A NOTES ON R-IDEALS OVER SEMIRING AND FUZZY SOFT SEMIRING HOMOMORPHISM

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ABSTRACT

Aim of this paper, We introduce the concept of R-fuzzy soft ideals over semiring, fuzzy soft semiring homomorphism, fuzzy soft semiring, fuzzy soft-ideals and fuzzy R-ideals. And also we study some of their properties of R-fuzzy soft ideals and properties of homomorphic image of fuzzy soft semiring.

KeyWords:-Fuzzy semiring, Fuzzy subsemiring, Fuzzy left and right ideals, Fuzzy homomorphism, Fuzzy endomorphism.

1.INTRODUCTION

Notion of a semiring was introduced by Vandiver [10] in 1934. Semiring is a well known universal algebra. An universal algebra $(\mathcal{S},+,\cdot)$ is called a semiring if and only if $(\mathcal{S},+),(\mathcal{S},\cdot)$ are semigroups which ar connected by disriutive laws $i.e., a(b+c)=ab+ac, (a+b)c=ac+bc,$ for all $a,b,c\in\mathcal{S}$.Though semiring is a generalization of ring,ideals of semiring do not coincide with ring ideals . For example an ideal of semiring need not be the kernel of some semiring homomorphism. Semiring is very useful for solving problems in applied mathematics and information science because semiring provides an algebraic frame work for modling. Semiring play an important role in studing matrices and determinants. Molodtsov [8] introduced the concept of soft set theory as anw mathematical tool for dealing with uncertainties. Maji t al [7] extended soft set theory to fuzzy soft set theory. Feng et al [3] initiated the study of soft semirings.soft rings are defined by Acar et al [1] and JayanthGhosh et al [1] initiated the study of fuzzy soft rings and fuzzy soft ideals. Here introduce the notion of fuzzy soft semirings, Fuzzy soft ideals, Fuzzy soft R- ideals and R-fuzzy ideals over semiring and study some of their algebraic properties. We introduce the notion of fuzzy soft semiring homomorphism and study some properties of homomorphicimae of fuzzy soft semiring.

2. PRELIMINARIES

2.1 DefinitionA set $\mathcal{S}$ together with two associative binary operations called addition and mulitiplication (denoted by $+$ and $\cdot$ respectively) will called a semiring provided

(i) addition is a commutative operation.

(ii) mulitiplication distributes over addition both from the left and from the right.

(iii) there exists $0 \in \mathcal{S}$ such that $x + 0 = x$ and $x \cdot 0 = 0 \cdot x = 0$ for all $x \in \mathcal{S}$.

2.2Definition Let$\mathcal{S}$ be a semiring. A fuzzy subset μ of $\mathcal{S}$ is said to be fuzzy subsemiring of $\mathcal{S}$ if it satisfies the following conditions

(i) $\mu(x + y) \geq \min\{\mu(x), \mu(y)\}$

(ii) $\mu(xy) \geq \min\{\mu(x), \mu(y)\}$for all $x, y \in \mathcal{S}$.

2.3 DefinitionA fuzzy subset μ of semiring $\mathcal{S}$ is called a fuzzy left(right) ideal of $\mathcal{S}$ if for all $x, y \in \mathcal{S}$

(i) $\mu(x + y) \geq \min\{\mu(x), \mu(y)\}$

(ii) $\mu(xy) \geq \mu(x)(\mu(x))$

2.4DefinitionA fuzzy subset μ of semiring $\mathcal{S}$ is called a fuzzy ideal of $\mathcal{S}$ if for all $x, y \in \mathcal{S}$

(i) $\mu(x + y) \geq \min\{\mu(x), \mu(y)\}$

(ii)$\mu(xy) \geq max\{\mu(x), \mu(y)\}$

3 FUZZY SOFT SEMIRING AND FUZZY SOFT IDEAL OVER SEMIRING

In this section, the concepts of soft semiring, fuzzy soft semiring and fuzzy soft ideal over semiring are introduced an study the properties related to these notions.

