# AN APPLICATION OF INTERVAL SYSTEM OF LINEAR EQUATIONS IN CIRCUIT ANALYSIS

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**ABSTRACT**

*This paper deals with application of interval system of linear equations (ISLE) in circuit analysis (CA). In CA problems, each circuit consists of resistance, inductance and capacitance and each circuit has been mathematically modeled as a system of linear equations. An interval valued current or interval valued voltage in the circuit is more important measure relative to crisp/precise value because of some factors such as environmental conditions, tolerance in the units/elements and leakage of power harmonics. For this purpose introduction of interval system of linear equations (ISLE) are highly important. Here, an interval valued linear system has been introduced and a numerical example has been presented for illustration purpose*.

**Keyword:** - *Interval linear system, circuit analysis, interval number, current, voltage, Kirchhoff's Voltage Law*

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## 1. INTRODUCTION

System of linear equations plays an important role in many areas such as economics, physics, engineering, statistics and management science. Recently, in many applications some or all of the system parameters and their measures are considered using imprecise valued rather than precise valued [8]. To overcome the problem with imprecise measured generally stochastic, fuzzy and fuzzy-stochastic approaches are used. In stochastic approach, the parameters are assumed as random variables with known probability distribution. In fuzzy approach, the parameters are considered as fuzzy sets with known membership functions or fuzzy numbers [2,6, 9]. On the other hand, in fuzzy-stochastic approach, some parameters are viewed as fuzzy sets and other as random variables. However, for a design engineer to specify the suitable membership function for fuzzy approach and probability distribution for stochastic approach and both for fuzzy-stochastic approach is a very hard task. To avoid these difficulties for representation of imprecise numbers by several approaches, one may represent the same by interval number as it is the best representation among others [1, 4, 5]. Therefore, it is very important to develop mathematical models that would handle interval linear system. Any design of circuit may be represented in the form of interval system of linear equations. Studies of the circuit analysis problems where the current and/or voltage are fuzzy valued have already been initiated by some authors like [7, 10]. In this paper, we have considered interval valued current and/or voltage. For this purpose, we have introduced interval system of linear equations and its solution methodology has been presented. This proposed method has been illustrated with a numerical example arises in a circuit theory. As a special case considering the lower and upper bounds of interval valued voltage as same, the resulting problem becomes identical with the existing problem available in the literature.

## 2. ASSUMPTIONS AND NOTATIONS

A circuit analysis problem is formulated under the following assumptions and notations:

**Assumptions:**

(i) The current flow of a circuit is imprecise and interval valued.
(ii) The source voltage of a circuit is imprecise and interval valued.
(iii) The Resistance of a circuit is fixed and/or precise valued.

**Notations:**

| | |
|---|---|
| $I_i$ | Current flow of *i*-th mesh in a circuit |
| $V_i$ | Voltage of *i*-th source in a circuit |
| $R_i$ | *i*-th resistance in a circuit |
| $I_i^-$ | Lower limit of current flow of *i*-th mesh in a circuit |
| $I_i^+$ | Upper limit of current flow of *i*-th mesh in a circuit |
| $V_i^-$ | Lower limit of voltage of *i*-th source in a circuit |
| $V_i^+$ | Upper limit of voltage of *i*-th source in a circuit |
| KVL | Kirchhoff's Voltage Law |

## 3. FINITE INTERVAL ARITHMETIC

Interval arithmetic was introduced by Moore (Moore [4], Moore, Kearfott & Cloud [5]). In interval arithmetic, an imprecise variable $a$ is represented as an interval number denoted by $\bar{A}=[a^-,a^+]$ and is defined by $\bar{A}=[a^-,a^+]=\{a: a^- \le a \le a^+, a \in \mathbb{R}\}$, where $a^-$ and $a^+$ are the lower and upper limits respectively and $\mathbb{R}$ is the set of all real numbers. Actually, every real number can be treated as an interval, such as for all $a \in \mathbb{R}$, $a$ can be written as an interval with the form $[a,a]$ .The basic arithmetical operations of interval numbers $\bar{A}=[a^-,a^+]$ and $\bar{B}=[b^-,b^+]$ are defined as

