APPLICATION OF OMA ON THE BENCH-SCALE ALUMINUM BRIDGE USING MICRO TREMOR DATA

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ABSTRACT

In this study was investigated of possibility using the recorded micro tremor data on ground level as ambient vibration input excitation data for investigation and application Operational Modal Analysis (OMA) for bench-scale aluminum bridges. As known OMA methods (such as EFDD, SSI-UPC/SSI-PC/SSI-CVA and so on) are supposed to deal with the ambient responses. For this purpose, analytical and experimental modal analysis of a bench-scale aluminum bridge for dynamic characteristics was evaluated. 3D Finite element model of the building was evaluated SAP2000 for the bench-scale aluminum bridge based on the design drawing. Ambient excitation was provided from the recorded micro tremor ambient vibration data on ground level. Enhanced Frequency Domain Decomposition is used for the output only modal identification. From this study, very best correlation is found between mode shapes and frequencies. Natural frequencies and analytical frequencies in average (only) 2.67% are differences.

Keyword: - *OMA, EFDD, Bridge, Modal parameters, Micro tremor*

1. INTRODUCTION

In recent years, in the world and our country, the determination of the effect of vibrations on structures and structural behavior has become very important. Our country has many important structures in terms of its historical background and geographical location. In addition, the earthquakes that have frequently occurred in recent years in our country have increased the researches and studies on the experimental determination of the behavior of structures under vibration. Many buildings built in the past are known to suffer many damages due to faults in the design and manufacturing stages, as well as natural disasters and overload effects. Especially when our country is on an active earthquake zone, we have over 80 million inhabitants, considering our geographical position; Damage assessment and assessment are clearly visible in terms of our country. Structures are under constant vibration. Many factors such as wind, earthquake, wave, explosion, and vehicle load etc. cause vibration. These vibrations sometimes cause cracks and sometimes serious damage. Thus, the determination of the behavior of structures under vibrations directly affects the life of that structure. The behavior of the structure under the vibrations affecting it can only be determined by experimental studies. At the design of the structures, firstly analytical models are formed to represent the structures and static and dynamic analyzes are carried out for different loading situations on these models. But in most cases the analytical model created does not fully represent the actual behavior of the building. The comparison of dynamic parameters is used as a practical solution in determining and eliminating differences in building behavior. Calibration of the analytical model is very effective by making changes on the analytical model in order to provide the experimental results in case the dynamic parameters obtained by the experimental modal analysis methods belong to the actual state of construction. Thus, analytical models can be obtained that represent the real state of the structures. Experimental modal analysis method is used in determining the dynamic parameters of the structures. In this method, the vibration signals from the accelerometers placed in the structure are collected with the help of the data acquisition unit and the dynamic parameters are obtained by using the software. Experimental modal analysis method is two parts as forced vibration test and ambient vibration test. In the forced vibration test method, the structure is vibrated with the aid of a known and measurable effect, and the response of the structure is measured. In the ambient vibration test method, which is referred to as the operational modal analysis method, it is assumed that the structure is vibrated by environmental influences and the responses that the structure gives to these effects are measured. Different methods based on the frequency and time domain are used to measure and evaluate the reactions. The mathematical bases of the methods used are the same and data processing, equations solving techniques and matrix arrays are different from each other.

Ambient vibration testing (also called Operational Modal Analysis) is the most economical non-destructive testing method to acquire vibration data from large civil engineering structures for Output-Only Model Identification. General characteristics of structural response (appropriate frequency, displacement, velocity, acceleration rungs), suggested measuring quantity (such as velocity or acceleration) depends on the type of vibrations (Traffic, Acoustic, Machinery inside, Earthquakes, Wind…) are given in Vibration of Buildings (1990).