3.1 Definition Let$(f, A), (g, B)$ be fuzzy soft sets over U. The intersection of fuzzy soft sets over $(f, A) \cap (g, B) = (h, C)$. Where $C = A \cap B$ is defind as

$$h_c = f_c \cap g_c, \text{if} C \in A \cap B$$

3.2Definition Let$(f, A), (g, B)$ be fuzzy soft sets over U. The union of soft sets (f, A) and (g, B) is denoted by

$(f, A) \cup (g, B) = (h, C)$. Where $C = A \cup B$and it is defined as

$$h_c = \begin{cases} f_c & if\ C \in A \setminus B\ ; \\ g_C & if\ C \in B \setminus A; \\ f_c \cup g_c & if\ C \in A \cap B\ ; \end{cases}$$

3.3Definition Let$(f, A), (g, B)$ be fuzzy soft sets over U."$(f, A)\ OR\ (g, B)$" is denoted by $(f, A) \vee (g, B)$ and it is defined by

$(f, A) \vee (g, B) = (h, C)$. Where$C = A \times B$ and $h_c(x) = \max\{f_a(x), g_b(x)\}$ for all $C = (a, b) \in A \times B, x \in U$.

3.4Definition Let$(f, A), (g, B)$ be fuzzy soft ideals over semiring $\mathcal{S}$. The product (f, A) and (g, B) is defined as$((f \circ g), C)$ where $C = A \cup B$ and

$$f \circ g = \begin{cases} f_c(x), & if\ c \in A \backslash B; \\ g_c(x) & if\ c \in B \backslash A; \\ sup_{x=ab}\{min\{f_c(a), g_c(b)\}\}, & if\ c \in A \cap B. \end{cases}$$

For all $c \in A \cup B$ and $x \in \mathcal{S}$.

3.5 Definition Let (f, A) be a fuzzy soft semiring over $\mathcal{S}$. A non null fuzzy soft set (g, B) over $\mathcal{S}$ is called a fuzzy soft ideal of (f, A) if it satisfies the following conditions

(i) (g, B) is a fuzzy soft subsemiring of (f, A)

(ii) (g, B) is a fuzzy soft ideal over semiring $\mathcal{S}$.

3.6 Theorem Let (f, A) and (g, B) be fuzzy soft semirings over semiring $\mathcal{S}$. Then $(f, A) \vee (g, B)$ is fuzzy soft semiring over semiring $\mathcal{S}$.

Proof

Let (f, A) and (g, B) be fuzzy soft semirings over semiring $\mathcal{S}$.

By Definition 3.3, $(f, A) \vee (g, B) = (h, C)$ where $C = A \times B$. let $c = (a, b) \in C = A \times B$ and $x, y \in \mathcal{S}$. Then

$$h_c(x + y) = max\{f_a(x + y), g_b(x + y)\}$$

$$\leq \max\{\max\{f_a(x), f_a(y)\}, \max\{g_b(x), g_b(y)\}\}$$

$$= \max\{\max\{f_a(x), g_b(x)\}, \max\{f_a(y), g_b(y)\}\}$$

$$= \max\{h_c(x), h_c(y)\}$$

$$h_c(xy) \quad = \max\{f_a(xy), g_b(xy)\}$$

$$\leq \max\{\max\{f_a(x), f_b(y)\}, \max\{g_b(x), g_b(y)\}\}$$

$$= \max\{\max\{f_a(x), g_b(x)\}, \max\{f_a(y), g_b(y)\}\}$$

$$= \max\{h_c(x), h_c(y)\}$$

Hence h_c is a fuzzy subsemiring of $\mathcal{S}$. Therefore $(f, A) \vee (g, B)$ is a fuzzy soft semiring over semiring $\mathcal{S}$.