$\bar{A}+\bar{B}=[a^-+b^-, a^++b^+]$,

$\bar{A}-\bar{B}=[a^--b^+, a^+-b^-]$,

For any scalar $\lambda$, $\lambda\bar{A}=\begin{cases}[\lambda a^-,\lambda a^+] & \text{for } \lambda \ge 0\\ [\lambda a^+,\lambda a^-] & \text{for } \lambda < 0\end{cases}$,

$\bar{A}\times\bar{B}=[\min(a^-b^-,a^-b^+,a^+b^-,a^+b^+),\max(a^-b^-,a^-b^+,a^+b^-,a^+b^+)]$,

$\frac{\bar{B}}{\bar{A}}=\bar{B}\times\frac{1}{\bar{A}}=[b^-,b^+]\times[\frac{1}{a^+},\frac{1}{a^-}]$, provided $0 \notin [a^-,a^+]$ .

For more details one may refer to the works of Hansen and Walster [1].

## 4. INTERVAL LINEAR SYSTEM

Let us considered a linear system as follows:

$$AX=\bar{b} \tag{1}$$

where $A=\begin{pmatrix} a_{11} & a_{12} & \dots & a_{1n}\\ a_{21} & a_{22} & \dots & a_{2n}\\ \vdots & \vdots & \vdots & \vdots\\ a_{n1} & a_{n2} & \dots & a_{nn}\end{pmatrix}$ is a matrix of order $n\times n$ whose elements are crisp/precise valued and

$\bar{b}=\begin{pmatrix}[b_1^-,b_1^+]\\ [b_2^-,b_2^+]\\ \vdots\\ [b_n^-,b_n^+]\end{pmatrix}$ is a matrix of order $n\times 1$ whose elements are interval valued.

Then equation (1) is called an interval linear system where $A$ is crisp matrix and $\bar{b}$ is an interval matrix. Obviously $X=\bar{X}$ is also an interval matrix and our aim is to find out such $\bar{X}^*$ which satisfy the equation (1).

Now, transform equation (1) into $2n \times 2n$ matrix as follows:

$$\begin{aligned}
&c_{11}x_1^- + \ldots + c_{1n}x_n^- + c_{1(n+1)}(-x_1^+) + \ldots + c_{1(n+n)}(-x_n^+) = b_1^+ \\
&\vdots \\
&c_{n1}x_1^- + \ldots + c_{nn}x_n^- + c_{n(n+1)}(-x_1^+) + \ldots + c_{n(n+n)}(-x_n^+) = b_n^+ \\
&c_{(n+1)1}x_1^- + \ldots + c_{(n+1)n}x_n^- + c_{(n+1)(n+1)}(-x_1^+) + \ldots + c_{(n+1)(n+n)}(-x_n^+) = -b_1^+ \\
&\vdots \\
&c_{(n+n)1}x_1^- + \ldots + c_{(n+n)n}x_n^- + c_{(n+n)(n+1)}(-x_1^+) + \ldots + c_{(n+n)(n+n)}(-x_n^+) = -b_n^+
\end{aligned} \qquad (2)$$

Where $c_{ij} = c_{(i+n)(j+n)} = a_{ij}$, if $a_{ij} \geq 0$ and $c_{i(j+n)} = c_{(i+n)j} = -a_{ij}$, if $a_{ij} < 0$ (3)

and any $c_{ij}$ which is not determined by equation (3) is set as zero. Using matrix notation, equation (2) can be written as

$$CX_{new} = b_{new} \qquad (4)$$

where $C = \begin{pmatrix} c_{11} & c_{12} & \cdots & c_{1(n+n)} \\ c_{21} & c_{22} & \cdots & c_{2(n+n)} \\ \vdots & \vdots & \vdots & \vdots \\ c_{(n+n)1} & c_{n2} & \cdots & c_{(n+n)(n+n)} \end{pmatrix}$, $X_{new} = \begin{pmatrix} x_1^- \\ x_2^- \\ \vdots \\ x_n^- \\ -x_1^+ \\ -x_2^+ \\ \vdots \\ -x_n^+ \end{pmatrix}$ and $b_{new} = \begin{pmatrix} b_1^- \\ b_2^- \\ \vdots \\ b_n^- \\ -b_1^+ \\ -b_2^+ \\ \vdots \\ -b_n^+ \end{pmatrix}$

