theorem. Let N be 3-prime near-ring with nonzero derivations d1 and d2 such thatd1(x) d2(y) = −d2(x) d1(y) for all x, y ∈N . Then (N , +) is abelian.

Very recently Boua and Oukhtite [25] investigated some differential identities which force a 3-prime near-ring to be a commutative ring and also gave the suitable examples, proving the necessity of the 3-primeness condition.

Theorem Let N be a 3-prime near-ring. Suppose that N admits a nonzero derivation d satisfying the following property, i.e.; d ([x, y]) = ±[x, y] for all x, y
∈N. Then N is a commutative ring[25].

Theorem. ([22], Theorem 2.2-2.3). Let N be a 2-torsion free 3-prime near-ring. If N admits a nonzero derivation d satisfying any one of the following properties:

$$[d(x), y] = [x, d(y)] \; for \; all \; x, y \in N, [d(x), y] = \pm [x, d(y)] \; for \; all \; x, y \in N, [x, d(y)], = [d(y) \;\;)] \; for \; all \; x, y \in N,$$
$$[x, d(y)], = -[d(y) \;\;)] \; for \; all \; x, y \in N,$$

$$Then \; n \; is \; a \; commentative \; ring.$$

Generalized Derivations in Near-Rings

Matej Bresar [27] introduced the concept of generalized derivation in associative rings. This concept covers the concept of derivation already known to us for ring theory. Later a lot of study was done by Hvala, Golbasi, T. K. Lee etc. about generalized derivations in the setting of prime rings and semiprime rings and several known results for derivation in prime and semi prime rings were extended in the setting of generalized derivations in rings by above authors.

Motivated by the above concept, Golbasi[28] introduced the concept of generalized derivations in near-rings as given below and studied this in the setting of 3-prime and semi prime near-rings. Later in 2008, H. E. Bell[19] also studied this notion and derived some commutativity theorems of 3-prime near-rings equipped with generalized derivation. The above authors also generalized the several known results of derivations in 3-prime and semiprime near-rings.

Definition 3.1. Let N be a near-ring. An additive mapping f : N −→ N is called

(i) a right generalized derivation of N if there exists a derivation d of N such that f(xy) =f(x)y + xd(y) for all x, y ∈N .

(ii) (ii) a left generalized derivation of N if there exists a derivation d of N such that f(xy) = d(x)y + xf(y) for all x, y ∈N .

(iii) (iii) a generalized derivation of N if there exists a derivation d of N such that f(xy) =f(x)y+ xd(y) for all x, y ∈ N and f(xy) = d(x)y + xf(y) hold for all x, y ∈N .

On n- derivations in near-rings

Recently K. H. Park [36] introduced the notion of an n-derivation and symmetric n- derivation, where n is any positive integer in rings and extended several known results, earlier in the setting of derivations in prime rings and semiprime rings. Motivated by the above notion in rings the authors [5] introduced the notion of n-derivations in the setting of near-rings and generalized several known results obtained earlier in the setting of 3-prime near-rings and semiprime nearrings.

Lemma* :. *Let N be a nearring. Then D is a permuting n − derivation of N if and only if $D(x1x d' 1, x2, \cdots, xnx1 D(x'1, x2, \cdots, xn) + D(x1, x2, \cdots, xn)x'1$ ***for all*** $x1, x1', x2, \cdots, xn \in N$.

In a left near-ring N , right distributive law does not hold in general, however, the following partial distributive properties in N have been obtained in ([5], Lemma 2.4-2.6).

Theorem : Let N be a near-ring. Let D be a permuting n-derivation of N and d be thetrace of D. Then

(i) $\{D\ (1\ , x2, \ldots . Xn\ ) X, ;$ ‘ $+ \{D\ (x1\ , x2, \ldots . Xn\ ) X, ;$ ‘$)\ \}\}\ y\ =$

(ii) $\{D\ (1\ , x2, \ldots . Xn\ ) X, ;$ ‘ $+ \{D\ (x1\ , x2, \ldots . Xn\ ) X, ;$ ‘$)\ \}\}\ y\ =$

$for\ every\ x1, x1$.’, … $xny \in N.$

(iii) $\{\ , \ldots\ D\ , (x1\ , x2, \ldots . Xn\ ) X, ;$ ‘ +

$\{D\ (x1\ , x2, \ldots . Xn\ ) X, ;$ ‘$)\ \}\}\ y =$

(iv) $\{\ , 1\ \ldots\ D\ , (x1\ , x2, \ldots . Xn\ ) X, ;$ ‘ +

$\{D\ (x1\ , x2, \ldots . Xn\ ) X, ;$ ‘$)\ \}\}\ y =\ for\ every\ x, 1\ x, 2\ \ldots\ x, n\ y \in\ N.$

