A REVIEW ON (gg)*- CLOSED SETS, SD CLOSED SET, SD OPEN SET AND APPLICATION OF TOPOLOGY

Mukesh Lochab[1]

[1]*Assistant Professor, Dept. of Mathematics, SWIET,Ajmer Rajasthan.,India*

Abstract

In this review paper, we provide a introduction about generalization of generalized star closed sets (briefly (gg) - closed), SD –closed set and SD open sets by utilizing generalized closed sets and regular b-closed set.we also discuss about thiere properties. Also, we would like to discuss the applications of topology.*

KEY WORDS: *(gg)* - closed set, SD closed set, SD open set,DNA replication, biomathematics.*

INTRODUCTION

The concept of generalized closed sets in Topological spaces was introduced by N. Levine in 1970. D. E. Cameron and M. Stone introduced regular semi open sets and regular open sets respectively. In 2017, Basavaraj M. Ittanagi and Govardhana Reddy introduced and studied generalization of generalized closed sets in Topological spaces.

In this review paper we give a brief review on (gg)*-closed set which introduce by Christal Bai and T.Shyla Isac Mary .Also discuss about SD-closed and SD- open set which introduced by S.Divya priya and K.Amutha . Also, we would like to discuss the applications of topology in industries through different areas of sciences such as Biology, Chemistry, Physics, Computer Science, Business Economics and Engineering.
Throughout this paper, a space means a topological space on which no separation axioms are assumed unless otherwise mentioned. For a subset A of a space (X, τ), Cl(A) and Int(A) denote the closure of A and interior of A, respectively.X -A or A^c denotes the complement of A in X .

PRELIMINARIES

Definition.A subset A of a topological space (X, τ) is called a

(1) generalized- closed set (briefly g - closed) if cl (A) $\subseteq$ U whenever A $\subseteq$ U and U is open in X .

(2) regular semi open if there is a regular open set U such that U $\subseteq$ A $\subseteq$ cl (U).

(3) regular open set if $A = int(cl(A))$ and a regular closed set if $cl(int(A)) = A$.

(4) generalization of generalized closed set (briefly gg-closed) if gcl (A) $\subseteq$ U whenever A $\subseteq$ U and U is regular semi - open in X.

(5) semi - open set if A $\subseteq$ $cl(int(A))$ and a semi - closed set if $int(cl(A))$ $\subseteq$ A.

(6) pre - open set if A $\subseteq$ $int(cl(A))$ and pre - closed if $cl(int(A))$ $\subseteq$ A.

(7) semi - pre open set if A $\subseteq$ $cl(int(cl(A)))$ and semi pre - closed if $int(cl(int(A)))$ $\subseteq$ A.

(8) β-open set if A $\subseteq$ $cl(int(cl(A)))$, whenever A ⊆ U and U is open in X.

(9) α- open set if $A \subseteq int(cl(int(A)))$ and α - closed set if $cl(int(cl(A))) \subseteq A$.

(10) t - set iff $int(A) = int(cl(A))$.

(11) generalized semi - pre closed (briefly gsp - closed) if $spcl(A) \subseteq U$ whenever $A \subseteq U$ and U is open in X.

(12) generalized pre - closed set (briefly gp - closed) if $pcl(A) \subseteq U$ whenever $A \subseteq U$ and U is open in X.

(13) generalized semi - closed set (briefly gs - closed) if $scl(A) \subseteq U$ whenever $A \subseteq U$ and U is open in X.

(14) α -generalized closed set (briefly αg - closed) if $\alpha cl(A) \subseteq U$ whenever $A \subseteq U$ and U is open in X.

(15) regular generalized closed set (briefly rg - closed) if $cl(A) \subseteq U$ whenever $A \subseteq U$ and U is regular - open in X.

(16) generalized pre - regular closed set (briefly gpr - closed) if $pcl(A) \subseteq U$ whenever $A \subseteq U$ and U is regular open in X.

