A STUDY OF CERTAIN TOPOLOGICAL SPACES THROUGH IDEALS

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ABSTRACT

In this paper we are discussing a Study of Certain Topological Spaces through Ideals. Being so general, topological spaces are a central unifying notion and appear in virtually every branch of modern mathematics. The branch of mathematics that studies topological spaces in their own right is called point-set topology or general topology.

Keyword: - *Ideal, Spaces, Classic, Set, Operator etc.*

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1. INTRODUCTION

In topology and related branches of mathematics, a topological space may be defined as a set of points, along with a set of neighborhoods for each point, satisfying a set of axioms relating points and neighborhoods. The definition of a topological space relies only upon set theory and is the most general notion of a mathematical space that allows for the definition of concepts such as continuity, connectedness, and convergence.[1] Other spaces, such as manifolds and metric spaces, are specializations of topological spaces with extra structures or constraints. Being so general, topological spaces are a central unifying notion and appear in virtually every branch of modern mathematics. The branch of mathematics that studies topological spaces in their own right is called point-set topology or general topology.

The concept of ideals in general topological spaces is found in the classic text by Kuratowski [4] and also in [11]. A collection $I \subseteq P(X)$ is called an ideal on X if it satisfies the following two conditions: (i) $A \in I$ and $A \supseteq B \Rightarrow B \in I$, and (ii) $A \in I$, $B \in I \Rightarrow A$ S $B \in I$. A topological space (X, τ) with an ideal I on X is denoted by (X, τ, I), called an ideal topological space. For a subset A of X, an operator $(.) * : P(X) \to P(X)$ (where P(X) denotes the power set of X), called a local function[4] of A and denoted by $A*(I, \tau)$ or simply $A*$, is defined by the set $\{x \in X :$ U T A 6∈ I for every $U \in \tau(x)\}$, where $\tau(x) = \{U \in \tau : x \in U\}$. In [3,4], it was also shown that the operator $cl*(.)$, defined by $cl*(A) =$ A S $A*$, is a Kuratowski closure operator and hence generates a topology $\tau*(I)$ or simply $\tau*$ on X, called $*$-topology, finer than τ . The members of $\tau*$ are called $\tau*$ -open or simply $*$-open sets and the complement of a $*$-open set is called a $*$-closed set or equivalently, a subset A of X is called $*$-closed if $A* \subseteq A$. For a subset A of topological space (X, τ), H. Maki [6] introduced the following notations: $A\wedge =$ T $\{U : A \subseteq U$ and U is open $\}$ and $A\vee =$ S $\{F : F \subseteq A$ and F is closed $\}$. A subset A of X is said to be a $\wedge$-set ($\vee$-set) if $A = A\wedge$ (resp. $A = A\vee$). A subset A of X is said to be g-closed [5] if $cl(A) \subseteq U$ whenever $A \subseteq U$ and U is open in X; and the complement of a g-closed subset in X is called a g-open set in X. For further details regarding g-closed sets and similar such sets one may refer to [8–10]. A subset A of an ideal space (X, τ, I) is said to be $*$-g-closed [7] if $cl(A) \subseteq U$ whenever $A \subseteq U$ and U is $*$-open. A subset A of X is said to be $*$-g-open if $X \setminus A$ is $*$-g-closed. It was shown in [7] that the the class of $*$-g-closed sets lies strictly between the class of closed sets and the class of g-closed sets. It was also shown in the same paper that the class of g-closed sets in (X, τ) is not same as that of $*$-g-closed sets in an ideal topological space (X, τ, I)

Theorem. Let (X, τ, I) be an ideal topological space and $A \subseteq X$. If $A^{\wedge *}$ is $*$-g-closed then A is $*$-g-closed. **Proof.** Let $A^\wedge *$ be $*$-g-closed. Suppose that $A \subseteq U$, where U is $*$-open. Then $A\wedge * \subseteq U$. Now $A\wedge *$ is $*$-g-closed $\Rightarrow cl(A^\wedge{}_*) \subseteq U \Rightarrow cl(A) \subseteq cl(A^\wedge{}_*) \subseteq U \Rightarrow A$ is a $*$-g-closed set.