This structures Response characteristics gives a general idea of the preferred quantity and its rungs to be measured. A few studies the analysis of ambient vibration measurements of buildings from 1982 until 2015 are discussed in Brincker and Ventura (2015) [13]. Last ten years Output-Only Model Identification studies of buildings are given in appropriate references structural vibration solutions. For the modal updating of the structure it is necessary to estimate sensitivity of reaction of examined system to change of parameters of a building. Kasimzade (2006) System identification is the process of developing or improving a mathematical representation of a physical system using experimental data investigated in HO and Kalman (1966), Kalman (1960), Ibrahim and Miculcik (1977), Ibrahim (1977), Bendat (1998), Ljung (1999), Juang (1994), Van Overschee and De Moor (1996), and system identification applications in civil engineering structures are presented in works Trifunac (1972), Huang and Chen (2017), (Li *et al.* 2016), (Park *et al.* 2016), (Ni *et al.* 2015) (Brincker *et al.* 2000), Roeck (2003), Peeters (2000), (Cunha *et al.* 2005), Wenzel and Pichler (2005), Kasimzade and Tuhta (2007a, b), (2009). Extracting system physical parameters from identified state space representation was investigated in references. Alvin and Park (1994), Balmes (1997), (Juang *et al.*1988), Juang and Pappa (1985), (Lus *et al.* 2003), (Phan *et al.* 2003), Sestieri and Ibrahim (1994), (Tseng *et al.* 1994). The solution of a matrix algebraic Riccati equation and orthogonality projection more intensively and inevitably used in system identification was deeply investigated in works of Aliev (1998). In engineering structures there are three types of identification: modal parameter identification; structural-modal parameter identification; control-model identification methods are used. In the frequency domain the identification is based on the singular value decomposition of the spectral density matrix and it is denoted Frequency Domain Decomposition (FDD) and its further development Enhanced Frequency Domain Decomposition (EFDD). In the time domain there are three different implementations of the Stochastic Subspace Identification (SSI) technique: Unweighted Principal Component (UPC); Principal component (PC); Canonical Variety Analysis (CVA) is used for the modal updating of the structure Friswell and Mottershead (1995), Marwala (2010). It is necessary to estimate sensitivity of reaction of examined system to change of random or fuzzy parameters of a structure. Investigated measurement noise perturbation influences to the identified system modal and physical parameters. Estimated measurement noise border, for which identified system parameters are acceptable for validation of finite element model of examine system. System identification is realized by observer Kalman filter (Juang *et al.* 1993) and Subspace Overschee and De Moor (1996), algorithms. In special case observer gain may be coincide with the Kalman gain. Stochastic state-space model of the structure are simulated by Monte-Carlo method [12], [35], [36], [37].

The bench-scale aluminum bridge ideal for teaching structural dynamics, system identification topics related to earthquake, aerospace and mechanical engineering and widely used in applications [9]. In this study was investigated of possibility using the recorded micro tremor data on ground level as ambient vibration input excitation data for investigation and application Operational Modal Analysis (OMA) for bench-scale aluminum bridges [11].

For this purpose, analytical and experimental modal analysis of a bench-scale aluminum bridge for dynamic characteristics was evaluated. 3D Finite element model of the building was evaluated for the bench-scale aluminum bridge based on the design drawing. Ambient excitation was provided from the recorded micro tremor ambient vibration data on ground level. Enhanced Frequency Domain Decomposition is used for the output only modal identification.

2. MODAL PARAMETER EXTRACTIONS

The (FDD) ambient modal identification is an extension of the Basic Frequency Domain (BFD) technique or called the Peak-Picking technique. This method uses the fact that modes can be estimated from the spectral densities calculated, in the case of a white noise input, and a lightly damped structure. It is a non-parametric technique that determines the modal parameters directly from signal processing. The FDD technique estimates the modes using a Singular Value Decomposition (SVD) of each of the measurement data sets. This decomposition corresponds to a Single Degree of Freedom (SDOF) identification of the measured system for each singular value (Brincker *et al.* 2000).

The Enhanced Frequency Domain Decomposition technique is an extension to Frequency Domain Decomposition (FDD) technique. This t technique is a simple technique that is extremely basic to use. In this technique, modes are easily picked locating the peaks in Singular Value Decomposition (SVD) plots calculated from the spectral density spectra of the responses. FDD technique is based on using a single frequency line from the Fast Fourier Transform analysis (FFT), the accuracy of the estimated natural frequency based on the FFT resolution and no modal damping is calculated. On the other hand, EFDD technique gives an advanced estimation of both the natural frequencies, the mode shapes and includes the damping ratios (Jacobsen *et al.* 2006). In EFDD technique, the single degree of freedom (SDOF) Power Spectral Density (PSD) function, identified about a peak of resonance, is taken back to the time domain using the Inverse Discrete Fourier Transform (IDFT). The natural frequency is acquired by defining the number of zero crossing as a function of time, and the damping by the logarithmic decrement of the correspondent single degree of freedom (SDOF) normalized auto correlation function Peeters (2000).