(17) generalized semi - pre regular - closed set (briefly gspr - closed) if $spcl(A) \subseteq U$ whenever $A \subseteq U$ and U is regular open in X.

(18) generalized star pre closed (briefly g*p - closed) if $pcl(A) \subseteq U$ whenever $A \subseteq U$ and U is g -open in X.

(19) weakly closed set (briefly w - closed) if $cl(A) \subseteq U$ whenever $A \subseteq U$ and U is semi- open in X.

(20) tgr - closed set if $rcl(A) \subseteq U$ whenever $A \subseteq U$ and U is a t -set.

(21) regular w-closed (briefly rw - closed) if $cl(A) \subseteq U$ whenever $A \subseteq U$ and U is regular semi - open in X.

(22) regular gereralized α - closed set (briefly rgα - closed) if $\alpha cl(A) \subseteq U$ whenever $A \subseteq U$ and U is regular α - open in X
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(23) gereralized α- closed set (briefly gα - closed) if $\alpha cl(A) \subseteq U$ whenever $A \subseteq U$ and U is α- open in X.

(24) Semi - generalized closed set (briefly sg - closed) if $scl(A) \subseteq U$ whenever $A \subseteq U$ and U is semi open in X.

(25) R*- closed set if $rcl(A) \subseteq U$ whenever $A \subseteq U$ and U is regular semi - open in X.

(26) $R^{\#}$ - closed set if $gcl(A) \subseteq U$ whenever $A \subseteq U$ and U is R* -open in X.

(27) βg* - closed set if $gcl(A) \subseteq U$ whenever $A \subseteq U$ and U is β - open in X.

(28) r ^ g - closed set if $gcl(A) \subseteq U$ whenever $A \subseteq U$ and U is regular - open in X.

(29) g**- closed set if $cl(A) \subseteq U$ whenever $A \subseteq U$ and U is g*- open in X.

(30) g* - closed set if $cl(A) \subseteq U$ whenever $A \subseteq U$ and U is g - open in X.

(31) generalized regular closed set (briefly gr - closed) if $rcl(A) \subseteq U$ whenever $A \subseteq U$ and U is open in X.

(32) generalized regular star closed (briefly gr *- closed) if *rcl*

(A) □ U whenever A □ U and U is g - open in X.

The complements of the above closed sets are their open sets and vice versa.

Definition. The regular closure of a subset A □ X is the set *rcl*(A) = ∩ { B □ X : B is regular closed and A □ B}

GENERALIZATION OF GENERALIZED STAR - CLOSED SETS

Definition. A subset A of a topological space (X, τ) is called generalization of generalized star closed sets (briefly (gg)*- closed) if *rcl* (A) □ U whenever A □ U and U is gg - open.

Example. Let X= {1,2,3,4}, and τ = {□ , { 3}, {4}, {3,4}, {1, 3, 4}, X}
gg-open = { □ , {1, 3, 4}, {3, 4}, {1, 4}, {1, 3}, {1, 2}, {4}, {2}, {3}, {1},X}
(gg)* - closed = { □ , {1, 2}, {2, 3}, {2, 4}, {1, 2, 3}, {1, 2, 4}, {2, 3, 4}, X}.

Proposition. Every regular closed set is (gg)* - closed.

Proof: Let A be a regular closed set in X such that A □ U and U is gg - open. Then *rcl*(A) = A. Hence *rcl* (A) □ U. Therefore A is (gg)*- closed.

Remark . The converse of the above proposition need not be true as shown in the following example.

Example. Let X = {1, 2, 3, 4}, and τ = { □ , {3}, {4}, {3, 4}, {1, 3, 4}, X} Then the set {1,2} is (gg)* - closed but not regular closed.

INDEPENDENCY OF (gg)*-CLOSED SETS WITH OTHER CLOSED SETS.

The following example shows that (gg)*-closed sets are independent from α-closed, regular semi -closed, gα -closed, rgα - closed, w -closed, rw- closed, sg - closed, pre - closed, g*s- closed, R* - closed, tgr - closed.

