# A STUDY ON SOFT SETS AND SOFT SUBLATTICES

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**ABSTRACT***This paper presents the concept of soft sets and soft sublattices. Also it discuss the related properties and theorems with examples.*

**Keywords -** *Soft Sets,Soft Lattices,Soft Sublattices*

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**INTRODUCTION**To solve complex problems in economy, engineering environmental science and social science , the methods in classical mathematics may not be succesfully modeled because of various types of uncertainities.There are some mathematical theories for dealing with uncertanities such as fuzzy soft theory,soft set theory [9],fuzzy soft set theory and so on.

Soft set theory was introduced by Molodstov [10] in 1999 as a mathematical tool for dealing with uncertainity. Maji[9] defined some operations on soft sets and proved related properties . Irfan Ali et[6] studied some new operations on soft setsLi[8],Nagarajan etal[11] defined the soft lattice using soft sets .Faruk Karaaslan et al [7] defined the concept of soft latttice over a collection of soft sets by using the operations of soft sets defined by cagman et al[2].In this paper, we define the concept of principle of duality in soft lattices and discuss some related propertiesof modular and distributive soft lattice .We also illustrate them with some examples.In addition, we establish characterization theorems for modular and distributive soft lattice by their soft sublattices.

In this paper we define concept of fuzzy soft lattice . we then study fuzzy soft sublattice , modular fuzzy soft lattice, distributive fuzzy soft lattice with examples.

## SOME CONCEPT IN SOFT SETS AND SOFT LATTICE

**DEFINITION 2.1** Afunction $f_A: E \rightarrow P(U)$ such that $f_A(x) = \emptyset$ if $x \notin A$ is called a soft set over .The set of all soft sets over $U$ is denoted by $S(U)$.

**DEFINITION 2.2**

Let $f_A \in S(U)$ . If $f_A(x) = \emptyset$ if $x \in E$ ,then $f_A$ is called an empty soft set denoted by $f_\emptyset$.If $f_A(x) = U$ for all $x \in A$ , then $f_A$ is called A- universal soft set denoted by $f_{\tilde{A}}$ .If $A = E$ then A-universal soft set is called A-universal soft set denoted by $f_{\tilde{E}}$ .

**DEFINITION 2.3**

Let $f_A, f_B \in S(U)$ .Then $f_A$of softsubset of$f_B$, deoted by $f_A \tilde{\subseteq} f_B$for all$x \in E$.$f_A$ and $f_B$ are equal , denoted by $f_A = f_B$, if and only if $f_A(x) = f_B(x)$ for all $x \in E$.

**DEFINITION2.4** Let$f_A \in S(U)$.Then soft complement of $f_A$ is defined by $f_A^{\tilde{c}} = f_{A^{\tilde{c}}}$ such that$f_{A^{\tilde{c}}}(x) = f_A^{\tilde{c}}(x) = U \backslash f_A(x)$ for all $x \in E$.

**DEFINITION 2.5**

Let $f_A, f_B \in S(U)$ .Then ,soft union of $f_A$ and $f_B$ is defined by $f_A \tilde{\cup} f_B = f_{A\tilde{\cup}B}$ such that $f_{A\tilde{\cup}B}(x) = f_A(x) \cup f_B(x)$ for all $x \in E$.

**DEFINITION2.6** Let $f_A , f_B \in S(U)$ .Then ,soft intersection of $f_A$ and $f_B$ is defined by$f_A \tilde{\cap} f_B = f_{A\tilde{\cap}B}$ such that $f_{A\tilde{\cap}B}(x) = f_A(x) \cap f_B(x)$ for all $x \in E$.

**DEFINITION 2.7**

Let $L \subseteq S(U)$ and $\vee$ and $\wedge$ be two binary operations on $L$.If the set $L$together with $\vee$ and $\wedge$ satisfies the following conditions then $(L,\vee.\wedge)$ is called a *soft lattice*

1. $f_A \vee f_B = f_B \wedge f_A$ and $f_A \wedge f_B = f_B \vee f_A$ for all$f_A$,$f_B \in L$.
2. $f_A \vee (f_B \vee f_C) = (f_A \vee f_B) \vee f_C$ and $f_A \wedge (f_B \wedge f_C) = (f_A \wedge f_B) \wedge f_C$ for all $f_A, f_B, f_C \in L$
3. $f_A \vee (f_A \wedge f_B) = f_A$ and$f_A \wedge (f_A \vee f_B) = f_A$ for all $f_A, f_B \in L$

