A STUDY ON WHOLE INTIMIDATION NUMBER AND CHROMATIC NUMBER OF A FUZZY GRAPH

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ABSTRACT

The study of fuzzy unit notations has been growing at an exponential rate. A development is a subset S of V; each within mathematics and it is a dominance set in G when every vertex V-S applies. The traditional ties form is adjacent to at least one vertex in S. Logic and topology are examples of mathematical disciplines. A dominating set is claimed to be fuzzy algebra,evaluation etc.As a result, the fuzzy total dominating set has energised as a capacity zone next to a minimum of one vertex in S of multidisciplinary research, and the fuzzy graph minimum cardinality taken over all total concepts is of fresh importance.

Keywords

Fuzzy TDN Chromatic Number, Clique, Fuzzy Graphs.

INTRODUCTION

Any vertex u in G has a degree.The observe of dominating units in is the number of intersecting edges with u and graphs become began through Cockayne is denoted by $d(u)$ and Hedetniemi. A Mathematical the minimum and maximum degree framework to explain the phenomena of a the name of vertex is $\delta(G)$ and $\Delta(G)$ uncertainty in actual international scenario is first counseled through L.A.Zadeh in 1965.

The study of fuzzy unit notation has grown at an accelerating rate, both within mathematics and in its applications.

This tiers from conventional mathematical topics like logic, topology, algebra, evaluation etc. therefore fuzzy arithmetic has emerged as capacity region of interdisciplinary studies and fuzzy graph idea is of new interest.

PRELIMINARES

If X is a generically denoted collection of items, by x, then a Fuzzy set $\tilde{A}$ in X is a set of ordered pairs: $\tilde{A} = \{(x, \mu_{\tilde{A}}(x))/x \epsilon X\}, \mu_{\tilde{A}}(x)$ is called the membership Grade of x in $\tilde{A}$ that connects X to M's membership region . Let E stand for the (sharp) set of nodes.

A fuzzy graph is $\tilde{G}(x_i, x_j) = \{(x_i, x_j), \mu_{\tilde{G}}(x_i, x_j)(x_i, x_j)\epsilon E \times E\}$.

$H(x_i, x_j)$ is a Fuzzy Sub graph of $G(x_i, x_j)$ if $\mu_A(x_i, x_j) \le \mu_G(x_i, x_j) \forall (x_i, x_j) \epsilon E \times E, H(x_i, x_j)$ is a spanning fuzzy sub graph of $G(x_i, x_j)$ if the node set of $H(x_i, x_j)$ and $G(x_i, x_j)$are equal, that is if they differ only in there are weights.

Let $G(\mu, \sigma)$ be simple undirected fuzzy graph. The frequency of edges incident with any vertex u in G is denoted by $d(u)$ the grade of the vertex.

The minimum and maximum degree of a vertex is denoted by $\delta(G)$ and $\Delta(G)$ respectively, P_n denotes the path on n vertices. The vertex connectivity $k(G)$ of a graph G is the smallest number of vertices that results in a discontinuous graph when they are removed. The chromatic number is defined as a specific number of colours required to colour every one of the vertices while ensuring that no two neighbouring vertices have the same colour. A clique of G is a fully connected subgraph of any graph G. G's clique number is the number of vertices in the greatest clique.

A subset S of V is called a dominating set in G, if every vertex in $V - S$ is adjacent to at least one vertex is S. The minimum cardinality taken over all minimal dominating sets in G is called the dominating set in G is called the domination number of G and is denoted by γ. If each vertex in V is neighboring to at least one vertex in, the dominating set is said to be fuzzy total dominating set. The fuzzy total domination number is defined as the minimum cardinality across all total dominating sets and is represented by $\gamma_{ft}(G)$. We use the following previous results

MAIN RESULTS

Theorem:

For any connected fuzzy graph $G, \gamma_t(G) + \chi(G) = 2n - 3$ and the equality holds if and only if $G \cong P_3, K_4$

Proof:

Assume that $\gamma_t(G) + \chi(G) = 2n - 3$. This is possible only $\gamma_t(G) = n$
and $\chi(G) = n - 3$ (or) $\gamma_t(G) = n - 1$
and $\chi(G) = n - 2$ (or) $\gamma_t(G) = n - 2$
and $\chi(G) = n - 1$ (or) $\gamma_t(G) = n - 3$
and $\chi(G) = n$.