In this study modal parameter identification was implemented by the Enhanced Frequency Domain Decomposition. The relationship between the input and responses in the EFDD technique can be written as, In this method, unknown input is represented with $x(t)$ and measured output is represented with $y(t)$

$$\left[G_{yy}(j\omega)\right]=[H(j\omega)]^{*}[G_{xx}(j\omega)][H(j\omega)]^{T}$$
(1)

Where $G_{xx}(j\omega)$ is the $r\ x\ r$ Power Spectral Density (PSD) matrix of the input. $G_{yy}(j\omega)$ is the $m\ x\ m$ Power Spectral Density (PSD) matrix of the output, $H(j\omega)$ is the $m\ x\ r$ Frequency Response Function (FRF) matrix, and * and superscript T denote complex conjugate and transpose, respectively. The FRF can be reduced to a pole/residue form as follows:

$$[H(\omega)]=\frac{[Y(\omega)]}{[X(\omega)]}=\sum_{k=1}^{m}\frac{[R_k]}{j\omega-\lambda_k}+\frac{[R_k]^{*}}{j\omega-\lambda_k^{*}}$$
(2)

Where n is the number of modes λ_k is the pole and, R_k is the residue. Then Eq. (1) becomes as:

$$G_{yy}(j\omega)=\sum_{k=1}^{n}\sum_{s=1}^{n}\left[\frac{[R_k]}{j\omega-\lambda_k}+\frac{[R_k]^{*}}{j\omega-\lambda_k^{*}}\right]$$
$$G_{xx}(j\omega)\left[\frac{[R_s]}{j\omega-\lambda_s}+\frac{[R_s]^{*}}{j\omega-\lambda_s^{*}}\right]^{\overline{H}}$$
(3)

Where s the singular values, superscript is H denotes complex conjugate and transpose. Multiplying the two partial fraction factors and making use of the Heaviside partial fraction theorem, after some mathematical manipulations, the output PSD can be reduced to a pole/residue form as fallows;

$$\left[G_{yy}(j\omega)\right]=\sum_{k=1}^{n}\frac{[A_k]}{j\omega-\lambda_k}+\frac{[A_k]^{*}}{j\omega-\lambda_k^{*}}+\frac{[B_k]}{-j\omega-\lambda_k}+\frac{[B_k]^{*}}{-j\omega-\lambda_k^{*}}$$
(4)

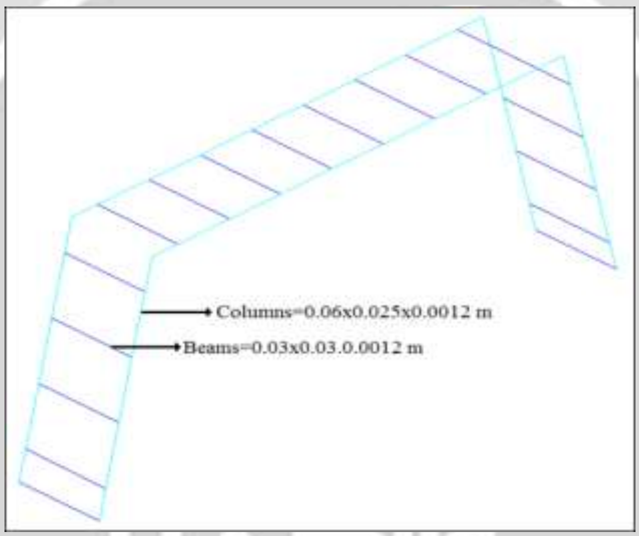
Where A_k is the k th residue matrix of the output PSD. In the EFDD identification, the first step is to estimate the PSD matrix. The estimation of the output PSD known at discrete frequencies is then decomposed by taking the SVD (singular value decomposition) of the matrix;

$$G_{yy}(j\omega_i)=U_i S_i U_i^{\bar{H}} \tag{5}$$

Where the matrix $U_i=[u_{i1},u_{i2},\dots,u_{im}]$ is a unitary matrix holding the singular vectors u_{ij} and s_{ij} is a diagonal matrix holding the scalar singular values. The first singular vector u_{ij} is an estimation of the mode shape. PSD function is identified around the peak by comparing the mode shape estimation u_{ij} with the singular vectors for the frequency lines around the peak. From the piece of the SDOF density function obtained around the peak of the PSD, the natural frequency and the damping can be obtained.