The structure of $C$ in equation (4) indicates that $c_{ij}, 1 \leq i, j \leq n$, and that $C = \begin{pmatrix} U & V \\ V & U \end{pmatrix}$ where $U$ contains the positive entries of $A$, $V$ the absolute values of the negative entries of $A$ and $A = U - V$. Now we find out $C^{-1}$ (if exist) and obtain the following:

$$X_{new} = C^{-1}b_{new} \qquad (5)$$

**Theorem 1:** If $C^{-1}$ exists it must have the same structure as $C$, i.e. $C^{-1} = \begin{pmatrix} E & F \\ F & E \end{pmatrix}$

**Theorem 2:** The unique solution $X^*_{new}$ of interval linear system (4) is a interval vector iff the inverse matrix of $C$ exists and nonnegative.

**Statement 1: Kirchhoff's Voltage Law (KVL)**

Kirchhoff's Voltage Law or KVL states that "in any closed loop network, the total voltage around the loop is equal to the sum of all the voltage drops within the same loop" which is also equal to zero.

## 5. PROBLEM FORMULATION: (Circuit Analysis with interval variables)

A linear time invariant electric circuit with constant coefficients and interval sources has a system of interval linear equations that can be expressed in the form as follows:

$$\begin{aligned}
&a_{11}I_1 + a_{12}I_2 + \ldots + a_{1n}I_n = [b_1^-, b_1^+] \\
&a_{21}I_1 + a_{22}I_2 + \ldots + a_{2n}I_n = [b_2^-, b_2^+] \\
&\vdots \\
&a_{n1}I_1 + a_{n2}I_2 + \ldots + a_{nn}I_n = [b_n^-, b_n^+]
\end{aligned}$$

Where $a_{ij}$ is a constant coefficient and $[b_i^-, b_i^+]$ is an interval source and $I_i$ can be the flow of current of *i*-th mesh in the circuit.

## 6. NUMERICAL EXAMPLE

To illustrate the proposed method for solving electrical circuit problem with interval valued sources, we have solved a numerical example. It is to be noted that for solving the said problem with interval valued sources, the voltage of each source is taken as interval valued. For this purpose, we have taken the interval version example from the book written by John O' Malley [3].

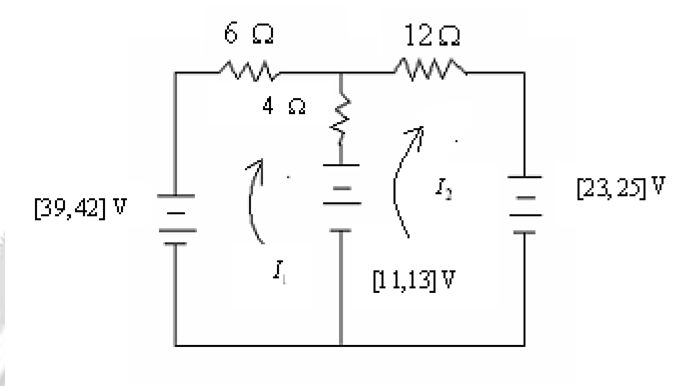


Fig. 1: A circuit with Interval current and Interval source

Here, the self-resistance of mesh 1 is $(6+4)\Omega = 10\Omega$, the mutual resistance of mesh 2 is $4\Omega$ and the sum of the voltage rises in the direction of $I_1$ is $([39,42]-[11,13])V = [26,31]V$. Therefore, for mesh 1 KVL equation is $10I_1 - 4I_2 = [26,31]$

For mesh 2 the self-resistance is $(4+12)\Omega = 16\Omega$ and the mutual resistance is $4\Omega$ and the sum of the voltage rises from voltage source is $([23,25]+[11,13])V = [34,38]V$. So for mesh 2, KVL equation is $-4I_1 + 16I_2 = [34,38]$.