**EXAMPLE 2.8**

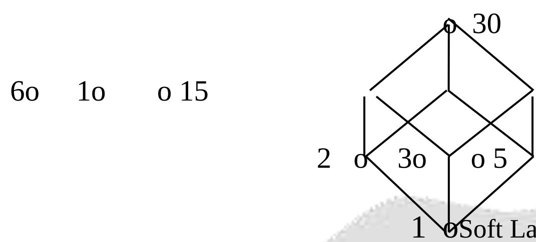
Let $(D_{30}, |)$ is a lattice.

**SOLUTION**

Where $D_{30}$ is a set of all divisors of 30 viz{1,2,3,5,6,10,15,30} and m|n if m divides n such that $m \vee n$ =least common multiple of$(m, n)$and $m \wedge n$ =greatest common divisor of$(m, n)$.

We can represent a lattice geometrically by a diagram called the Hasse diagram It is easy to find the $lub$ and $glb$ from the diagram.

Traverse upwards from the vertices representing $a$ and $b$ and reach a meeting point of the two paths the corresponding element is $a \vee b$ .By traversing downwards we can get $a \wedge b$ similarly.The Hasse diagram is



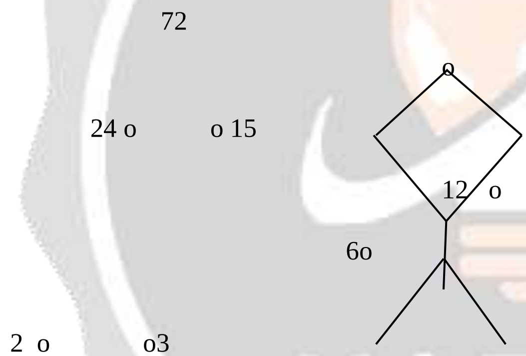

Soft Lattice structure

## EXAMPLE2.9

Let the poset A={2,3,6,12,24,36,72} and let divisibility be the relation on A.

**SOLUTION**

Hasse diagram of the poset $A$.since no lower bound and no greater lower bound exist for the pair of elements 2,3.The given poset is not a lattice because every pair of elements must have a unique least upper bound and unique greater bound.Hasse diagram of the poset $A$is given by




Not a Soft Lattice structure

**THEOREM 2.10**

Let $(L,\vee,\wedge)$ be a soft lattice and $f_A, f_B \in L$.Then $f_A \wedge f_B = f_A \iff f_A \vee f_B = f_B$.**PROOF**For all $f_A, f_B \in L$.such that$f_A \vee f_B = (f_A \wedge f_B)\vee f_B = (f_B \vee f_A) \wedge f_B = (f_B \vee f_B) \wedge f_A = f_B \wedge f_A = f_B$ ,since $f_A \vee f_B = f_B$.Conversely, $f_A \wedge f_B = f_A \wedge (f_A \vee f_B) = (f_A \vee f_A) \wedge f_B = f_A \wedge f_B = f_A$,hence,$f_A \wedge f_B = f_A$.Hence the proof.

**THEOREM2.11**Let $(L,\vee,\wedge)$ be a soft lattice and $f_A, f_B \in L$. Then a relation $\leq$ that is defined by$f_A \leq f_B \iff f_A \wedge f_B = f_A$or $f_A \vee f_B = f_B$ is an ordering relation on $L$.**PROOF**For all $f_A, f_B$and $f_C \in L$ i)$\leq$ is reflexive , $f_A \leq f_B$ , $f_A \wedge f_A = f_A$by theorem 2.8 and definition 2.7ii) $\leq$ is antisymmetric ,let $f_A \leq f_B$ and $f_B \leq f_A$,then by theorem 2.8 and definition2.7we can say that $f_A \wedge f_B = f_A$or $f_A \vee f_B = f_B$ .such that$f_A = f_A \wedge f_B = f_B \vee f_A = f_B$. iii) $\leq$ is transitive , let $f_A \leq f_B$ and $f_B \leq f_C$.Then

$f_A \wedge f_C = (f_A \wedge f_B) \wedge f_C = f_A \wedge (f_B \wedge f_C) = f_A \wedge f_B = f_A$ such that $f_A \wedge f_C = f_A$ By theorem 2.8 we can say that, $f_A \leq f_C$ **.** Hence the proof.