Example: Let X= {1, 2, 3, 4}, and τ = { □ , {3}, {4}, {3, 4}, {1, 3, 4}, X }.Then

(1) {2,3} is (gg)*- closed but not α-closed and {2} is α-closed but not (gg)*- closed.

(2) {2,3,4} is (gg)*- closed but not gα-closed and {1} is gα-closed but not (gg)*-

(3) {1,2} is (gg)*- closed but not regular semi - closed and {3} is regular semi -closed but not (g g)*- closed.

(4) {2,3} is (gg)*- closed but not rgα-closed and {1, 3, 4} is rgα-closed but not (gg)*- closed.

(5) {2,4} is (gg)*- closed but not w -closed and {2} is w -closed but not (gg)*- closed.

(6) {2,3} is (gg)*- closed but not rw - closed and {3, 4} is rw - closed but not (gg)*- closed.

(7) {2,3,4} is (gg)*- closed but not sg - closed and {1,4} is sg - closed but not (gg)*- closed.

(8) {2,3,4} is (gg)*- closed but not g*s - closed and {4} is g*s - closed but not (gg)*- closed.

(9) {1,2} is (gg)*- closed but not pre- closed and {3} is pre- closed but not (gg)*- closed.

(10) {2,4} is (gg)*- closed but not R* - closed and {1,3,4} is R* - closed but not (gg)*- closed.

(11) {2,3} is (gg)*- closed but not tgr- closed and {3, 4} is tgr- closed but not (gg)*- closed.

Remark:

From the above discussions and known results the relationship between (gg)*-closed sets and other existing generalizations of closed sets are implemented in Figure:

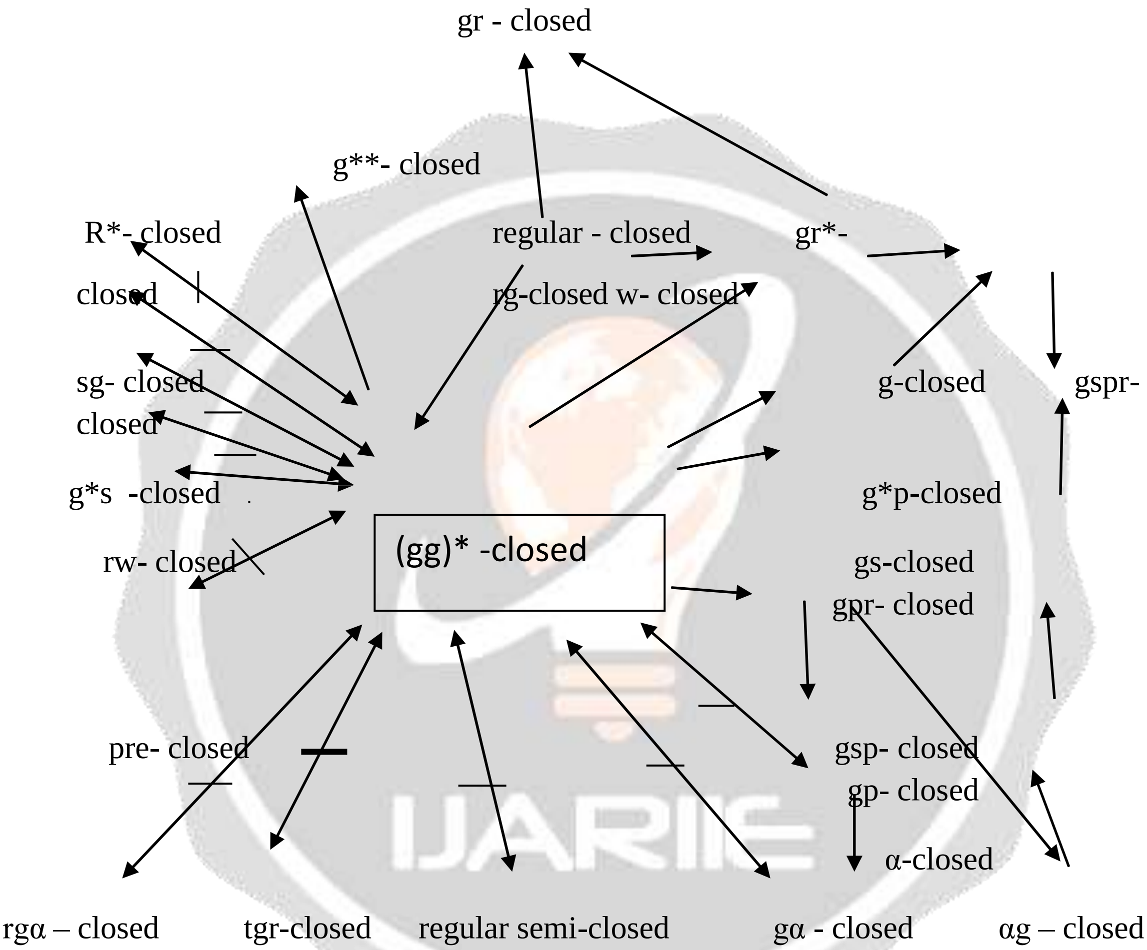

Figure:

In the above figure A B means the set A implies the set B but not conversely and

A B means the set A and B are independent of each other.

SD CLOSED SET:

Definition : A generalized closed (briefly g-closed) set if cl(A)□□U whenever A □ U and U is open in (X, τ) The complement of g-closed set is called g-open set.

Definition: A subset A of a space X is said to be regular b-closed (briefly rb-closed) if rcl (A)□□ U whenever A □ U and U is b-open in X. The complement of regular b-closed set is called regular b-open set.

Definition : A Set A is said to be SD closed set if

(i) A is generalized closed set and (ii) A is regular b-closed

Example:

Let X ={1,2,3}, τ = {□ ,X,{2}}, τ^C = {□,X,{1,3}} and U=X.

SD closed set = {{1},{2},{3},{1,2},{2,3},{3,1},□ ,X}.

Proposition: Let (X,

. Prove that if A is regular closed set then A is SD closed set in X. Converse is need not be true. (U=X).

Proof: Let A be a regular closed then A=Cl(Int(A)).

□ A is regular b-closed set. … (1)

Since every regular closed set is regular b-closed and also Every regular closed set is regular open set. Now Every regular open is open. Therefore Int (A) = A.

A = Cl(Int(A)) = Cl(A)

□ Cl(A) = A.

□ Cl(A) □ A whenever A □ U and U is open in X. Then A is generalized closed (g-closed). … (2)

From (1) and (2) A is SD closed set.

Converse :

Example:

Let X = {1,2,3}, τ □□{ □, X ,{2}} , τ^c □{ □, X ,{1,3}} and U □X

(i) Regular closed set = {{1},{2,3},□, X}

(ii) SD closed set = {{1},{2},{3},{1,2},{2,3},{3,1},□, X}

Here the element {3, 1} is in SD set but not in regular closed set.

PROPERTIES OF SD CLOSED SETS:

Let (X,□) be a topological space. Let A and B be the closed set ,

1.Int(SD closed set) □□Cl(SD closed set)

2.Int(SD closed set) □□Cl(SD closed set)

3. A □ B= □(A)

4. A □ B= □(A)

5. A □ B= A □ B

6. $(A □ B)^C$ = □

□□(A □ $B)^C$ = □

□□(A □ $B)^C$ = (A □ $B)^C$

9. $(A □ B)^C = A^C$ □□B^C

10. $(A □ B)^C = A^C$ □ B^C

REMARK: The above properties are true for SD open sets.

APPLICATIONS OF GENERAL TOPOLOGY

Here we would like to discuss in brief the use of general topology in industries through by some areas of sciences.