3. DESCRIPTION OF BENCH-SCALE ALUMINUM BRIDGE

Bench-scale aluminum bridge is 1.27 m height. Shape of bridge is trapezoid. Top and bottom width respectively 2.15 m-2.30 m. Dimensions of elements are shown in Fig. 1.



a)

b)

Fig -1a, b: Illustration of bench-scale aluminum bridge

Fig -2: View of bench-scale aluminum bridge

4. ANALYTICAL MODAL ANALYSIS OF BENCH-SCALE ALUMINUM BRIDGE

A finite element model was generated in SAP2000 (1997). Beams and columns were modeled as 3D beam-column elements (in Fig.3 shown by the black color). Structure modeled as an absolutely rigidity floor (rigid diaphragm). The selected structure is modeled as a space frame structure with 3D elements. Beams and columns were modeled as 3D beam-column elements which have degrees of freedom. At the base of the structure in the model, the ends of every element were fixed against translation and rotation for the 6 degree of freedom (DOF) then creating finite element model of the structure in SAP2000. The following assumptions were taken into account. Bench-scale aluminum bridge is modeled using an equivalent thickness and shell elements with isotropic property. All supports are modeled as fully fixed. The members of aluminum frame are modeled as rigidly connected together at the intersection points. In modeling of beams and columns the modulus of elasticity E=6.960E10 N/m^2, Poisson ratio μ=0.33, mass per unit volume ρ=26601 N/m^3.

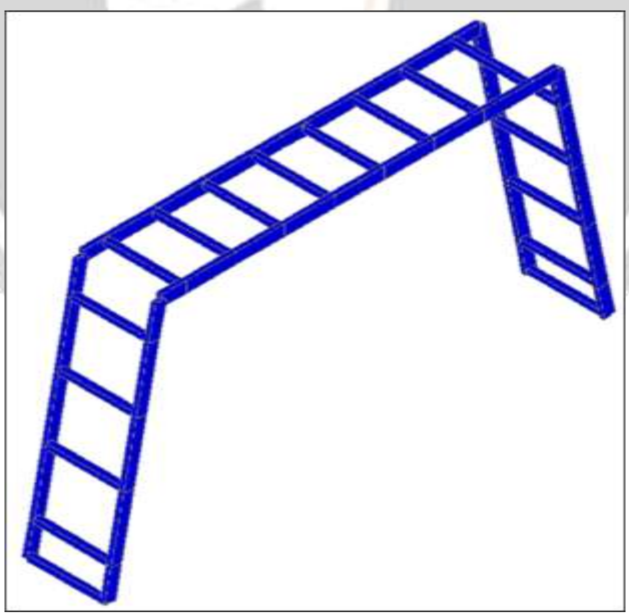
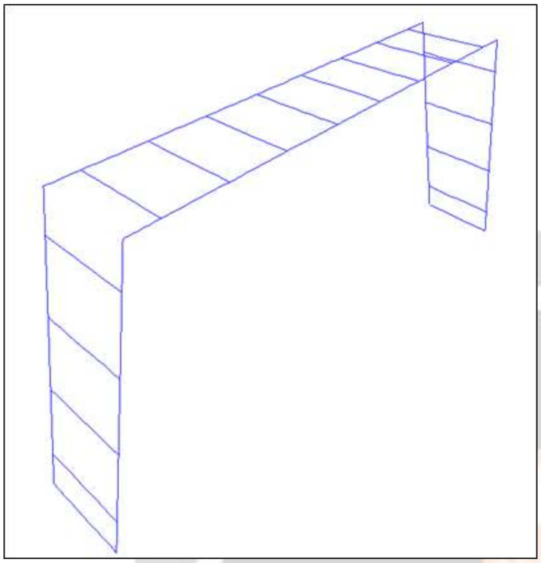


Fig -3: Finite element model of bench-scale aluminum bridge

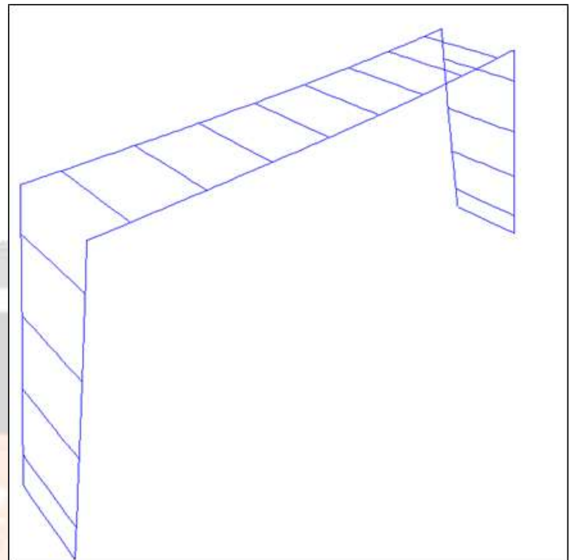
Natural frequencies and vibration modes are concerned a significant impact on the dynamic performance of buildings is an important dynamic property. A total of five natural frequencies of the structure are attained which range between 4 and 8 Hz. The first five vibration mode of the structure is shown in Figure 4. Analytical modal analysis results at the finite element model are shown in Table 1.

