APPLICATIONS OF DIFFERENTIAL EQUATIONS TO ENGINEERING

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ABSTRACT

The laws of physics are generally written down as differential equations. Therefore, all of science and engineering use differential equations to some degree. Understanding differential equations is essential to understanding almost anything you will study in your science and engineering classes. You can think of mathematics as the language of science, and differential equations are one of the most important parts of this language as far as science and engineering are concerned . This paper presents a systematic and comprehensive introduction to ordinary differential equations for engineering students and practitioners. Mathematical concepts and various techniques are presented in a clear, logical, and concise manner. Various visual features are used to highlight focus areas. Complete illustrative diagrams are used to facilitate mathematical modeling of application problems. Thus the emphasis is given on the relevance of differential equations through their applications in various engineering disciplines. Studies of various types of differential equations are determined by engineering applications. Detailed step-by-step analysis is presented to model the engineering problems using differential equations from physical . Such a detailed, step-by-step approach, especially when applied to practical engineering problems, helps the readers to develop problem-solving skills.

Keyword: *-differential equations, step by step analysis, comprehensive,*

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1. INTRODUCTION:

In general, modeling of the variation of a physical quantity, such as temperature, pressure, displacement, velocity, stress, strain, current, voltage, or concentration of a pollutant, with the change of time or location, or both would result in differential equations. Similarly, studying the variation of some physical quantities on other physical quantities would also lead to differential equations. In fact, many engineering subjects, such as mechanical vibration or structural dynamics, heat transfer, or theory of electric circuits, are founded on the theory of differential equations. This paper will enable you to develop a more profound understanding of engineering concepts and enhance your skills in solving engineering problems. In other words, you will be able to construct relatively simple models of change and deduce their consequences. By studying these, you will learn how to monitor and even control a given system to do what you want it to do

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2. APPLICATIONS:

2.1 Applications to Projectile Motion of a body:

- The first important area which comes to mind is ball games and sports. Footballs are heavy enough to follow a nearly parabolic trajectory, without spin, with the effect of spin often being spectacular.

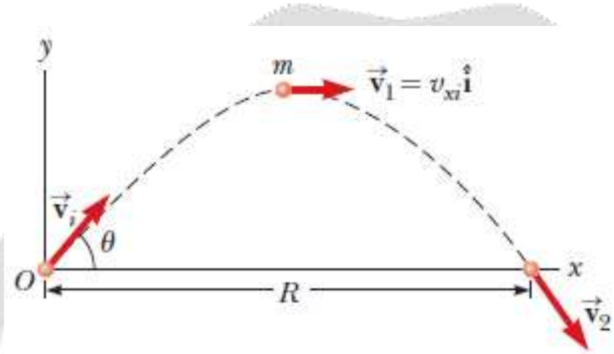
- A cricket ball is small and dense enough to follow a nearly parabolic path, and it is up to the batsman to judge this, in playing his shot. But, the extraordinary thing is that a cricket ball can swing in the air, when bowled in a certain way, making the batsman's job much more difficult. The parabolic trajectory of the ball is also very important when a fielder tries to catch a very high, long ball, on the boundary. . Getting a feel for parabolic flight is essential.

- With tennis and table tennis, the constant use of spin, even with lob shots, makes parabolic trajectories less important. This is also true in games such as golf, where spin predominates, and in rugby, where the shape of the ball affects its motion

- Scoring from distance in basketball is an example of the pure judgement of parabolic flight, and can only be mastered by constant practice.

- Another general and important area which is to do with projectile motion, is projectile weapons, of which there are and have been many. For instance, arrows and thrown spears are projectile weapons, where the angle of projection which

  The general graph of projectile motion is as below:



The equations of motion in the X and Y direction respectively are given by-

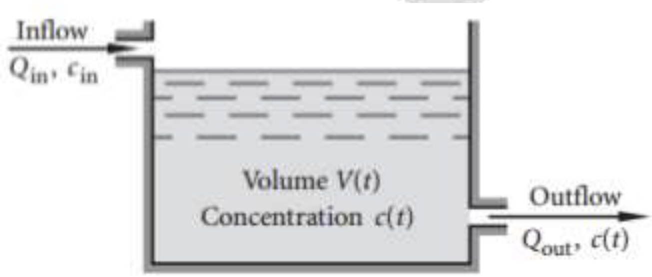
$$m\ddot{x}(t) + \beta\dot{x}(t) = 0$$

$$m\ddot{y}(t) + \beta\dot{y}(t) = -mg$$

in which the initial conditions are at time t =0: x(0)=0, y(0)=0, x˙(0)=v0 cos θ0, y˙(0)=v0 sin θ0. The equations of motion are two equations involving the first- and second-order derivatives x˙(t), y˙(t), x¨(t), and y¨(t). These equations are called, as will be defined later, a system of two second-order ordinary differential equations.