(v) $\{D\ (x)\ x\ , \ldots . 1\ x \ldots 2\ X2\ \ \ldots .)\ xi\ \ \ldots .) =\ D\ (x, 1\ x, 2)\ +$

$D(\ X, \ldots \ldots \ldots X2\ )$’ $x, \ldots \ldots \ldots \ldots\ )$

(vi) $\{\ 1\ D\ (x)\ x1\ , \ldots . 1\ x \ldots 2\ X2\ \ \ldots .)\ xi\ \ \ldots .)\ =\ D\ (x, 1\ x, 2)\ +$

$D(\ X, \ldots \ldots \ldots X2\ )$’ $x, \ldots \ldots \ldots \ldots\ for\ every\ x, .\ .X, .\ . \in N.$

(vii) $If\ N\ is\ 3 - prim, D\ \neq$

$0, and\ xD\ (n, N, \ldots \ldots \ldots \ldots . N\ )\ x\ \{0\}\ where\ x \in N.\ then\ x, = 0$.

(viii) $If\ N\ is\ 3 - prim, D\ \neq$

$0, and\ xD\ (n, n, \ldots \ldots \ldots \ldots . N\ )\ x\ \{0\}\ where\ x \in N.\ then\ x, = 0$.

(ix) $(9) f\ N\ is\ 3 -\ prime\ , D\ \neq\ 0, and\ xC\ (n, C\ , \ldots C\ \ \ldots \ldots \ldots . C\ ) \neq$

$x\ \{0\}\ where\ C\ where\ C \neq \{0\}.$

Recently Öztürk and Jun ([35], Lemma 3.1) proved that in a 2-torsion free 3-prime near-ring which admits a symmetric bi-additive mapping D if the trace d of D is zero, then D = 0. Further, this result was generalized by K.H. Park and Y.S. Jun ([37], Lemma 2.2) for permuting tri-additive mapping in 3!-torsion free 3-prime near-ring. We have extended this result, as below, for permuting n-additive mapping in a n!-torsion free 3-prime near-ring under some constraints.

On Generalized n-Derivations in Near-rings

Motivated by the concept of generalized derivation in rings and near-rings the authors [10] generalized the concept of n-derivation of near-rings by introducing the notion of generalized derivations in near-rings.

Definition 5.1.

Let n be a fixed positive integer. An n-additive mapping F : N ×N ×· · ·×N −→ N is called a right generalized n-derivation of N with associated n-derivation D if the relations

$$F(x1, x2, \ldots, xi-, x, x, i+, \ldots\ldots xn)$$
$$= F(x1, x2, \ldots, xi-, x, x, i+, \ldots\ldots xn)x, n$$
$$+ +x, 1\, x, D\,(X1, x2, \ldots\ldots xi\, x, i..+1, \ldots\ldots, X\,n,)$$

hold for all $(x1, x2, \ldots, xi-, x, i\;\; x, i, x, i+, \ldots\ldots xn) =$ $xn, \in, i = 1, 2, 3, \ldots, n,$

If in addition, both F and D are permuting maps then all the above conditions are equivalent and in this case F is called a permuting right generalized n-derivation of N with associated permuting n-derivation D. An n-additive mapping F: N × N × · · · × N −→ N is called a left generalized n-derivation of N with associated n-derivation D if the relations

$$F(x1, x2, \ldots, xi-, x, x, i+, \ldots\ldots xn)$$
$$= F(x1, x2, \ldots, xi-, x, x, i+, \ldots\ldots xn)x, n$$
$$+ +x, 1\, x, D\,(X1, x2, \ldots\ldots xi\, x, i..+1, \ldots\ldots, X\,n,)$$

Hold for all $(x1, x2, \ldots, xi-, x, i\;\; x, i, x, i+, \ldots\ldots xn) =$ $xn, \in, i = 1, 2, 3, \ldots, n,$

If in addition, both F and D are permuting maps then all the above conditions are equivalent and in this case F is called a permuting left generalized n-derivation of N with associated permuting n-derivation D. An n-additive mapping $F : N \times \times \cdots \times N \longrightarrow$
$N$is called a generalized n-derivation of N with associated n-derivation D if it is both aright generalized n-derivation as well as a left generalized n-derivation of N with associated n-derivation D. If in addition, both F and D are permuting maps then F is called a permuting generalized n-derivation of N with associated permuting n-derivation D (see [10] for further reference). If N is a commutative ring, then it is trivial to see that the set of all leftgeneralized n-derivations of N is equal to the set of all right generalized n-derivations of N .