**LEMMA2.12**Let $(L,\vee,\wedge)$ be a soft lattice and $f_A, f_B \in L$. Then $f_A \wedge f_B$, $f_A \vee f_B$ areleast upper bound and the greatest lower bound of $f_A$ and $f_B$ respectively.**PROOF**From theorem,“ Let $(L,\vee,\wedge)$ be a soft lattice . Then

i) $f_A \wedge f_B \leq f_A$, and $f_A \wedge f_B \leq f_B$
ii) $f_A \leq f_A \vee f_B$ and $f_B \leq f_A \vee f_B$”

We can say that $f_A \wedge f_B$ and $f_A \vee f_B$ are lower bound and upper bound $f_A$ and $f_B$ respectively. Assume that $f_A \wedge f_B$ is not a greatest lower bound of $f_A$ and $f_B$ .Then $f_C \in L$ exists.such that ,$f_A \wedge f_B \leq f_C \leq f_A$, and $f_A \wedge f_B \leq f_C \leq f_B$.Also by theorem“ Let $(L,\vee,\wedge)$ be a fuzzy soft lattice.Then $f_A \leq f_B$,and $f_C \leq f_D \Rightarrow f_A \wedge f_C \leq f_B \wedge f_D$ ”.Therefore $f_C \leq f_A \wedge f_B$ that is $f_C = f_A \wedge f_B$ .which is contradiction.For $f_A \vee f_B$ the proof can be made similarly.Hence the proof

**THEOREM 2.13** A soft lattice $(L,\vee,\wedge)$ is a poset.**PROOF**From the previous lemma “Let $(L,\vee,\wedge)$ be a soft lattice and $f_A, f_B \in L$. Then $f_A \wedge f_B$, $f_A \vee f_B$ are least upper bound and the greatest lower bound of $f_A$ and $f_B$ respectively”We can say that A soft lattice $(L,\vee,\wedge)$ is a poset

**THEOREM2.14**Let $L \subseteq S(U)$ .Then the algebraic structure $(L,\vee,\wedge,\leq)$ is a soft lattice.**PROOF**For all $f_A, f_B, f_C \in L$.From lemma 2.10$f_A \wedge f_B \leq f_A$, and $f_A \wedge f_B \leq f_B$. Also by theorem$f_A \wedge f_B \leq f_B \wedge f_A$.similarly $f_B \wedge f_A = f_A \wedge f_B$ then$f_A \wedge f_B \leq f_B \wedge f_A$.By the same way ,the proof of $f_A \vee f_B = f_B \vee f_A$ can be made.From lemma 2.10 $(f_A \wedge f_B) \wedge f_C \leq f_A \wedge f_B \leq f_B$ and $f_A \wedge f_B \wedge f_C \leq f_C$ .By theorem$f_A \wedge f_B = f_A$, $f_A \vee f_B = f_B$such that

$f_A \wedge f_B \wedge f_C \leq f_B \wedge f_C \longrightarrow 1$also$f_A \wedge f_B \wedge f_C \leq f_B \wedge f_C \longrightarrow 2$from 1 and 2 we can say that $f_A \wedge f_B \wedge f_C \leq f_A \wedge f_B \wedge f_C$ . similarly $f_A \wedge f_B \wedge f_C \leq f_A \wedge f_B \wedge f_C$ , then$(f_A \wedge f_B) \wedge f_C = f_A \wedge f_B \wedge f_C$by the same way the proof of $(f_A \vee f_B) \vee f_C = f_A \vee (f_B \vee f_C)$can be made.From theorem “ Let $(L,\vee,\wedge)$ be a soft lattice . Then

i) $f_A \wedge f_B \leq f_A$, and $f_A \wedge f_B \leq f_B$
ii) $f_A \leq f_A \vee f_B$and $f_B \leq f_A \vee f_B$”

we can say that $f_A \leq f_A \vee f_B$and$f_A \leq f_A$. Also by theorem“ Let $(L,\vee,\wedge)$ be a fuzzy soft lattice.Then $f_A \leq f_B$ , and $f_C \leq f_D \Rightarrow f_A \wedge f_C \leq f_B \wedge f_D$ ”.since $f_A \leq f_A \vee f_B \vee f_C$ . similarly $f_A \vee f_B \wedge f_C \leq f_A$ .Then $f_A \wedge (f_A \vee f_B) = f_A$ by the same way ,the proof of $f_A \vee (f_B \vee f_C) = f_A$ can be made.Hence the proof.