2.2 Application to Mixing problems:

- These **problems** arise in many settings, such as when combining solutions in a chemistry lab .
- Adding ingredients to a recipe.e.g. A lemonade mixture problem may ask how tartness changes when pure water is added or when different batches of lemonade are combined.



Inflow: Solution of concentration c_{in} grams/liter flows in at a rate of r1 liters/minute

In tank: A(t)- amount of chemical in the tank at time t

V(t) -volume of solution in the tank at time t

c(t)= A(t)/ V(t) - concentration of chemical in the tank at time t

Out flow: Solution of concentration c(t) grams/liter flows out at a rate of r2 liters/minute

at time t =0, the volume of the liquid is V_0 with a pollutant concentration of c_0.

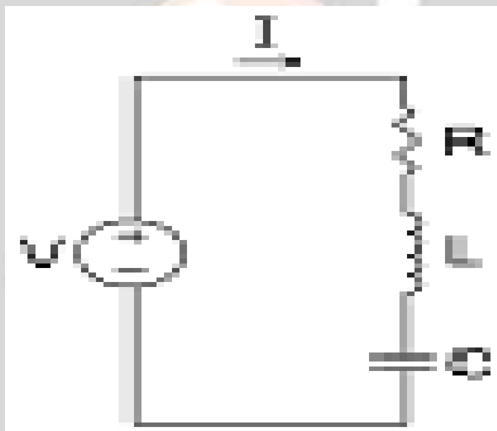
The equation governing the pollutant concentration c(t) is given by

$$[V_0 + (Q_{in} + Q_{out})t]\frac{dc(t)}{dt} + Q_{in}c(t) = Q_{in}c_{in}$$

with initial condition $c(0)=c_0$. This is a first-order ordinary differential equation

2.3 Application to RLC circuits:

The RLC circuit is the electrical circuit consisting of a resistor of resistance R, a coil of inductance L, a capacitor of capacitance C and a voltage source arranged in series. If the charge on the capacitor is Q and the current flowing in the circuit is I, the voltage across R, L and C are RI, LdI/ dt and Q /C respectively.



By the Kirchhoff's law that says that the voltage between any two points has to be independent of the path used to travel between the two points,

$LI'(t) + R\,I(t) + 1/C\,Q(t) = V(t)$……………………………………………………………..(1)

Assuming that R, L, C and V are known, this is still one differential equation in two unknowns, I and Q. However the two unknowns are related by $I(t) = dQ/dt$,so that

$LQ''(t) + RQ'(t) + 1/C\,Q(t) = V(t)$ or,………………………………………………………..(2)

Differentiating with respect to t and then substituting $dQ/dt = I(t)$,

$LI''(t) + RI'(t) + 1/C\,I(t) = V'(t)$ ……………………………………………………………….(3)

For an ac voltage source, choosing the origin of time so that $V(0) = 0$, $V(t) = E_0 \sin(\omega t)$ and the differential equation becomes $LI''(t) + RI'(t) + 1\,C\,I(t) = \omega E_0 \cos(\omega t)$

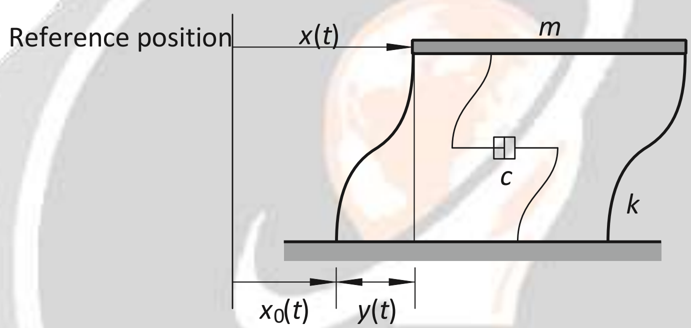
The equations (1),(2) and (3) represents the same system.

Also one can form the equations containing only RL OR only LC circuits

2.4 Application to tuned mass dampers (TMD):

- tuned mass dampers (TMDs) can be successfully employed to control steady-state vibration problems of other composite floor systems. ANTHONY C. WEBSTER and RIMAS VAICAITIS [2]
- Tuned Mass Damper Design for Optimally Minimizing Fatigue Damage. Hua-Jun Li et al[47](2002).
- However the problems such as insufficient control force capacity and excessive power demands encountered by current technology in the context of structural control against earthquakes are unavoidable and need to be overcome

The optimum parameters of tuned mass dampers (TMD) that result in considerable reduction in the response of structures to seismic loading are presented. The criterion used to obtain the optimum parameters is to select, for a given mass ratio, the frequency (tuning) and damping ratios that would result in equal and large modal damping in the first two modes of vibration. The parameters are used to compute the response of several single and multi-degree-of-freedom structures with TMDs to different earthquake excitations. The results indicate that the use of the proposed parameters reduces the displacement and acceleration responses significantly. The method can also be used in vibration control of tall buildings using the so-called ‘mega-substructure configuration’, where substructures serve as vibration absorbers for the main .