Case (i).

Let $\gamma_t(G) = n$ and $\chi(G) = n - 3$.
Since $\chi(G) = n - 3, G$ contains a clique K on n-3 vertices.
Let $S = \{x, y, z\} \in V - S$.
Then $S \ge K_3, \acute{K}_3, K_2 \cup K_1, P_3$

Sub case (i)

Let $S \ge K_3$. Since G is connected, x is adjacent to some u_i of K_{n-3}.
Then $\{x, u_i\}$ is γ_t −set, so that $\gamma_t(G) = 2$ and hence $n = 2$.
But $\chi(G) = n - 3$ =negative value Which is a contradiction.
Hence no fuzzy graph exists.

Sub case (ii)

Let $S \ge \acute{K}_3$ since G is connected, one of the vertices of K_{n-3} say u_i is adjacent to all the vertices of S or two vertices of S or one vertex of S. if u_i for some I is adjacent to all the vertices of S, then $\{u_i\}$ in K_{n-3} is a γ_t −set of G, so that $\gamma_t(G) = 1$ and
hence $n = 1$.
But $\chi(G) = n - 3$ =negative value. Which is a contradiction.

Hence no fuzzy graph exists. Since G is connected u_i for some i is adjacent to two vertices of S say x and y and z is adjacent to u_i for $i \ne j$ in K_{n-3}, then $\{u_i, u_j\}$ in K_{n-3} is γ_t −set of G, so that $\gamma_t(G) = 2$ and
hence $n = 2$.
But $\chi(G) = n - 3$ =negative value. Which is a contradiction.

Hence no fuzzy graph exists. If u_i for some i is adjacent to x and u_j is adjacent to y and u_k is adjacent to z, then $\{u_i, u_j, u_k\}$ for $i \ne j \ne k$ in K_{n-3} is a γ_t −set of G.
so that $\gamma_t(G) = 3$ and hence $n = 3$. But$\chi(G) = n - 3 = 0$. This is a contradiction.
Hence no fuzzy graph exists.

Sub case (iii)

Let $S > P_3 = \{x, y, z\}$. Since G is connected, x (or equivalently z) is adjacent to u_i for some i in K_{n-3}.

Then$\{x, y, u_i\}$ is a γ_t −set of G. so that $\gamma_t(G) = 3$ and hence $n = 3$.
But $\chi(G) = n - 3 = 0$. This is a contradiction
Hence no fuzzy graph exists. If u_i is adjacent to y then $\{u_i, y\}$ is a γ_t −set of G. so that $\gamma_t(G) = 2$ and hence $n = 2$.
But $\chi(G) = n - 3$ =negative value Which is a contradiction.
Hence no fuzzy graph exists.

Sub case (iv)

Let $S > K_2 \cup K_1$.

Let xy represent the edge and z represent the isolated vertex of $K_2 \cup K_1$. Since G is connected, there exists a u_i in K_{n-3} is adjacent to x and z. Then $\{u_i\}$ is γ_t −set of G, so that $\gamma_t(G) = 1$ and hence $n = 1$.
But $\chi(G) = n - 3$ =negative value Which is a contradiction
Hence no fuzzy graph exists. If z is adjacent to u_i for some $i \neq j$ then $\{u_i, u_j\}$ for $i \neq j$ is γ_t −set of G, so that $\gamma_t(G) = 2$ and hence $n = 2$.
But $\chi(G) = n - 3$ =negative value which is a contradiction
Hence no fuzzy graph exists.