3.7 Theorem If (f, A) is a fuzzy soft semiring over semiring $\mathcal{S}$ and for each $\tilde{a} \in A, f_{\tilde{a}}^{+}$ is defined by $f_{\tilde{a}}^{+}(x) = f_{\tilde{a}}(x) + 1 - f_{\tilde{a}}(0)$ for all $x \in \mathcal{S}$ then (f^{+}, A) is a normal fuzzy soft semiring over $\mathcal{S}, i.e., f_{\tilde{a}}^{+}$ is a normal fuzzy semiring over $\mathcal{S}$ for all $\tilde{a} \in A$ and (f, A) is a subset of (f^{+}, A).

Proof

Let (f, A) be a fuzzy soft semiring over semiring $\mathcal{S}$. For each $\tilde{a} \in A, f_{\tilde{a}}^{+}$ is defined by $f_{\tilde{a}}^{+}(x) = f_{\tilde{a}}(x) + 1 - f_{\tilde{a}}(0)$ for all $x \in \mathcal{S}$. Suppose $x, y \in \mathcal{S}$ and $\tilde{a} \in A$.

Then

$$f_{\tilde{a}}^{+}(x + y) = f_{\tilde{a}}(x + y) + 1 - f_{\tilde{a}}(0)$$
$$\geq \min\{f_{\tilde{a}}(x), f_{\tilde{a}}(y)\} + 1 - f_{\tilde{a}}(0)$$
$$= \min\{f_{\tilde{a}}(x) + 1 - f_{\tilde{a}}(0), f_{\tilde{a}}(y) + 1 - f_{\tilde{a}}(0)\}$$
$$= \min\{f_{\tilde{a}}^{+}(x), f_{\tilde{a}}^{+}(y)\}$$

$$f_{\tilde{a}}^{+}(xy) = f_{\tilde{a}}(xy) + 1 - f_{\tilde{a}}(0)$$
$$\geq \max\{f_{\tilde{a}}(x), f_{\tilde{a}}(y)\} + 1 - f_{\tilde{a}}(0)$$
$$= \max\{f_{\tilde{a}}(x) + 1 - f_{\tilde{a}}(0), f_{\tilde{a}}(y) + 1 - f_{\tilde{a}}(0)\}$$
$$= \max\{f_{\tilde{a}}^{+}(x), f_{\tilde{a}}^{+}(y)\}$$

If $x = 0$, then $f_{\tilde{a}}^{+}(0) = 1$ and $f_{\tilde{a}} \subset f_{\tilde{a}}^{+}$. Hence (f^{+}, A) is a normal fuzzy soft semiring over semiring $\mathcal{S}$ and (f, A) is asubset of (f^{+}, A).

3.8Theorem Let(f, A) and (g, B) be two fuzzy soft ideals over semiring $\mathcal{S}$. Then $(f, A) \cup (g, B)$ is a fuzzy soft ideal over semiring $\mathcal{S}$.

Proof

Let (f, A) and (g, B) be two fuzy soft ideals over semiring $\mathcal{S}$.
By definition 3.3we have $(f, A) \cup (g, B) = (h, C)$,
Where $C = A \cup B$

$$h_c = \begin{cases} f_c if C \in A \setminus B; \\ g_c \quad if\ C \in B \setminus A; \\ f_c \cup g_c \quad if\ C \in A \cap B. \end{cases}$$

Case (i)

If $C \in A \setminus B$ then $h_c = f_c, h_c$ is a fuzzy ideal of $\mathcal{S}$. Since (f, A) is a fuzzy soft ideal over semiring $\mathcal{S}$.

Case (ii)

If $C \in B \setminus A$ then $h_c = g_c, h_c$ is a fuzzy ideal of $\mathcal{S}$.Since(g, B) is a fuzzy soft ideal over $\mathcal{S}$.