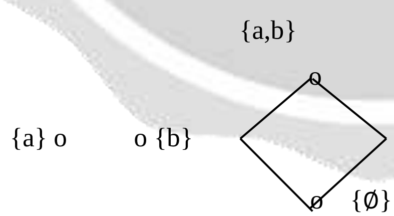
**DEFINITION 2.15** Let $(L,\vee,\wedge,\leq)$ be a soft lattice and $S \subseteq L$.If $S$ is a soft lattice with the operations of$L$,then $S$is called soft sublattice of $L$.

**THEOREM2.16** Let $(L,\vee,\wedge,\leq)$ be a soft lattice and $S \subseteq L$.If$f_A \vee f_B \in S$ and $f_A \wedge f_B \in S$ for all$f_A, f_B \in S$ then $S$ is a softsublattice.**PROOF**For all $f_A, f_B, f_C \in L$.From lemma 2.10$f_A \wedge f_B \leq f_A$, and $f_A \wedge f_B \leq f_B$. Also by theorem$f_A \wedge f_B \leq f_B \wedge f_A$ .similarly $f_B \wedge f_A = f_A \wedge f_B$ then $f_A \wedge f_B \leq f_B \wedge f_A$ .By the same way ,the proof of $f_A \vee f_B = f_B \vee f_A$ can be made.From lemma 2.10 $(f_A \wedge f_B) \wedge f_C \leq f_A \wedge f_B \leq f_B$ and$f_A \wedge f_B \wedge f_C \leq f_C$.By theorem$f_A \wedge f_B = f_A$, $f_A \vee f_B = f_B$ such that $f_A \wedge f_B \wedge f_C \leq f_B \wedge f_C \longrightarrow 1$ also$f_A \wedge f_B \wedge f_C \leq f_B \wedge f_C \longrightarrow 2$from 1 and 2 we can say that $f_A \wedge f_B \wedge f_C \leq f_A \wedge f_B \wedge f_C$, similarly $f_A \wedge f_B \wedge f_C \leq f_A \wedge f_B \wedge f_C$then( $f_A \wedge f_B) \wedge f_C = f_A \wedge f_B \wedge f_C$ by the same way the proof of ( $f_A \vee f_B) \vee f_C = f_A \vee ( f_B \vee f_C)$ can be made. From theorem " Let $(L,\vee,\wedge)$ be a soft lattice . Then

i) $f_A \wedge f_B \leq f_A$, and $f_A \wedge f_B \leq f_B$
ii) $f_A \leq f_A \vee f_B$and $f_B \leq f_A \vee f_B$"

we can say that $f_A \leq f_A \vee f_B$ and $f_A \leq f_A$. Also by theorem " Let $(L,\vee,\wedge)$ be a fuzzy soft lattice.Then $f_A \leq f_B$, and $f_C \leq f_D \Rightarrow f_A \wedge f_C \leq f_B \wedge f_D$ "since $f_A \leq f_A \vee f_B \vee f_C$. similarly $f_A \vee f_B \wedge f_C \leq f_A$ .Then $f_A \wedge (f_A \vee f_B) = f_A$ .By the same way ,the proof of $f_A \vee (f_B \vee f_C) = f_A$ canbe made.Hence the proof.

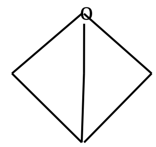
**EXAMPLE2.17**Let $(L,\leq)$ be a lattice where $L = \wp(a,b,c)$ and the partial ordering relation is$\subseteq$.Let$A = \{\emptyset,\{a\}\{b\}\{a,b\}\}$ then $(A,\leq)$ is a sub lattice of $(L,\leq)$.Then the Hasse diagram for soft sublattice is given by




soft sublattice structure

**EXAMPLE 2.18**
Let $(L,\leq)$ be a lattice where $L = \wp(a,b,c)$ and the partial ordering relation is $\subseteq$ .Let $B = \{\emptyset,\{a\}\{b\}\{a,b,c\}\}$ then( $B,\leq)$ is not a sub lattice of $(L,\leq)$ .Since $\{a\} \vee \{b\} = \{a,b\}$ and $\{a,b\} \notin B$.Then the Hasse diagram for not sublattice is given by