Case (ii)

Let $\gamma_t(G) = n - 1$ and $\chi(G) = n - 2$.
Since $\chi(G) = n - 2$, G contains a clique K on n-2 vertices.
Let $S = \{x, y\} \in V - S$. Then $S \geq K_2$ or $\acute{K_2}$

Sub case (a)

Let $S \geq K_2$ since G is connected, x(or equivalently y) is adjacent to some u_i of K_{n-2}.
Then $\{x, u_i\}$ is γ_t −set, so that $\gamma_t(G) = 2$ and hence $n = 3$.
But $\chi(G) = n - 2 = 1$ for which G is totally disconnected, which is a contradiction.
Hence no fuzzy graph exists.

Sub case (b)

Let $S \geq \acute{K_2}$ since G is connected, x is adjacent to some u_i of K_{n-2}. Then y is adjacent to the same u_i of K_{n-2}.
Then y is adjacent to the same u_i of K_{n-2}. Then $\{u_i\}$ is γ_t −set, so that $\gamma_t(G) = 1$ and hence $n = 2$.
But $\chi(G) = n - 2 = 0$. Which is a contradiction
Hence no fuzzy graph exists.
Otherwise x is adjacent to u_i of K_{n-2} for some I and y is adjacent to u_i of K_{n-2} for $i \neq j$. In this $\{u_i, u_j\}\gamma_t$ −set,
so that $\gamma_t(G) = 2$ and hence $n = 3$.
But $\chi(G) = 1$ for which G is totally disconnected.
Which is a contradiction.
There is no fuzzy graph in this situation as well.

Case (iii)

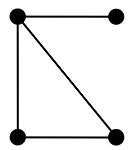
Let $\gamma_t(G) = n - 2$ and $\chi(G) = n - 1$.
Since $\chi(G) = n - 1, G$ contains a clique K on n-1 values.
Let x be a point in the graph K_{n-1}.
Since G is connected the vertex x is adjacent to one vertex u_i for some i in K_{n-1} so that $\gamma_t(G) = 1$, we have $n = 3$ and $\chi = 2$.
Then $K = K_2 = uv$. If x is adjacent to u_i then $G \cong P_3$.

Case (iv)

Let $\gamma_t(G) = n - 3$ and $\chi(G) = n$
Since $\chi(G) = n, G = K_n, \gamma_t(G) = 1$,
so that $n = 4, \chi = 4$.
Hence $\cong K_4$. converse is obvious.

Theorem:

For any connected fuzzy graph G, $\gamma_t(G)+\chi(G)=2n-4$ and the equality holds if and only if $G\cong P_4,K_5$ or the graph is figure



Proof:

Assume that $\gamma_t(G)+\chi(G)=2n-4$. This is possible only if $\gamma_t(G)=n$
and $\chi(G)=n-4$ or $\gamma_t(G)=n-1$
and $\chi(G)=n-3$ (or) $\gamma_t(G)=n-2$
and $\chi(G)=n-2$ (or) $\gamma_t(G)=n-3$
and $\chi(G)=n-1$ (or) $\gamma_t(G)=n-4$
and $\chi(G)=n$.

Case (i)

Let $\gamma_t(G)=n$ and $\chi(G)=n-4$.
Since $\chi(G)=n-4, G$ contains a clique K on n-4 vertices.
Let $S=\{V_1,V_2,V_3,V_4\}$. Then the induced subgraph $S>$ has the following possible cases $K_4,\acute{K}_4,P_4,P_3\cup K_1,K_2\cup K_2,K_3\cup K_1,K_{1,3}$
This can be verified that no proposed fuzzy graphs exist in all of the following circumstances.

Case (ii)

Let $\gamma_t(G)=n-1$ and $\chi(G)=n-3$.
Since $\chi(G)=n-3, G$ contains a clique K on n-3 vertices.
Let$S=\{x,y,z\}\in V-S$.
Then $S\geq K_3,\acute{K}_3,K_2\cup K_1,P_3$

Sub case (i)

Let$S>K_3$. Since G is connected, x is adjacent to some u_i of K_{n-3}.
Then $\{x,u_i\}$ is γ_t −set, so that $\gamma_t(G)=2$ and hence $n=3$.
But $\chi(G)=n-3=0$. Which is a contradiction.
Hence no fuzzy graph exists.

