AN OPTIMIZATION OF FRACTIONAL FUNCTION AND THEIR APPLICATIONS

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ABSTRACT

Linear Fractional Programming Problems deal with the problem of optimizing the ratio of two linear functions subject to a set of linear constraints and non-negativity constraints on the variables. Several methods such as Charnes-Cooper Method, Hartmut Wolf Parametric Method, Kanti Swarup Method, Bitran and Novae's Method and Ratio Algorithm are available to solve these kinds of problem. These methods depend on the simplex type method which is based on vertex information and may have difficulties as the problem size increases and may require much time for computer to get the final optimum solution.

Keyword: - *Linear Fractional Programming1, Optimizing2, Algorithm3, and Linear Functions4.*

1. INTRODUCTION

Mostly, linear fractional programming (LFP) problems solve by using the simplex type method. In this thesis, a new approach for solving linear fractional programming problem is investigated. The approach adopted is based mainly upon solving the problem algebraically using the concept of duality in two ways. In first aproach, we assume an initial feasible point which will help in transferring a linear fractional programming problem into linear programming problem and there after solve the problem algebraically using the concept of duality. On other hand, in second approach the problem is solving by algebraically using the concept of duality and partial fractions. We represent the objective function in linear fractional function, whereas constraint functions in from of linear inequalities.

2. ALGORITHM FOR SOLVING LINEAR FRACTIONAL PROGRAMMING PROBLEMS

The steps to follow for solving the linear fractional programming problems are summarizes here:

Step 1: Compute $T_1 = p(p^T p)$, and T_2 matrix as equation (4.15)

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Step 2: Find the matrix Q of non-negative entries such that $QGT_2 = 0$

⇓

Step 3: Find a sub matrix $\bar{Q}$ of the given matrix Q satisfying $\bar{Q}GT_1 = 1$

⇓

Step 4: In the rows of $\bar{Q}$ for every positive entry, determine the corresponding active constraint in the given matrix GT_1

⇓

Step 5: Solve an n*n system of linear equations for these set of active constraints to get the optimal solution y*.

⇓

Step 6: Then use (4.13) to get the optimal solution of the Linear Fractional Programming (LFP) problem defined by Equation (4.1)

3. NUMERICAL APPROACH

Example: Maximize $Z(x) = \frac{2x_1 + 2x_2 + 6}{2x_2 + 2}$

Subject to,

$$x_1 + x_2 \leq 6$$

$$-x_2 \leq 0$$

$$-x_1 \leq 0$$

Solution: For this linear fractional programming we have c^t =(1 1), d^t= (0 1), b_i = 3, g_i = 1 , hwere c and d are matices

Then we have

$$T_1 = \binom{1/5}{-2/5}, T_2 = \binom{2}{1}, \text{ and } GT = \begin{pmatrix} 10 \\ -2 \\ -1 \end{pmatrix},$$

Hence,

$$Q = \begin{matrix} 1 & 5 & 0 \\ 1 & 0 & 10 \end{matrix}$$

The second row in Q satifices $\bar{Q}GT_1 = 1$

This implies that first and third constraints in G are the active constraints .

Hence by solving

$$y_1 + 8y_2 = 6$$

$$y_2 = 0$$

Therefore we get $y^{t*} = (6 \quad 0)$ which is the optimal solution for the equivalent problem.

Hence by using equation (4.13) we get the optimum solution of the linear fractional program with optimal value z^* =9**.**

4. CONCLUSION

The proposed method in the paper can be able to solve the problem of Multi-Objective Programming of industries in production planning. Such types of problems are intrinsically multi objective fractional programming problems. During the past few years the concept of Multi-Objective Programming (MOP) has become popular among researchers due to the fact that many single objective optimization methods are not able to help practitioners reach desirable solutions [8-10].

5. REFERENCES

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