{a,b,c}



{a}o {b}o o{c}

o {∅}

Not a sublattice

**MODULAR AND DISTRIBUTIVE SOFT LATTICES**

**DEFINITION2.19**Let $(L,\vee,\wedge,\leq)$ be a soft lattice . Then $L$ is said to be a modular lattice if$f_A \leq f_c \Longrightarrow (f_A \vee f_B) \wedge f_C = f_A \vee (f_B \wedge f_C)$ for all $f_A, f_B, f_c \in L$or equivalently,$f_A \geq f_c \Longrightarrow f_A \wedge (f_B \vee f_C) = (f_A \wedge f_B) \vee f_C$.

**LEMMA 2.20** The soft sublattice(dual) of a modular soft lattice is modular.

**PROOF** Let $L$ be a modular soft lattice .Then $f_A \leq f_c \Longrightarrow (f_A \vee f_B) \wedge f_C = f_A \vee (f_B \wedge f_C)$ for all $f_A, f_B, f_c \in L$ .Let $L$ be a soft sublattice of $L$ .Take $f_A, f_B, f_c \in S$ .since $S \subseteq L$ , $f_A, f_B, f_c \in L$. Therefore $f_A \leq f_c \Longrightarrow (f_A \vee f_B) \wedge f_C = f_A \vee (f_B \wedge f_C)$ true for all $f_A, f_B, f_c \in S$ .Hence the soft sublattice $S$ is modular soft lattice. Therefore the soft sublattice of a modular soft lattice is modular.Let $L$ be a modular soft lattice. Then $f_A \leq f_c \Longrightarrow (f_A \vee f_B) \wedge f_C = f_A \vee (f_B \wedge f_C)$ for all $f_A, f_B, f_c \in L$The dual of $L$ is $f_A \geq f_c \Longrightarrow f_A \wedge (f_B \vee f_C) = (f_A \wedge f_B) \vee f_C$.Therefore ,the dual of the modular soft lattice is modular.

**DEFINITION 2.21** Let $(L,\vee,\wedge,\leq)$ be a soft lattice . Then $L$ is said to be a distributive soft lattice if for all $f_A, f_B, f_c \in L$

1.$f_A \vee (f_B \wedge f_C) = (f_A \vee f_B)\,(f_A \vee f_C)$ for all $f_A, f_B \in L$

2.$f_A \wedge (f_B \vee f_C) = (f_A \wedge f_B)(f_A \wedge f_C)$

**LEMMA 2.22** The soft sublattice(dual) of a distributive soft lattice is distributive.

**PROOF** Let $L$ be distributive soft lattice .Then$f_A \wedge (f_B \vee f_C) = (f_A \wedge f_B)(f_A \wedge f_C)$ for all $f_A, f_B, f_c \in L$ .Let $S$ be a soft sublattice of $L$ .Take $f_A, f_B, f_c \in S$ .since $S \subseteq L$ , $f_A, f_B, f_c \in L$ . Therefore$f_A \wedge (f_B \vee f_C) = (f_A \wedge f_B)(f_A \wedge f_C)$ true for all $f_A, f_B, f_c \in S$.Hence the soft sublattice $S$ is distributive soft lattice. Therefore the soft sublattice of adistributive soft lattice is distributive . Let $L$ be a distributive soft lattice. Then $f_A \wedge (f_B \vee f_C) = (f_A \wedge f_B)(f_A \wedge f_C)$ for all $f_A, f_B, f_c \in L$ .

The dual of $L$ is $f_A \vee (f_B \wedge f_C) = (f_A \vee f_B)\,(f_A \vee f_C)$ .Therefore ,the dual of the distributive soft lattice is distributive .

**CONCLUSION** Soft set theory has been applied to many fields from theoretic to practical. In this paper we first define cocept of Soft Lattices also we have given the study of Soft SubLattices, Modular Soft Lattices, Distributive Soft Lattices with their properties and examples. More over based on this paper we have studying about some of these soft lattices and are expected to give some more results.

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