Sub case (ii)

Let $S\geq\acute{K}_3$ since G is connected, one of the vertices of K_{n-3} say u_i is adjacent to all the vertices of S or two vertices of S or one vertex of S.

If u_i for some i is adjacent to all the vertices of S, then $\{u_i\}$ in K_{n-3} is γ_t −set of G. so that $\gamma_t(G)=1$ and hence $n=2$.

But $\chi(G)=n-3$ =negative value. Which is a contradiction.

Hence no fuzzy graph exists. If u_i for some i is adjacent to two vertices of S say x and y then G is connected, z is adjacent to u_j for $i\neq j$ in K_{n-3}, then $\{u_i,u_j\}$ in K_{n-3} is γ_t −set of G,
so that $\gamma_t(G)=2$ and hence $n=3$.
But $\chi(G)=n-3=0$. Which is a contradiction.

Hence no fuzzy graph exists. If u_i for some i is adjacent to x and u_j is adjacent to y and u_k is adjacent to z, then $\{u_i,u_j,u_k\}$ for $i\neq j\neq k$ in K_{n-3} is γ_t −set of G. so that $\gamma_t(G)=3$ and
hence $n=4$. But $\chi(G)=1$ for which G is totally disconnected. Which is a contradiction.
Hence no fuzzy graph exists.

Sub case (iii)

Let $S>P_3=\{x,y,z\}$. Since G is connected, x (or equivalently z) is adjacent to u_i for some i in K_{n-3}.
Then $\{x,y,u_i\}$ is γ_t −set of G so that $\gamma_t(G)=3$ and hence $n=4$.
But $\chi(G)=n-3=1$. Which is a contradiction.
Hence no fuzzy graph exists. If u_i is adjacent to y then $\{u_i,y\}$ is γ_t −set of G. so that $\gamma_t(G)=2$ and hence $n=3$. But $\chi(G)=n-3=0$. Which is a contradiction.
Hence no fuzzy graph exists.

Sub case (iv)

Let $S \geq K_2 \cup K_1$. Let xy be the edge and z be a isolated vertex or $K_2 \cup K_1$. Since G is connected, there exists a u_i in K_{n-3} is adjacent to x and z also adjacent to same u_i.

Then $\{u_i\}$ is a γ_t −set of G. so that $\gamma_t(G) = 1$ and hence $n = 2$.

But $\chi(G) = n - 3$ =negative value. Which is a contradiction.

Hence no fuzzy graph exists. If z is adjacent to u_j for some $i \neq j$ then $\{u_i, u_j\}$ for $i \neq j$ is a is a γ_t −set of G. so that $\gamma_t(G) = 2$ and hence $n = 3$.

But $\chi(G) = n - 3 = 0$. Which is a contradiction.

Hence no fuzzy graph exists.

Case (iii)

Let $\gamma_t(G) = n - 2$ and $\chi(G) = n - 2$.

Since $\chi(G) = n - 2, G$ contains a clique K on n-2 vertices. Let $S = \{x, y\} \in V - S$.

Then $S \geq K_2$ or $\acute{K}_2$

Sub case (a)

Let $S > K_2$. Since G is connected, x (or equivalently y) is adjacent to some u_i of K_{n-2}.

Let $\gamma_t(G) = n - 3$ and $\chi(G) = n - 1$.

Since $\chi(G) = n - 1, G$ contains a clique $u_i\}$ is γ_t −set, so that $\gamma_t(G) = 2$ and hence $n = 4$.

But $\chi(G) = n - 2 = 2$.

Then $G \cong P_4$.

Case (iv)

K on n-1 vertices. Let x be a point in the graph. $G - K_{n-1}$.

Since G is connected the vertex x is adjacent to one vertex u_i for some I in K_{n-1}, so that $\gamma_t(G) = 1$, we have $n = 4$ and $\chi = 3$. Then $K = K_3$ let u_1, u_2, u_3 be the vertices of K_3.