Semigroup ideals and generalized n-derivations in near-rings

A nonempty subset A of N is called semigroup left ideal (resp. semigroup right ideal) if N A ⊆ A (resp. AN ⊆ A) and if A is both a semigroup left ideal and a semigroup right ideal, it will be called a semigroup ideal. Recently many authors have studied commutativity of addition and ring behavior of 3-prime near-rings satisfying certain properties and identities involving derivations and generalized derivations on semigroup ideals (see [2],[18],[32][33], where further references can be found). In the present section we study
the commutativity of addition and ring behavior of 3-prime near-rings satisfying certain properties and identities involving generalized n-derivations on semigroup ideals. In fact, the results presented in this section generalize, extend, compliment and improve several results obtained earlier on derivations, generalized derivations, permuting n-derivations and generalized n-derivations for 3-prime near-rings; for example Theorem 1.2 of [2], Theorems 3.2–3.4&3.7 of [5], Theorems 3.1, 3.11, 3.15, 3.16 of [10], Theorems 3.2 − 3.3 of [18] etc.- to mention a few only. We begin with the following theorem obtained in ([12], Theorem3.1).

Theorem 6.1. Let N be a 3-prime near-ring and A1, A2, ..., An be nonzero semigroup ideals of N . If it admits a nonzero generalized n-derivation F with associated n-derivation D of N such **that** $(A1, A2, \ldots, An) \subseteq$Z, then N is a commutative ring. Corollary 6.2. ([10], Theorem 3.1). Let N be a 3-prime near-ring admitting a nonzero generalized n-derivation F with associated n-derivation D of N . If F(N , N , · · · , N) ⊆ Z, then N is a commutative ring. The following example demonstrates that N to be 3-prime is essential in the hypothesis of the above theorem.

Theorem 6.4. ([12], Theorem 3.2). Let N be a 3-prime near-ring and A1, A2, · · · , An nonzero semigroup ideals of N . If it admits generalized n-derivations F and G with associated nonzero n-derivations D and H of N respectively such that

$$F(x1, x2,\cdot\cdot\cdot, xn)H(y1, y2,\cdot\cdot\cdot, yn) = -G(x1, x2,\cdot\cdot\cdot, xn)D(y1, y2,\cdot\cdot\cdot, yn)$$

$$\textit{for all } x1, y1 \in A1;\ x2, y2 \in A2;\cdot\cdot\cdot;\ xn, yn \in An, \textit{ then } (N, +) \textit{ is abelian}.$$

Corollary 6.5. ([10], Theorem 3.15). Let F and G be generalized n-derivations of 3-prime nearring N with associated nonzero n-derivations D and H of N respectively such that

$$F(x1, x2,\cdot\cdot\cdot, xn)H(y1, y2,\cdot\cdot\cdot, yn) = -G(x1, x2,\cdot\cdot\cdot, xn)D(y1, y2,\cdot\cdot\cdot, yn)$$

$$\textit{for all } x1, x2,\cdot\cdot\cdot, xn, y1, y2,\cdot\cdot\cdot, yn \in N. \textit{ Then } (N, +) \textit{ is an abelian group}.$$

Let X and Y be nonempty subsets of N and a ∈N . By the notations [X, Y] and [X, a] we mean the subsets of N defined by [X, Y] = {[x, y] | x ∈ X, y ∈ Y } and [X, a] = {[x, a] | x ∈ X} respectively. Very recently A. Ali et al. ([2], Theorem 12) proved that if N is a 3-prime near-ring, admitting a nonzero generalized derivation f with associated nonzero derivation d
such that [f(A), f(A)] = {0}, where A is a nonzero semigroup ideal of N , then (N , +) is abelian. We have improved and extended this result for generalized n-derivation in the setting of 3-prime near-rings. In fact we obtained the following. Theorem 6.6. ([12], Theorem 3.3). Let N be a 3-prime near-ring and A1, A2, · · · , An nonzero semigroup ideals of N . If it admits generalized n-derivations F1 and F2 with associated nonzero n-derivations D1 and D2 of N respectively such that [F1(A1, A2, · · · , An), F2(A1, A2, · · · , An)] = {0}, then (N , +) is abelian. Corollary 6.7. ([10], Theorem 3.16). Let F1 and F2 be generalized n- derivations of 3-prime near-ring N with associated nonzero n-derivations D1 and D2 of N respectively such that [F1(N , N , · · · , N), F2(N , N , · · · , N)] = {0}. Then (N , +) is an abelian group. The following example shows that the restriction of 3-primeness imposed on the hypotheses of Theorems 6.2 & 6.3 is not superfluous.

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