Applications of Conformal Mapping in Fluid Mechanics: A Review

Mohammed Mukhtar Mohammed Zabih
Maulana Azad College of Arts, Commerce and Science,
Rouza Bagh, Aurangabad.431001

ABSTRACT

Conformal mapping transforms the curves sustaining their angle is a great tool in complex analysis to map the domain (of non interest) to sought after domain. In this article we relieved the applications and importance of conformal mapping in the fluid mechanics, in particular in an irrotational flow of incompressible flow i.e. the flow of an ideal fluid in the complex plane.

Introduction

There are infinitely many problems in fluid mechanics that are difficult to solve in their original form in the given domain. Conformal mapping easily translates an equation and a domain from its original form into another, after some mathematical manipulations we get the solution and the solution can be sent back into the original form. For solving such problems in engineering and physics that are expressible as a function of a complex variable, but that show clumsy geometry, conformal mapping is vital. By choosing suitable function, the inconvenient geometry can be transformed into a much more suitable one. For example, one may be interested of calculating the velocity potential of flow of an ideal fluid [1-9] in an elliptic pipe that make certain angle. This problem is quite awkward to solve in its original form. However, by using a simple conformal mapping, the inconvenient curved angle is mapped to one of precisely π^c, it means that the two curved boundaries are transformed to a straight line and in the new region, the problem of calculating the velocity potential is easy.

Preliminaries

Conformal Mapping [10-16]

Let $w = f(z)$ be a complex mapping defined in a domain D and let z_0 be a point in D. Then we say that $w = f(z)$ is **conformal** at z_0 if for every pair of smooth oriented curves C_1 and C_2 in D intersecting at z_0 the angle between C_1 and C_2 at z_0 is equal to the angle between the image curves C'_1 and C'_2 at $f(z_0)$ in both magnitude and sense.

Theorem

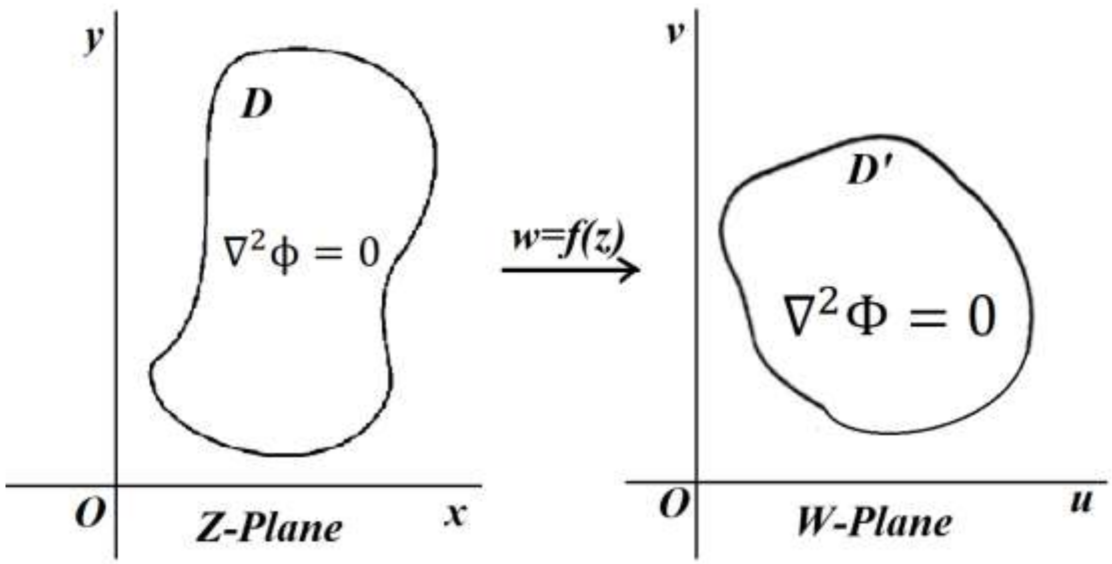
If f is an analytic function in a domain D containing z_0, and if $f'(z_0) \neq 0$, then $w = f(z)$ is a conformal mapping at z_0.

Theorem

$Let\ f(z) = u(x,y) + iv\,(x,y)$ be an analytic mapping of a domain D in the z-plane onto a domain D' in the w-plane. If the function $\Phi(u,v)$ is harmonic in D', then the function $\phi(x,y) = \Phi(u(x,y), v\,(x,y))$ is harmonic in D.

Discussion

There are lot of features of conformal mapping that can be used for various practical applications in distinct field although the essence remains the same. It preserves the shape and angle locally and maps of harmonic functions to the harmonic functions. These properties of conformal mapping make it beneficial in complex conditions, specifically complex velocity potential problems for general systems. However, the conformal mapping approach is restricted to problems that are two dimensional and to problems involving high degrees of symmetry. It is sometimes impossible to apply this technique when there is no symmetry. Also, this technique is applicable only for irrotational flow of an ideal fluid, as it depends on the harmonicity of the velocity potential which comes from the Laplace equation, an equation of continuity of an incompressible fluid. Furthermore, it gives the complex velocity potential of flow of an ideal fluid in the region that can be either mapped onto horizontal strip or vertical strip under the one-to-one, onto, conformal mapping. Following figure shows that the conformal mapping of harmonic function to harmonic function from one domain to another domain.



Conclusion

Conformal mapping helps in producing flows of an ideal fluid when the flow is two-dimensional with no trade off in the laws of fluid mechanics. However, it does not work for the practical three-dimensional flows found in nature.

References

1. A. Majda, Vorticity and incompressible flow, Cambridge University Press. Cambridge, England (2002)
2. J. S. Marshall, Inviscid incompressible flow. Wiley, New York (2001)
3. A. K. Mohanty, Fluid mechanics (2nd ed.), PHI Learning, New Delhi, India (2009)
4. B. R. Munson, B. R. (2002). Fundamentals of fluid mechanics (5th ed.), Wiley India, New Delhi, India (2002)
5. K. Muralidhar, Advanced engineering fluid mechanics (2nd ed.), Narosa Publishing House New Delhi, India (1996)
6. K. Muralidhar, Computational fluid flow and heat transfer (2nd ed.), Narosa Publishing House, New Delhi, India (1995)
7. S. Narasimhan, First course in fluid mechanics. Universities Press, Hyderabad, India (2006)
8. P. A. Narayana, Engineering fluid mechanics, Narosa Publishing House, New Delhi, India (2004)
9. . P. N. Narayana, Principles of fluid mechanics and fluid machines (3rd ed.). Universities Press, Hyderabad, India (2009)
10. L.V. Ahlfors, complex Analysis, McGraw-Hill Book Co. (1966)
11. W. Rudin, Real and complex Analysis, McGraw-Hill Book Co. (1966)
12. R. V. Churchrill, J. Brown and R. Verhey, Complex variable and application, McGraw Hill Co, New York, (1974)
13. J.B. Conway, Functions of One Complex Variable I, Springer-Verlag New York, second edition, (1978)
14. D. T.Piele, M. W. Firebaugh and R. Manulik, Application to conformal mapping to potential theory through computer graphics, American Mathematical month, (1977), 677-692.

15. I. N. Sneddon, Elements of Partial Differential Equations, McGraw- Hill Book Company, INC. Kogakusha Company, LTD. Tokyo. (1957)
16. D. G. Zill and Patrick D. Shanahan, A first Course in Complex Analysis with Applications, Second Edition. Jones and Bartlett Publishers. (2010)