Then only one vertex of y must be near to x. $G - K_3$. Allow x to be near to y without losing generality. u_i. If $d(x) = 1$, then $G \cong G_1$. (in Fig 2.1)

Case (v)

Let $\gamma_t(G) = n - 4$ and $\chi(G) = n$

Since $\chi(G) = n, G = K_n$.

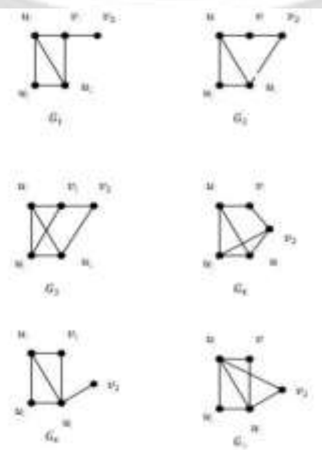
But $\gamma_t(G) = 1$,

so that $n = 5, \chi = 5$.

Hence $G \cong K_5$. Converse is obvious.

Theorem 3.6

For any connected fuzzy graph G, $\gamma_t(G) + \chi(G) = 2n - 5$ for any $n > 4$, if and only if G is isomorphic to $K_6, K_3(P_3), K_3(1,1,0), P_5, K_4(1,0,0,0), K_{1,3}$ (or) any one of the following fuzzy graphs in figure



If G is any one or one graphs in the theorem, then it can be verified that $\gamma_t(G) + \chi(G) = 2n - 5$.

Conversely set

$\gamma_t(G) + \chi(G) = 2n - 5$

then $\gamma_t(G) = n$ and $\chi(G) = n - 5$ (or)

$\gamma_t(G) = n - 1$ and $\chi(G) = n - 4$ (or)

$\gamma_t(G) = n - 2$ and $\chi(G) = n - 3$ (or)

$\gamma_t(G) = n - 3$ and $\chi(G) = n - 2$ (or)

$\gamma_t(G) = n - 4$ and $\chi(G) = n - 1$ (or)

$\gamma_t(G) = n - 5$ and $\chi(G) = n$.

Case (i):

Let $\gamma_t(G) = n$ and $\chi(G) = n - 5$

Since $\chi(G) = n - 5, G$ contains a clique K in n-5 vertices (or) does not contain a clique K on n-5 vertices.

Let $S = \{V_1, V_2, V_3, V_4, V_5\}$. Then the induced subgraph $S >$ has the following possible cases. $K_5, \acute{K}_5, P_5, P_3 \cup P_2, P_3 \cup \acute{K}_2, K_4 \cup K_1, P_4 \cup K_{1,} K_3 \cup K_2, K_3 \cup \acute{K}_2$.

It can be verified that no new fuzzy graph exists in all of the above scenarios.

Case (ii):

Let $\gamma_t(G) = n - 1$ and $\chi(G) = n - 4$.

Since $\chi(G) = n - 4G$ contains a clique K on n-4 vertices.

Let $S = \{V_1, V_2, V_3, V_4\}$. The induced subgraph may then have the following case. $K_4, \acute{K}_4, P_4, P_3 \cup K_1, K_2 \cup K_2, K_3 \cup K_1$ It can be verified that no new fuzzy graph exists in all of the above circumstances.

Case (iii):

Let $\gamma_t(G) = n - 2$ and $\chi(G) = n - 3$

Since $\chi(G) = n - 3, G$ contains a clique with n-3 vertices.

Let $S = \{V_1, V_2, V_3\}$. Then the induced subgraph $S >$ has the following possible cases $K_3, \acute{K}_3, K_2 \cup K_1, P_3$.

Sub case (i):

Let $S \geq K_3$. Since G is connected there exists a vertex u_i in K_{n-3} which is adjacent to any one of $\{V_1, V_2, V_3\}$ without loss of generality let u_i be adjacent to V_1, then $\{v_1, u_j\}$ is γ_t −set of G,

so that $n = 4$. But $\chi(G) = 1$ which is a contradiction. Hence no graph exists.