Case(iii)

If $C \in A \cap B$ then for all $x, y \in \mathcal{S}$,

$$h_c(x + y) = f_c \cup g_c(x + y)$$
$$= \max\{f_c(x + y), g_c(x + y)\}$$
$$\geq \max\{\min\{f_c(x), f_c(y)\}, \min\{g_c(x), g_c(y)\}\}$$
$$= \min\{\max\{f_c(x), g_c(x)\}, \max\{f_c(y), g_c(y)\}\}$$
$$= \max\{(f \cup g)_c(x), (f \cup g)_c(y)\}$$
$$h_c(xy) = (f_c \cup g_c)(xy)$$
$$= \max\{f_c(xy), g_c(xy)\}$$
$$\geq \max\{\max\{f_c(x), f_c(y)\}, \max\{g_c(x), g_c(y)\}\}$$
$$= \max\{max\{f_c(x), g_c(x)\}, \max\{f_c(y), g_c(y)\}\}$$

Hence h_c is a fuzzy ideal of $\mathcal{S}$.

Therefore (h, c) is a fuzzy soft idealover semiring $\mathcal{S}$.

3.9 Theorem Let(f, A) and (g, B) be fuzzy soft ideals over semiring $\mathcal{S}$. Then $(f, A) \vee (g, B)$ is a fuzzy soft ideal over semiring $\mathcal{S}$.

Proof

Let (f, A) and (g, B) be two fuzzy soft ideals over semiring $\mathcal{S}$.
By Definition 3.3 $(f, A) \vee (g, B) = (h, C)$. Where $C = A \times B$,
let$c = (a, b) \in C = A \times B$ and $x, y \in \mathcal{S}$. Then

$$h_c(x + y) = f_a \vee g_b(x + y)$$
$$= \max\{f_a(x + y), g_b(x + y)\}$$
$$\leq \max\{\max\{f_a(x), f_a(y)\}, \max\{g_b(x), g_b(y)\}\}$$
$$= \max\{\max\{f_a(x), g_b(x)\}, \max\{f_a(y), g_b(y)\}\}$$
$$= \max\{h_c(x), h_c(y)\}$$

$$h_c(xy) \quad = f_a \vee g_b(xy)$$
$$= \max\{f_a(xy), g_b(xy)\}$$

$$\leq \max\{\min\{f_a(x), f_a(y)\}, \min\{g_b(x), g_b(y)\}\}$$
$$= \min\{\max\{f_a(x), g_b(x)\}, \max\{f_a(y), g_b(y)\}\}$$
$$= \min\{h_c(x), h_c(y)\}$$

Hence h_c is a fuzzy soft ideal of $\mathcal{S}$. Therefore $(h, A \times B)$ is a fuzzy soft ideal over $\mathcal{S}$.

3.10 Theorem If(f, A) and (g, B) are fuzzy soft ideals over semiring $\mathcal{S}$ with an identity element then $(f \circ g, A \cup B)$ is a fuzzy soft ideal over semiring $\mathcal{S}$.

Proof

Let (f, A) and (g, B) be fuzzy soft ideals over semiring $\mathcal{S}$. Then for any $C = A \cup B$ and $x, y \in \mathcal{S}$

Case (i)

If $C \in A \setminus B$ then by definition 3.4$(f \circ g)_c = f_c$

Which is a fuzzy ideal of$\mathcal{S}$.

Case (ii)

If $C \in B \setminus A$ then by definition 3.4$(f \circ g)_c = g_c$

Which is a fuzzy ideal of$\mathcal{S}$.

Case (iii)

If $C \in A \cap B$ and $x, y \in \mathcal{S}$ then by definition 1.7

$$(f \circ g)_c(y) = sup_{y=ab}\min\{f_c(a), g_c(b)\}$$
$$\leq sup_{xy=xab}\min\{f_c(a), g_c(b)\}$$
$$= sup_{xy=lm}\min\{f_c(l), g_c(m)\}$$
$$= (f \circ g)_c(xy)$$

Therefore$(f \circ g)_c(xy) \geq (f \circ g)_c(y)$

Similarly we can prove that $(f \circ g)_c(xy) \geq (f \circ g)_c(x)$ and

$$(f \circ g)_c(xy) \geq \min\{(f \circ g)_c(x), (f \circ g)_c(y)\}$$

Let e be an identity element of $\mathcal{S}$.