2. $*$-$T_{1/2}$ –SPACES

Dunham [2] defined a kind of separation axiom viz. T $_{1/2}$ -property in a topological space. It is shown in [2] that the class of T 1 2 -spaces lies between the classes of T0-spaces and T1-spaces. The intent of this section is to introduce a similar type of separation axiom, termed $*$-T $_{1/2}$ -property which is strictly stronger than T $_{1/2}$ -property, but is weaker than the T1-axiom. Such a separation axiom is characterized here in terms of the types of sets introduced in earlier sections. We begin by recalling the definition of T $_{1/2}$ -spaces as given in [2]

Definition.
A topological space (X, τ) is said to be a T $_{1/2}$ -space if every g-closed set is closed in X. Our proposed definition of $*$-T $_{1/2}$ -spaces goes as follows.

Definition. An ideal topological space (X, τ, I) is said to be a $*$-T $_{1/}$2 -space if every $*$-g-closed set is closed in X.
This axiomatization is due to Felix Hausdorff. Let X be a set; the elements of X are usually called points, though they can be any mathematical object. We allow X to be empty. Let N be a function assigning to each x (point) in X a non-empty collection N(x) of subsets of X. The elements of N(x) will be called neighbourhoods of x with respect to N (or, simply, neighbourhoods of x). The function N is called a neighbourhood topology if the axioms below[6] are satisfied; and then X with N is called a topological space.
If N is a neighbourhood of x (i.e., N ∈ N(x)), then x ∈ N. In other words, each point belongs to every one of its neighbourhoods.
If N is a subset of X and includes a neighbourhood of x, then N is a neighbourhood of x. I.e., every superset of a neighbourhood of a point x in X is again a neighbourhood of x.
The intersection of two neighbourhoods of x is a neighbourhood of x.
Any neighbourhood N of x includes a neighbourhood M of x such that N is a neighbourhood of each point of M.
The first three axioms for neighbourhoods have a clear meaning. The fourth axiom has a very important use in the structure of the theory, that of linking together the neighbourhoods of different points of X.
A standard example of such a system of neighbourhoods is for the real line R, where a subset N of R is defined to be a neighbourhood of a real number x if it includes an open interval containing x.
Given such a structure, a subset U of X is defined to be open if U is a neighbourhood of all points in U. The open sets then satisfy the axioms given below. Conversely, when given the open sets of a topological space, the neighbourhoods satisfying the above axioms can be recovered by defining N to be a neighbourhood of x if N includes an open set U such that x ∈ U.[7]

3. Examples

1. Given X = {1, 2, 3, 4}, the collection τ = {{}, {1, 2, 3, 4}} of only the two subsets of X required by the axioms forms a topology of X, the trivial topology (indiscrete topology).
2. Given X = {1, 2, 3, 4}, the collection τ = {{}, {2}, {1, 2}, {2, 3}, {1, 2, 3}, {1, 2, 3, 4}} of six subsets of X forms another topology of X.
3. Given X = {1, 2, 3, 4} and the collection τ = P(X) (the power set of X), (X, τ) is a topological space. τ is called the discrete topology.
4. Given X = Z, the set of integers, the collection τ of all finite subsets of the integers plus Z itself is not a topology, because (for example) the union of all finite sets not containing zero is infinite but is not all of Z, and so is not in τ.

4. EXAMPLES OF TOPOLOGICAL SPACES

A given set may have many different topologies. If a set is given a different topology, it is viewed as a different topological space. Any set can be given the discrete topology in which every subset is open. The only convergent sequences or nets in this topology are those that are eventually constant. Also, any set can be given the trivial topology (also called the indiscrete topology), in which only the empty set and the whole space are open. Every sequence and net in this topology converges to every point of the space. This example shows that in general topological spaces, limits of sequences need not be unique. However, often topological spaces must be Hausdorff spaces where limit points are unique.

5. METRIC SPACES

Metric spaces embody a metric, a precise notion of distance between points.