Table -1: Analytical modal analysis result at the first at the Finite Element (FE) model

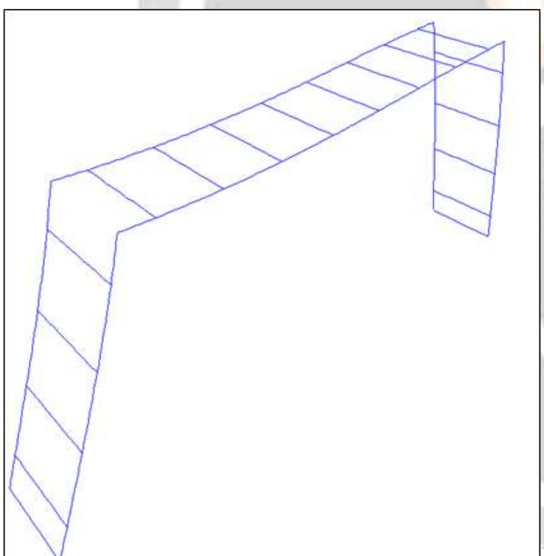
Mode number	1	2	3	4	5
Frequency (Hz)	4.880	5.222	5.740	6.502	7.084



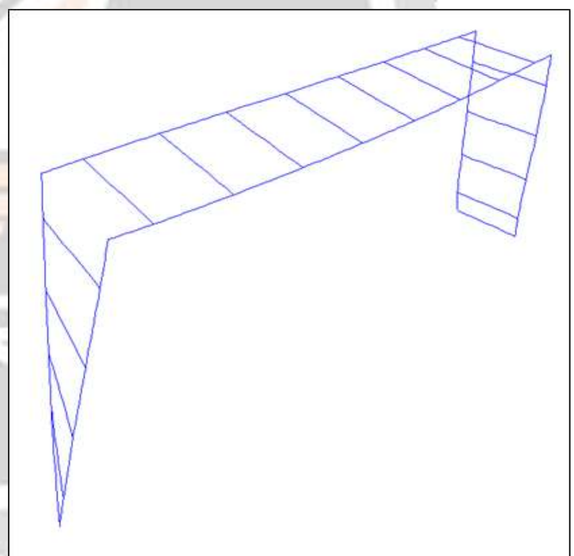
1st Mode Shape (f=4.880 Hz)



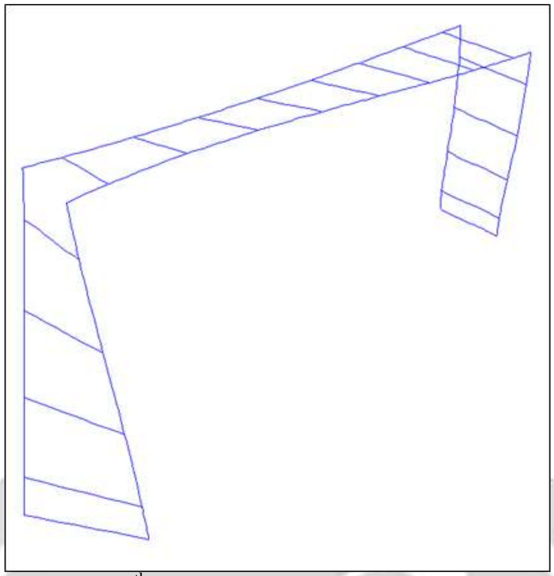
2nd Mode Shape (f=5.222 Hz)



3rd Mode Shape (f=5.740 Hz)



4th Mode Shape (f=6.502 Hz)