Then

$$(f \circ g)_c(x + y) = sup_{(x+y)e}\min\{f_c(x + y), g_c(e)\}$$
$$\geq sup_{(x+y)e}\min\{\min\{f_c(x), f_c(y), g_c(e)\}\}$$
$$= sup_{(x+y)e}\min\{\min\{f_c(x), g_c(e)\}, \min\{f_c(y), g_c(e)\}$$
$$= \min\{sup_{xe}\{\min\{f_c(x), g_c(e)\}\}, sup_{ye}\min\{f_c(y), g_c(e)\}\}$$
$$= \min\{(f \circ g)_c(x), (f \circ g)_c(y)\}$$

Therefore $(f \circ g)_c$is a fuzzy ideal of $\mathcal{S}$.Hence $(f \circ g, C)$, where $C = A \cup B$ is a fuzzy soft ideal over semiring $\mathcal{S}$.

4. FUZZY SOFT R-IDEAL AND R-FUZZY SOFT IDEAL

In this section the concept of fuzzy soft R- ideal, R-fuzzy soft ideal and fuzzy soft ideal of a fuzzy soft semiring is introduced and study the properties related to them.

4.1 Definition Let (f, A) be a fuzzy-soft ideal over semiring $\mathcal{S}$. If f_a is a fuzzy R-ideal of semiring $\mathcal{S}$, for all $a \in A$ then (f, A) is said to be fuzzy soft R-ideal over semiring $\mathcal{S}$.

4.2 DefinitionA fuzzy soft ideal (f, A) over semiring $\mathcal{S}$ is said to be R-ideal over semiring $\mathcal{S}$ if f_a is a R-fuzzy ideal of semiring $\mathcal{S}$, for all $a \in A$.

4.3 Theorem Let(f, A) and (g, B) be two fuzzy soft R-ideals over semiring $\mathcal{S}$. Then $(f, A) \cap (g, B)$ is a fuzzy soft R-ideal if it is non null.

Proof

Let (f, A) and (g, B) be two fuzzy soft R-ideals over semiring

By Theorem 3.2,$(f, A) \cap (g, B)$ is a fuzzy soft ideal over$\mathcal{S}$.

Let $(f, A) \cap (g, B) = (h, C)$

Where $C = A \cap B$. if $c \in A \cap B$ then $h_c = f_c \cap g_c$and

$$h_c(x) = (f_c \cap g_c)(x)$$
$$= \min\{f_c(x), g_c(x)\}$$
$$\geq \min\{\min\{f_c(x+y), f_c(y)\}, \min\{g_c(x+y), g_c(y)\}\}$$
$$= \min\{\min\{f_c(x+y), g_c(x+y)\}, \min\{f_c(y), g_c(y)\}\}$$
$$= \min\{f_c \cap g_c(x+y), f_c \cap g_c(y)\} \text{for all } x, y \in \mathcal{S}.$$

Hence $f_c \cap g_c$ is a fuzzy R-ideal of $\mathcal{S}$.

Therefore $(f, A) \cap (g, B)$ is a fuzzy soft R-ideal over semiring $\mathcal{S}$.

4.4 Theorem Every fuzzy soft R-ideal over semiring is a R-fuzzy soft ideal over semiring.

Proof

Let (f, A) be a fuzzy soft R-ideal over semiring $\mathcal{S}$ and $\tilde{a} \in A$. Then $f_{\tilde{a}}$ is a fuzzy R-ideal since (f, A) is fuzzy soft R-ideal. Let $x, y \in \mathcal{S}$ such that

$f_{\tilde{a}}(x+y) = f_{\tilde{a}}(0)andf_{\tilde{a}}(y) = f_{\tilde{a}}(0)$. Then

$$f_{\tilde{a}}(x) \geq \min\{f_{\tilde{a}}(x+y), f_{\tilde{a}}(y)\}$$
$$\geq \min\{f_{\tilde{a}}(0), f_{\tilde{a}}(0)\}$$
$$= f_{\tilde{a}}(0)$$

Therefore $f_{\tilde{a}}(x) \geq f_{\tilde{a}}(0)$. Also, we have $f_{\tilde{a}}(0) \geq f_{\tilde{a}}(x)$.