Every metric space can be given a metric topology, in which the basic open sets are open balls defined by the metric. This is the standard topology on any normed vector space. On a finite-dimensional vector space this topology is the same for all norms.

There are many ways of defining a topology on R, the set of real numbers. The standard topology on R is generated by the open intervals. The set of all open intervals forms a base or basis for the topology, meaning that every open set is a union of some collection of sets from the base. In particular, this means that a set is open if there exists an open interval of non zero radius about every point in the set. More generally, the Euclidean spaces Rn can be given a topology. In the usual topology on Rn the basic open sets are the open balls. Similarly, C, the set of complex numbers, and Cn have a standard topology in which the basic open sets are open balls.

7. CONNECTED IDEAL TOPOLOGICAL SPACES

In this section, we introduce the definitions of $\tau *$ -separated sets, $\tau *$ -connected sets in terms of cl∗ which is different from cl in general topology as $cl*(A) = A \cup A*$ and $A*$ is superset of Ad ∗ i.e. derived set of A in $\tau *$ -topology.

Definition 1. Let (X, τ, I) be an ideal topological space. Nonempty subsets A, B of X are called $\tau *$ -separated if $cl*(A) \cap B = \varphi = A \cap cl*(B)$.

Definition 2. Let (X, τ, I) be an ideal topological space. A space X is said to be $\tau *$ -connected if X cannot be expressed as the disjoint union of two nonempty $\tau *$ -open sets. Otherwise X is called $\tau *$ -disconnected, equivalently X is $\tau *$ -disconnected if $X = A \cup B$, where $cl*(A) \cap B = \varphi = A \cap cl*(B)$. If $I = \varphi$, then $\tau *$ -connected and connected coincide.

Remark. From definitions 1 and 2, we have the following implications, but none of the implications is reversible as shown in example 1. Separated ⇒ ∗-separated ⇒ $\tau *$ -separated, $\tau *$ -connected ⇒ ∗-connected ⇒ connected.

Theorem. Let (X, τ, I) be an ideal topological space. If A and B are non empty disjoint $\tau *$ -open sets, then A and B are $\tau *$ -separated.

Proof. Let $A \cap B = \varphi$ therefore $A \subset X - B$ implies $cl*(A) \subset cl*(X - B) = X-B$ implies $cl*(A) \cap B = \varphi$. Also $B \subset X-A$ and $cl*(B) \subset cl*(X-A) = X-A$ implies $cl*(B) \cap A = \varphi$. Hence the proof

Example 1. Let $X = \{a, b, c, d\}$ with topology $\tau = \{\varphi, X, \{a\}, \{c\}, \{a, c\}, \{a, b, d\}\}$ and $I = \{\varphi, \{a\}\}$. Let $A = \{b, d\}$ we have $A* = \{b, d\}$, $cl*(A) = \{b, d\}$, $cl(A) = \{b, d\}$ and for $B = \{a, c\}$ we have $B* = \{c\}$, $cl*(B) = \{a, c\}$, $cl(B) = X$. X is $\tau *$ -separated but neither ∗-separated nor separated. Also (X, τ) is connected but neither ∗-connected nor $\tau *$ -connected.

6. CONCLUSION

The relationship of certain objects that are inactive under a certain type of change, especially those properties that are immutable under a certain type of equivalent, and it is the study of those properties of geometric configurations that are one-to-one of these configurations. Remain unchanged when there is one. Bisexual transformations or homomorphisms. Topology operates with more general concepts than analysis. The differential properties of a given transformation are ineffective for topology, but bicopturity is necessary. As a result, the topology is often suitable for the resolution of problems, for which analysis cannot respond.

Although the concept of topology has been recognized as a difficult area in mathematics, we have taken it as a challenge and cherished this research study. Ideal topology is

Generalization of topology in classical mathematics, but also has its own distinct characteristics. It can further enhance the understanding of the basic structure of classical mathematics and provides new methods and results to achieve important results of classical mathematics. Apart from this it is also in some important areas of science and technology

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