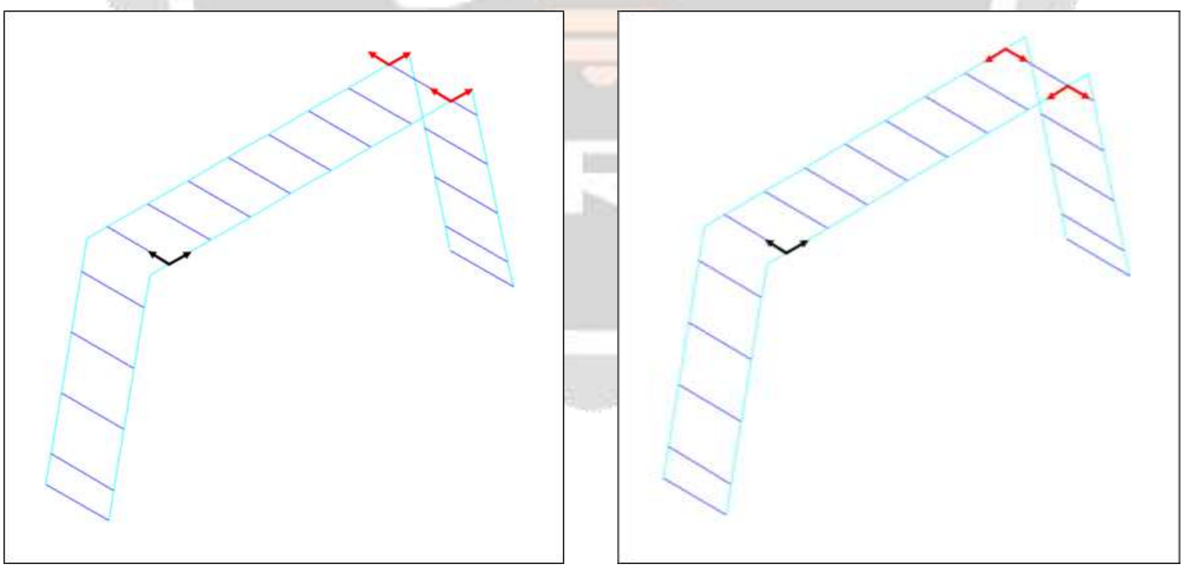


5th Mode Shape (f=7.084 Hz)

Fig -4: Analytically identified mode shapes of bench-scale aluminum bridge

5. EXPERIMENTAL MODAL ANALYSIS OF BENCH-SCALE ALUMINUM BRIDGE

Ambient excitation was provided by the recorded micro tremor data on ground level. Three accelerometers (with both x and y directional measures) were used for the ambient vibration measurements (Fig. 6), one of which were allocated as reference sensor always located in the first floor (they are shown by the red line in Fig. 5a, b). Two accelerometers were used as roving sensors (they are shown by the black line in Fig. 5a, b). The response was measured in two data sets (Fig. 5 a, b). For two data sets were used 3 and 5 degree of freedom records respectively (Fig. 5 a, b). Every data set (Fig. 5 a, b) was measured 100 min. The selected measurement points and directions are shown in Fig. 5 a, b.



a) First setup b) Second setup

Fig -5: Accelerometers location of experimental model in the 3D view

The data acquisition computer was dedicated to acquiring the ambient vibration records. In between measurements, the data files from the previous setup were transferred to the data analysis computer using a software package. This arrangement allowed data to be collected on the computer while the second, and faster, computer could be used to process the data in site. This approach maintained a good quality control that allowed preliminary analyses of the collected data. If the data showed unexpected signal drifts or unwanted noise or for some unknown reasons, was corrupted, the data set was discarded and the measurements were repeated.

Fig -6: Ambient vibrations recorded by the seismometer

Before the measurements could begin, the cable used to connect the sensors to the data acquisition, equipment had to be laid out. Following each measurement, the roving sensors were systematically located from floor to floor until the test was completed. The equipment used for the measurement includes three sensebox accelerometers (triaxial measures) and geosig seismometer, matlab data acquisition toolbox (wincon). For modal parameter estimation from the ambient vibration data, the operational modal analysis (OMA) software ARTeMIS Extractor (1999) is used.