Hence $f_{\tilde{a}}(x) = f_{\tilde{a}}(0)$. By definition of R-ideals, $f_{\tilde{a}}$ is a R-ideal of $\mathcal{S}$.

Hence (f, A) is a R-fuzzy soft ideal over semiring $\mathcal{S}$.

4.5 Theorem Let (f, A) and (g, B) be two fuzzy soft ideals of a fuzzy soft semiring (h, C) over semiring $\mathcal{S}$.Then$(f, A) \cup (g, B)$ is a fuzzy soft ideal of (h, C) if it is non null.

Proof

Let (f, A) and (g, B) be two fuzzy soft ideals of fuzzy soft semiring (h, C) over semiring $\mathcal{S}$.

By theorem 3.6,$(f, A) \cup (g, B) = (h, C)$

Where $C = A \cup B \ \forall x, y \in \mathcal{S}$ then $h_c = f_c \cup g_c$

$h_c(x+y) = f_c \cup g_c(x+y)$

$$h_c(x+y) = \max\{f_c(x+y), g_c(x+y)\}$$
$$\leq \max\{\min\{f_c(x), f_c(y)\}, \min\{g_c(x), g_c(y)\}\}$$
$$= \min\{\max\{f_c(x), g_c(x)\}, \max\{f_c(y), g_c(y)\}\}$$
$$= \max\{(f \cup g)_c(x), (f \cup g)_c(y)$$
$$h_c(xy) = (f_c \cup g_c)(xy)$$
$$= \max\{f_c(xy), g_c(xy)\}$$
$$\leq \max\{\min\{f_c(x), f_c(y)\}, \min\{g_c(x), g_c(y)\}\}$$
$$= \min\{\max\{f_c(x), g_c(x)\}, \max\{f_c(y), g_c(y)\}\}$$
$$= \min\{f_c \cap g_c(x), f_c \cap g_c(y)\} \qquad = \min\{h_c(x), g_c(y)\}$$

Hence h_c is a fuzzy ideal of $\mathcal{S}$. Thus $(f, A) \cup (g, B)$ is a fuzzy soft ideal over semiring $\mathcal{S}$.

4.6 Theorem Let(f, A) and (g, B) be fuzzy soft semirings over semiring $\mathcal{S}$, (f_1, C) and (g_1, D) be fuzzy soft ideals of (f, A) and (g, B) respectively. Then $(f_1$,C)$\cap (g_1, D)$ is a fuzzy soft ideal of $(f, A) \cap (g, B)$ if it is non null.

Proof

Let (f_1, C) and (g_1, D) be fuzzy soft ideals of (f, A) and (g, B) respectively.

Since (f_1, C) and (g_1, D) are fuzzy soft ideals of (f, A) and (g, B) respectively, we have Definition

$(f, C)$and(g, D) are fuzzy soft ideals over semiring $\mathcal{S}$.

By Theorem

Let (f, A) and (g, B) be two fuzzy soft ideals over semiring $\mathcal{S}$.Then$(f, A) \cap (g, B)$ is a fuzzy soft ideal over semiring $\mathcal{S}$.

$(f_1, C) \cap (g_1(D)$is a fuzzy soft ideal over semiring $\mathcal{S}$. and$(f, A) \cap (g, B)$ is a fuzzy soft semiring over semiring $\mathcal{S}$.

Hence $(f_1, C) \cap (g_1, D)$ is a fuzzy soft ideal of $(f, A) \cap (g, B)$.