Application in BIOLOGY

In recent years, topologists have developed the discrete geometric language of knots to a fine mathematical art one of the most interesting new scientific application of topology is the use of knot theory in analysis of DNA experiments.One of the important issues in molecular biology in the 3-dimensional structure of proteins and DNA in solution in the Cell and the relationship between structure and functions. Generally, protein and DNA structures are determined by X-rays crystallization and the manipulation required preparing a specimen for electron microscope. The DNA molecules are long and thin and the packing of DNA molecules in the cell nucleus is very complex. The biological solution to this entanglement problem is the existence of enzymes, which convert DNA from one topological form to another and appear to have a preformed role in the central genetic events of DNA replication, recombination and transcription. The topological approach to enzymology aims to exploit knot theory directly to reveal the secrets of enzyme action. How recent results in 3-dimensional topology have proved to be of use in the description and quantization of the action of cellular enzymes on DNA is best described by D.W.Sumners in his research paper published in 1995.

Application in CHEMISTRY

As a natural continuation of classical knot theory, chemists have been trying to synthesize and measure molecules with topologically interesting structures. The idea of molecules made of linked rings as a realistic possibility, was discussed at least as early as 1912.The most important tools in the topological method of making chemical predictions are known as indices. They derive from algorithms of procedures for converting the topological structure of a molecule into a single characteristic number. For example, an index might involve adding together the total number of rings in a molecule, or a number of atoms that are connected to three or more other atoms. The topological method has found applications beyond the simple prediction of chemical properties. It has the potential to help in modeling the behavior of gases, liquids and solids and of both organic & inorganic species, in developing new anesthetics and psychoactive drugs, in predicting the degree to which various pollutants might spread in the environment and the harm they might do once they spread, in estimating the cancer causing potential of certain chemicals and even in developing in beer with a well balanced taste.

Application in Physics

According to Normal Howes – uniform structures are the most important constructs from the physicist's point of view. The importance of uniform spaces from the physicist's points of view is also well brought out by the proceedings of the Nashville Topological Conference .In fact; topology has intrigued particle physicists for a long time. Recall that Donaldson used the Yand Mills field equations of mathematical physics, themselves generalizations of Maxwell's equations to study in 4 – space, there by reversing tradition by applying methods from physics to the understanding of topology.

Application in Computer Science

Recent developments in topology are penetrating other fields is best illustrated by the topics discussed at an extra ordinary research conference which was held at Barkley in 1990 in honor of the great topologist Stephen Samale's 60th birthday.The proceeding were published with title "Form topology to Computation: Unity and diversity in mathematical sciences" edited by Hirsch, Marsden and Shub. There seems to not many examples of the use of topology in computer science, perhaps because it is not clear hot it is related to the fundamental questions. However, in recent years, there have been some interesting results. The problem of the minimal number of conditional statements in an algorithm, to solve a particular problem, seems particularly well suited for the topological approach.

Application in BusinessEconomics

Topology has had tremendous effect on developments in economics. The study of conflicts of interest between individuals is what makes economics interesting and mathematically complex. Indeed we now know that the space of all individual preferences, which define the individual optimization problems, is topologically nontrivial and that is topological complexity is responsible for the impossibility of treating several individual preferences as if they were one.Because it is not possible, in general, to define a single optimization problem. Because of the complexity arising from simultaneous optimization problems, economic differs from physics where many of the fundamental relations derive from a single optimization problem. The attempt to find solutions to conflicts among individual interests led to there different theories about how economics are organized and how theybehave.

Application in Engineering

Topology has also found applications in engineering. a problem of great importance to an electric industry, which had failed of solution by its own engineers, has been solved by using methods of set theoretic topology. In particular, Daniel R. Baker has established that topological techniques are used in several robotics applications. Topology has been applied to production and distribution problems as well.

CONCLUSION

The class of (gg)*-closed sets in topological spaces is defined using regular closure and gg-open sets. We have reviewed SD closed sets, SD-open sets and application of topology in various field.

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