Consider the vibration of a single-story shear building under the excitation of earthquake. The shear building consists of a rigid girder of mass *m* supported by columns of combined stiffness *k*. The vibration of the girder can be described by the horizontal displacement $x(t)$. The earthquake is modeled by the displacement of the ground $x_0(t)$ as shown. When the girder vibrates, there is a damping force due to the internal friction between various components of the building, given by $c\dot{x}(t)-\dot{x}0(t)$, where *c* is the damping= −coefficient.

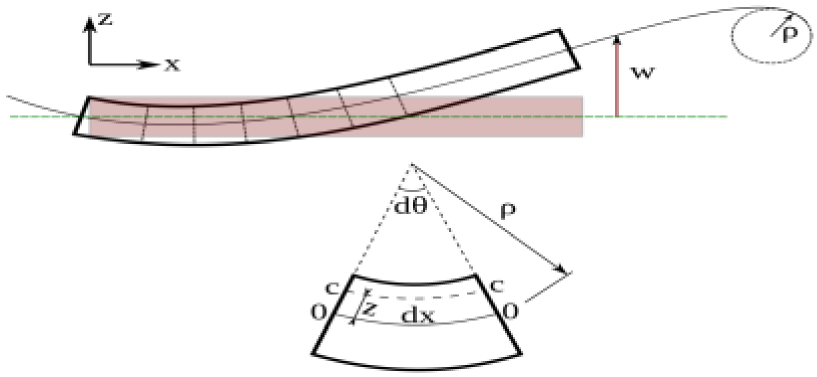
The relative displacement $y(t)$ $x(t)$ $x_0(t)$ between the girder and the ground is governed by the equation

$$m\frac{d^2y}{dt^2} + c\,\frac{dy}{dt} + ky(t) = -m\frac{d^2x_0}{dt^2}$$

which is a second-order linear ordinary differential equation.

2.5 Application to bending of beams:

The solution of bending and buckling problems is integral to the study of civil, mechanical and aerospace engineering. The academic introduction to the bending of beams with constant cross section is usually given to students of these disciplines early in the engineering curriculum in a course in mechanics of materials.



Because of the fundamental importance of the bending moment equation in engineering, we will provide a short derivation. We change to polar coordinates. The length of the neutral axis in the figure is ρdθ. The length of a fiber with a radial distance z below the neutral axis is (ρ+z)*d*θ Therefore, the strain of this fiber is ρ/z.

The stress of this fiber is $E * (\rho/z)$. where E is the elastic modulus in accordance with Hooke's Law. The differential force vector, *d*F resulting from this stress is given by, $dF = E * (\rho/z) dA e_x$

This is the differential force vector exerted on the right hand side of the section shown in the figure. We know that it is in the e_x direction since the figure clearly shows that the fibers in the lower half are in tension. *d*A is the differential element of area at the location of the fiber. The differential bending moment vector, *d*M associated with *d*F is given by

$$dM = -zdz\, X\, dF = e_y E(z^2 / \rho) dA$$

This expression is valid for the fibers in the lower half of the beam. The expression for the fibers in the upper half of the beam will be similar except that the moment arm vector will be in the positive z direction and the force vector will be in the -x direction since the upper fibers are in compression. But the resulting bending moment vector will still be in the -y direction since $e_z\, X - e_y = -e_x$ Therefore, we integrate over the entire cross section of the beam and get for M the bending moment vector exerted on the right cross section of the beam the expression

$$M = \int dM = e_y \frac{E}{\rho} \int z^2\, dA = -e_y \frac{EI}{\rho}$$

where I is the second moment of area. From calculus, we know that when $\frac{dw}{dx}$ is small as it is for an Euler–Bernoulli beam, $\frac{1}{\rho} = \frac{d^2w}{dx^2}$ (ρ is the radius of curvature). Therefore,

$$M = -e_y EI \frac{d^2w}{dx^2}$$

3. CONCLUSIONS

Differential equations are of great importance as almost every area of engineering, almost all real life situations can be expressed using differential equations. Here I had discussed very few of the applications.
I had only constructed the differential equations .These differential equations can be solved by any method either by analytically or by using numerical methods.

4. REFERENCES

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