5FUZZY SOFT SEMIRING HOMOMORPHISM

In this section, the concept of fuzzy soft semiring homomorphism is introduced and study their properties.

5.1 Definition Let (ϕ, ψ) be a fuzzy soft function from R to $\mathcal{S}$. The pre image of a fuzzy soft set (g, B) under the fuzzy soft function (ϕ, ψ) is denoted by $(\phi, \psi)^{-1}(g, B)$ and it is the fuzzy soft set defined by

$$(\phi, \psi)^{-1}(g, B) = (\phi^{-1}(g), \psi^{-1}(B)$$

5.2 Definition Let (f, A) and (g, B) be fuzzy soft semirings over semirings R and $\mathcal{S}$ respectively and (ϕ, ψ) be fuzzy soft function from R to $\mathcal{S}$. Then homomorphism if the following conditions hold.

(i) $\phi$is a homomorphism from R onto $\mathcal{S}$.

ii) $\psi$is a mapping from A to B

iii) $\phi(f_a) = g_\psi(a)$for all $a \in A$

5.3 Theorem Let (f, A) and (g, B) be fuzzy soft semirings over semirings$R$ and $\mathcal{S}$ respectively, and (ϕ, ψ) be a fuzzy soft semiring homomorphism from (f, A) onto (g, B). Then $(\phi(f), B)$ is a fuzzy soft semiring over $\mathcal{S}$.

Proof

Let (ϕ, ψ) be a fuzzy soft semiring homomorphism from (f, A) onto (g, B).

By Definition 5.2,ϕ is a homomorphism from R onto $\mathcal{S}$ and ψ is a mapping from A onto B.For each $b \in B$ there exists $a \in A$ such that $\psi(a) = b$.

Define $[(f)]_b = \phi f_a$.

Let $y_1, y_2 \in \mathcal{S}$. Then there exist $x_1, x_2 \in R$ such that $\phi(x_1) = y_{1,}$ $\phi(x_2) = y_2$ and $\phi(x_1 + x_2) = y_1 + y_2$ and $\phi(x_1 x_2) = y_1 y_2$.

Now$[\phi(f)]_{\psi(a)}(y_1 + y_2) = \phi(f_a)(y_1 + y_2)$

$$= f_a[x_1 + x_2]$$
$$\geq \min\{f_a(x_1), f_a(x_2)\}$$
$$= \min\{\phi(f_a(x_1), f_a(x_2)\}$$
$$= \min\{\phi(f_a)(y_1), \phi(f_a)(y_2)\}$$
$$= \min\{\phi(f)_{\psi(a)}, \phi(f)_{\psi(a)}(y_2)\}$$

$$[\phi(f)]_{\psi(a)}(y_1y_2) = \phi(f_a)(y_1y_2)$$
$$= f_a(x_1x_2)$$
$$\geq \min\{f_a(x_1), f_a(x_2)\}$$
$$= \min\{\phi(f_a)\,(y_1), \phi(f)_{\psi(a)}(y_2)\}$$

Therefore $\phi(f_b)$ is a fuzzy subsemiring of $\mathcal{S}$. hence $(\phi(f), B)$ is a fuzzy soft semiring over $\mathcal{S}$.

5.4 Definition Let$\mathbb{S}$and $\mathcal{T}$be two sets and$\Phi: \mathbb{S} \rightarrow \mathcal{T}$be any function.A fuzzy subset μ of $\mathbb{S}$is called a Φinvariant if $\Phi(x) = \Phi(y) \Rightarrow \mu(x) = \mu(y)$.

5.5 Theorem Let$\mathbb{R}$ and $\mathbb{S}$ be semirings, ϕ: $\mathbb{R} \rightarrow \mathbb{S}$ be a homomorphism and f be a ϕ invariant fuzzy subset of $\mathbb{R}$. If $x = \phi(a)$then $\phi(f)(x) = f(a), a \in \mathbb{R}$.