It is to be noted that the symbol $V, \Omega$ represents the S.I unit of voltage and resistance respectively and their corresponding measures are volt and ohm. Hence, our problem is to find Interval current $I_1$ and $I_2$ which satisfies the following system of linear equations.

$$\begin{aligned} 10I_1 - 4I_2 &= [26,31] \\ -4I_1 + 16I_2 &= [34,38] \end{aligned} \tag{6}$$

Now, equation (6) can be written as follows:

$$\begin{aligned} 10[I_1^-, I_1^+] - 4[I_2^-, I_2^+] &= [26,31] \\ -4[I_1^-, I_1^+] + 16[I_2^-, I_2^+] &= [34,38] \end{aligned} \tag{7}$$

To solve equation (7), we have used equation (3) and we obtained the following:

$$\begin{pmatrix} 10 & 0 & 0 & 4 \\ 0 & 16 & 4 & 0 \\ 0 & 4 & 10 & 0 \\ 4 & 0 & 0 & 16 \end{pmatrix} \begin{pmatrix} I_1^- \\ I_2^- \\ -I_1^+ \\ -I_2^+ \end{pmatrix} = \begin{pmatrix} 26 \\ 34 \\ -31 \\ -38 \end{pmatrix} \tag{8}$$

Solving equation (8) we get $I_1^- = \frac{71}{18}$, $I_1^+ = \frac{79}{18}$, $I_2^- = \frac{116}{36}$ and $I_2^+ = \frac{121}{36}$

So, $I_1=[\frac{71}{18},\frac{79}{18}]=[3.94,4.39]$ and $I_2=[\frac{116}{36},\frac{121}{36}]=[3.22,3.36]$

**Special case:** If $V_1^-=V_1^+=40$, $V_2^-=V_2^+=12$ and $V_3^-=V_3^+=24$ then the circuit problem is same given in (John O' Malley, Theory and Problems of Basic Circuit Analysis, Second Edition, ISBN: 0-07-047824-4). Then the system of equations for this circuit is as follows:

$$\begin{aligned}10I_1-4I_2&=[28,28]\\ -4I_1+16I_2&=[36,36]\end{aligned} \tag{9}$$

To solve equation (9), we have used equation (3) and we get the following:

$$\begin{pmatrix}10&0&0&4\\0&16&4&0\\0&4&10&0\\4&0&0&16\end{pmatrix}\begin{pmatrix}I_1^-\\I_2^-\\-I_1^+\\-I_2^+\end{pmatrix}=\begin{pmatrix}28\\36\\-28\\-36\end{pmatrix} \tag{10}$$

Solving equation (10) we get $I_1^-=\frac{74}{18}$, $I_1^+=\frac{74}{18}$, $I_2^-=\frac{59}{18}$ and $I_2^+=\frac{59}{18}$ and finally, $I_1=[\frac{74}{18},\frac{74}{18}]=[4.11,4.11]$ and $I_2=[\frac{59}{18},\frac{59}{18}]=[3.28,3.28]$ which are same in existing problem. It is to be noted that, the S.I unit for measuring an electric current is Ampere.

From numerical results, it is seen that the flow of current in each mesh of the circuit is an interval valued. The current flow of mesh 1 is $I_1=[3.94,4.39]\Omega$ and current flow of mesh 2 is $I_2=[3.22,4.36]\Omega$. And when all the voltage is fixed valued then the current flow of mesh 1 is $4.11\Omega\in I_1$ and current flow of mesh 2 is $3.28\Omega\in I_2$.

## 7. CONCLUSIONS

In this paper, circuit analysis problem with interval valued current and voltage has been solved. In this circuit, current and voltage of each mesh has been considered as an imprecise number and this imprecise number has been represented by an interval number which is the more appropriate representation among other representations like, random variable representation with known probability distribution, fuzzy set with known fuzzy membership function or fuzzy number. For this purpose, we have introduced interval system of linear equations and its solution methodology has been discussed with a numerical example. From numerical result, it is clear that optimum current flow of each mesh is an interval valued. As a special case, when all the voltage is precise values, the optimum current flow obtained from the circuit in form of interval is same with the existing literature. Finally it can be summerized that the entire methodologies attempted in this paper well to solve any circuit design problems with interval parameters.

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