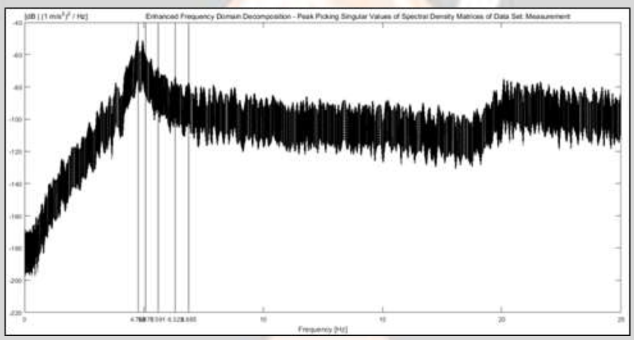


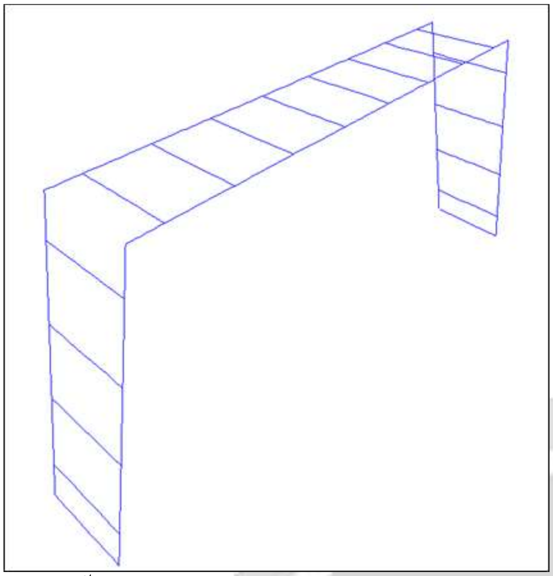
Fig -7: Singular values of spectral density matrices

The simple peak-picking method (PPM) finds the eigenfrequencies as the peaks of nonparametric spectrum estimates. This frequency selection procedure becomes a subjective task in case of noisy test data, weakly excited modes and relatively close eigenfrequencies. Also, for damping ratio estimation the related half-power bandwidth method is not reliable at all. Frequency domain algorithms have been the most popular, mainly due to their convenience and operating speed.

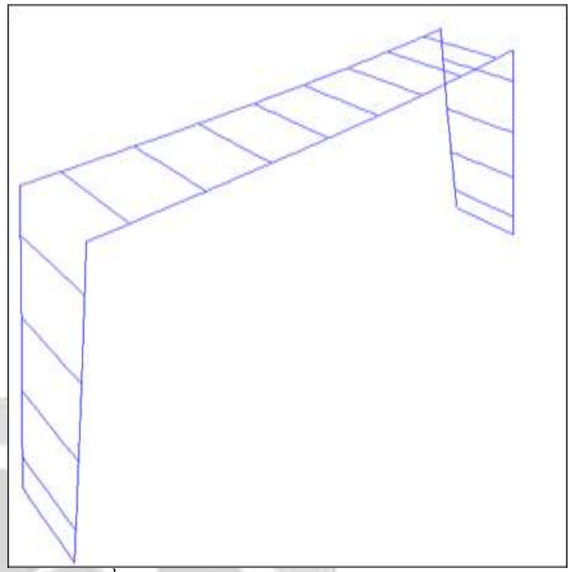
Singular values of spectral density matrices, attained from vibration data using PP (Peak Picking) technique are shown in Fig.7. Natural frequencies acquired from the all measurement setup are given in Table 2. The first five mode shapes extracted from experimental modal analyses are given in Fig. 8. When all measurements are examined, it can be seen that there are best accordance is found between experimental mode shapes. When the analytically and experimentally identified modal parameters are checked with each other, it can be seen that there is a best agreement between the mode shapes in experimental and analytical modal analyses (Table 3).

Table -2: Experimental modal analysis result at the bench-scale aluminum bridge

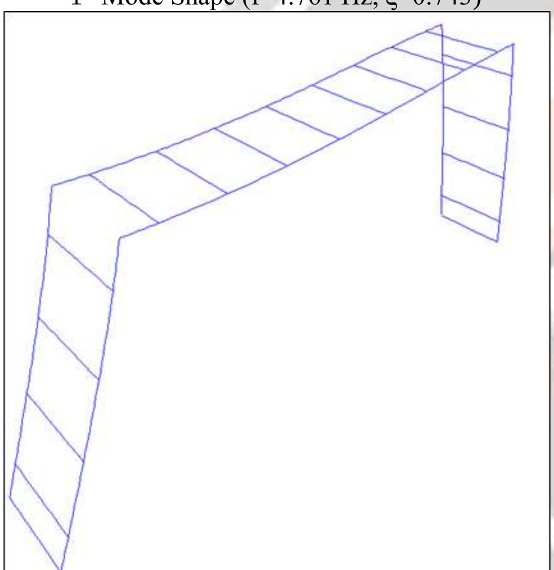
Mode number	1	2	3	4	5
Frequency (Hz)	4.761	5.078	5.591	6.323	6.885
Modal damping ratio (ξ)	0.743	0.696	0.632	0.559	0.513