Proof

Let $\mathbb{R}$ and $\mathbb{S}$ be semirings, ϕ: $\mathbb{R} \rightarrow \mathbb{S}$ be a homomorphism and f be a ϕ invariant fuzzy subset of $\mathbb{R}$.Suppose $a \in \mathbb{R}$and $\phi(a) = x$.

Then $\phi^{-1}(x) = a$. Let $t \in \phi^{-1}(x)$. Then $\phi(t) = x = \phi(a)$. Since f is a ϕ invariant fuzzy subset of $\mathbb{R}$, $f(t) = f(a)$.

Therefore $\phi(f)(x) = sup_{t\in\phi^{-1}(x)} f(t) = f(a)$and hence$\phi(f)(x) = f(a)$.

5.6 Theorem

Let (α, A) be a fuzzy soft left ideal over semiring R.and ϕ be a homomorphism from semiring R onto semiring S.For each $c \in A, \alpha_c$ is a ϕ invariant fuzzy left ideal of R. If $\beta_c = \phi(\alpha_c), c \in A$ then (β, A) is a fuzzy soft left ideal over semiring S.

Proof

Let (α, A) be a fuzzy soft left ideals over semiring R. ϕ be a homomorphism from semiring R onto semiring $S, x, y \in S$and $c \in A$.

Then there exists $a, b \in R$ such that $\phi(a) = x, \phi(b) = y, x + y = \phi(a + b), xy = \phi(ab)$. Since α_c is ϕ invariant and we have

$$\beta_c(x + y) = \phi(\alpha_c)(x + y)$$

$$= \alpha_c(a + b)$$

$$\geq \min\{\alpha_c(a), \alpha_c(b)\}$$

$$= \min\{\phi(\alpha_c)(x), \phi(\alpha_c)(y)\}$$

$$= \min\{\beta_c(x), \beta_c(y)\}$$

$$\beta_c(xy) \qquad = \phi(\alpha_c)(xy)$$

$$= \alpha_c(\phi(ab))$$

$$= \alpha_c[\phi(a)\phi(b)]$$

$$\geq \alpha_c(\phi(b))$$

$$= \phi(\alpha_c)(y)$$

$$= \beta_c(y)$$

Hence β_c is a left ideal of S.

Therefore (β, A) is a fuzzy soft left ideal over semiring S.

2.10 Theorem Let (α, A) be a fuzzy soft semiring over S, θ be an endomorphism of S and define $(\alpha\theta)_a = \alpha_a\theta$ for each $a \in A$.Then $(\alpha\theta, A)$ is a fuzzy soft semiring over semiring S.

Proof

Let $x, y \in S$, $a \in A$.Then

$$(\alpha\theta)_a(x+y) = \alpha_a(\theta(x+y))$$

$$= \alpha_a[\theta(x) + \theta(y)]$$

$$\geq \min\{\alpha_a(\theta(x)), \alpha_a(\theta(y))\}$$

$$= \min\{(\alpha\theta)_a(x), (\alpha\theta)_a(y)\}$$

$$(\alpha\theta)_a(xy) \quad = \alpha_a(\theta(xy))$$

$$= \alpha_a[\theta(x)\theta(y)]$$

$$\geq \min\{\alpha_a(\theta(x)), \alpha_a(\theta(y))\}$$

$$= \min\{(\alpha\theta)_a(x), (\alpha\theta)_a(y)\}$$

Hence $(\alpha\theta)_a$ is fuzzy subsemiring of S.Therefore $(\alpha\theta, A)$ is a fuzzy soft semiring over semiring S.

6 CONCLUSION

In this paper, The introduced concepts of R-fuzzy soft ideal over semiring and fuzzy soft semiringhomomorphism,fuzzy soft ideals and fuzzy R-ideals and also is studied properties of R-fuzzy soft ideals and homomorphic image of fuzzy soft semiring.Our future work of this paper we shall study prime ideals over semiring.

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