Sub case (ii):

Let $S > \acute{K}_3$. Let $\{V_1, V_2, V_3\}$ be the vertices of $\acute{K}_3$ are adjacent to one vertex say u_i in K_{n-3} (or) 2 vertices of $\acute{K}_3$ are adjacent to the vertex u_i and remaining one vertex of $\acute{K}_3$ is adjacent to the vertex u_j for $i \neq j$ in K_{n-3} (or) all the vertices of $\acute{K}_3$ are adjacent to the distinct vertices of K_{n-3}. If all the vertices of $\acute{K}_3$ are adjacent to one vertex say u_i in K_{n-3}, the $\{v_1, u_i\}$ is $\gamma_t(T)$ −set of G, so that $n = 4, \chi(G) = 1$ which is a contradiction. Hence no fuzzy graph exists.

If two vertices of $\acute{K}_3$ are adjacent to the vertex u_i and the remaining one is adjacent to $u_j i \neq j$ in K_{n-3} then $\{u_i, u_j, v_3\}$ is $\gamma_t(T)$ −set of G. So that $n = 4$ which is a contradiction. Hence no graphs exists. If all the vertices of $\acute{K}_3$ are adjacent to the vertices $u_i, u_j, u_k i \neq j \neq k$ respectively.

Then $\{u_i, u_j, v_3\}$ is γ_t −set of G. Hence $n = 5, \chi(G) = 2$ which is a contradiction.

Hence no graph exists.

Sub case (iii):

Let $S \geq P_3$. Since G is connected there exists a vertex u_i is adjacent to any one of $\{v_1, v_3\}$ or v_2.

If u_i is adjacent to any of the $\{v_1, v_3\}$, then $\{v_1, v_2, u_i\}$ is γ_t −set of G. Hence $n = 5$ so that $K = K_2$.

Let u_1, u_2 are the vertices of K_2. Let v_1 be adjacent to u_i and if $d(v_1) = d(v_2) = 2, d(v_3) = 1$ then $G \cong P_5$. If v_2 is adjacent to u_i then $\{u_i, v_2\}$ is γ_t −set of G.

Hence $n = 4\chi(G) = 1$ which is a contradiction.

As a result, no new fuzzy graphs are produced as the degree of the vertices is increased.

Sub case (iv):

Let $S > K_2 \cup K_1$. Let $\{v_1, v_2\}$ be the vertices of K_2 and v_3 be the isolated vertex. Since G is connected, there exists a vertex u_i is adjacent to any one of $\{v_1, v_2\}$ and u_j for $i \neq j$ is adjacent to v_3.

If u_i is adjacent to any one of $\{v_1, v_2\}$ and $v_3, \{v_1, v_2\}$ and $\{u_i, v_1\}$ is γ_t −set of G.

Hence $n = 4 \wedge \chi(G) = 1$ which is a contradiction.

Hence no graph exists.

If u_i is adjacent to any one of $\{v_1, v_2\}$ and u_j $j \neq i$ is adjacent to v_3.

In this case $\{v_1, u_i, u_j\}$ is γ_t −set of G.

Hence $n = 5\chi(G) = 2$.

Then $G \cong P_5$. There are no new fuzzy graphs if the degree of the vertices is increased.

Case (iv):

Let $\gamma_t(G) = n - 3, \chi(G) = n - 2$.

Since $\chi(G) = n - 2$, There is a clique in G K on $n - 2$ vertices.

If G has a clique K on n-2 of its vertices.

Let$S = \{v_1, v_2\} \in V(G) - V(K)$. Then the induced sub graph $S >$ has the following possible cases.

Hence $G \cong K_6$.

CONCLUSION

The upper bound of the sum of the number domination and chromatic number is established in this study. This result can be expanded to various domination parameters in the future. The graph structure presented in this paper can be employed in simulations and networks. With huge examples of graphs for which $\gamma_t(G) + \chi(G) = 2n - 8$the authors have gotten similar results

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