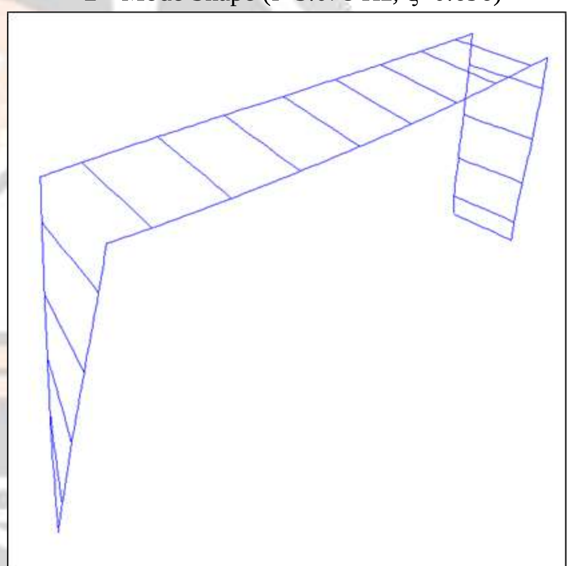
1st Mode Shape (f=4.761 Hz, ξ=0.743)



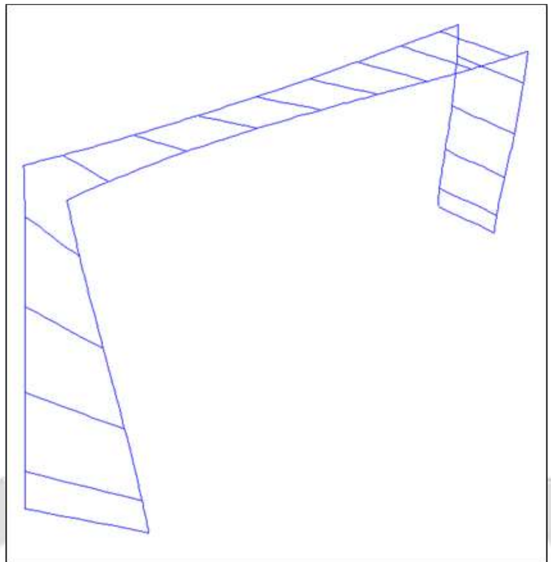
2nd Mode Shape (f=5.078 Hz, ξ=0.696)



3rd Mode Shape (f=5.591 Hz, ξ=0.632)



4th Mode Shape (f=6.323Hz, ξ=0.559)



5th Mode Shape (f=6.885 Hz, ξ=0.513)

Fig -8: Experimentally identified mode shapes of bench-scale aluminum bridge

Table -4: Comparison of analytical and experimental modal analysis results

Mode number	1	2	3	4	5
Analytical frequency (Hz)	4.880	5.222	5.740	6.502	7.084
Experimental frequency (Hz)	4.761	5.078	5.591	6.323	6.885
Difference (%)	2.438	2.757	2.595	2.752	2.809

6. CONCLUSIONS

In this paper, analytical and experimental modal analysis of bench-scale aluminum bridge was presented. Comparing the result of study, the following observation can be made:

From the finite element model of bench-scale aluminum bridge a total of 5 natural frequencies were attained analytically, which range between 4 and 8 Hz. 3D finite element model of bench-scale aluminum bridge is constructed with SAP2000 software and dynamic characteristics are determined analytically. The ambient vibration tests are conducted under provided from ambient vibration data on ground level. Modal parameter identification was implemented by the Enhanced Frequency Domain Decomposition (EFDD) technique. Comparing the result of analytically and experimentally modal analysis, the following observations can be made:

- From the finite element model of the bench-scale aluminum bridge, the first five mode shapes are attained analytically that range between 4 and 8 Hz.

- From the ambient vibration test, the first five natural frequencies are attained experimentally, which range between 4 and 7 Hz.

- When comparing the analytical and experimental results, it is clearly seen that there is very best agreement between mode shapes and frequencies.

- Analytical and experimental modal frequencies differences between 2.438%-2.809%.

Presented investigation results are shown and confirm of possibility using the recorded micro tremor data on ground level as ambient vibration input excitation data for investigation and application Operational Modal Analysis (OMA) for bench-scale aluminum bridges.

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