B-Boundary in Pythagorean Neutrosophic Fuzzy Topological Spaces

Ameerah Abu Surrah [(1)] & Fatimah Alhawuyil [(2)]

[(1)]Department of Mathematics, Faculty of Education, Abi Issa, [University of Zawia], Libya: [a.abusurrah@zu.edu.ly]

[(2)] Department of Mathematics, Faculty of Education, Abi Issa, [University of Zawia], Libya: [f.alhawuyil@zu.edu.ly]

Abstract:

In this paper, we explore the structure and behavior of Pythagorean Neutrosophic Fuzzy Topological Spaces (PNFTS), which generalize and fuzzy topologies by incorporating the concepts of indeterminacy and hesitation though the Neutrosophic and Pythagorean frameworks. We introduce and study the concept of b-boundary within this context and investigate its fundamental properties. The b-boundary operator plays a critical role in describing the topological boundaries of sets in uncertain environments and offers a deeper understanding of topological separation in neutrosophic fuzzy space. We provide a set of examples to illustrate and validate the theoretical findings. Furthermore, we examine several theorems related to the behavior of b-boundaries under various set operations. The results obtained demonstrate that the b-boundary in Pythagorean Neutrosophic Fuzzy topological space maintains desirable topological characteristics while offering greater flexibility in dealing with imprecise, vague, or inconsistent information. This work not only extends existing topological concepts to the neutrosophic domain but also opens the door to future application in areas where uncertainty and indeterminacy.

الملخص:

في هذه الورقة، نستعرض بنية وسلوك الفضاءات الطوبولوجية الضبابية النيوتروسوفية (PNFTS)، والتي تعد تعميما لكل من الفضاءات الطوبولوجية التقليدية والضبابية من خلال دمج مفاهيم عدم التحديد والتردد باستخدام الاطارين النيوتروسوفي والفيثاغوري. قمنا بتقديم ودراسة مفهوم الحد -b ضمن هذا السياق، كما قمنا بتحليل خصائصها الاساسية. يعد عامل الحد- b أداة هامة في وصف حدود المجموعات في البيئات غير المؤكدة، حيث يوفر فهما أعمق لمفهوم الفصل الطوبولوجي في الفضاءات الضبابية النيوتروسوفية. ولقد أرفقنا مجموعة من الأمثلة التطبيقية لتوضيح النتائج النظرية والتحقق من صحتها. علاوة على ذلك، تناولنا عددا من النظريات المتعلقة بسلوك الحد -b. وقد أظهرت النتائج أن هذا الحد يحتفظ بخصائص طوبولوجية مرغوبة، مع تقديم مرونة أكبر عند التعامل مع البيانات غير الدقيقة أو الغامضة أو المتناقضة. ان هذا العمل لا يقتصر فقط علي توسيع المفاهيم الطوبولوجية التقليدية الي المجال النيوتروسوفي, بل يفتح أيضا افاقا جديدة للتطبيقات في المجالات التي تتسم بالغموض وعدم اليقين.

Keyword: Pythagorean Neutrosophic Sets, Pythagorean Neutrosophic Fuzzy Topological Spaces, b-Boundary in Pythagorean Neutrosophic Fuzzy topological space.

1. Introduction:

The concept of fuzzy set was introduced by Zadeh [13], Since then, the fuzzy set theory has influenced almost all branches of mathematics. The idea of fuzzy topological spaces was further introduced and developed by Chang [5]. Later, Atanassov [2,3] proposed the notion of intuitionistic fuzzy set.

The concept of the Neutrosophic set was introduced by Smarandache, while Wang et.al. studied the notion of interval Neutrosophic sets [10]. Furthermore, Salama and Albowi defined the concept of crisp sets and extended it to Neutrosophic crisp topological spaces [9]. Building upon these ideas, Yager introduced the concept of Pythagorean membership grades [12]. Later, Shena and Nirmala proposed the concept of Pythagorean Netrosophic open sets and established some properties on Pythagorean Neutrosophic open set [8].

Granados introduced the notion of Pythagorean Neutrosophic b-open set and showed some properties on Pythagorean Netrosophic b open set [6].

In this paper, we used the notion of Pythagorean Neutrosophic Topological Spaces for introducing the concept of b-boundary Pythagorean Neutrosophic fuzzy topological Spaces. Moreover, some of their properties are proved.

Preliminaries:

In this section, we will review the basic concepts of Pythagorean Neutrosophic sets, and Pythagorean Neutrosophic Topological Spaces.

Definition 2.1: [10] Let X be a non-empty set. Then A is said to be a Neutrosophic set (NS) of X if it is an object having the form $A = \{\langle x, \mu_A, \sigma_A, \gamma_A \rangle : x \in X\}$. Where the function $\mu_A : X \to [0,1], \sigma_A : X \to [0,1]$, and $\gamma_A : X \to [0,1]$ denote the degree of membership (namely $\mu_A(x)$), degree of indeterminacy (namely $\sigma_A(x)$), and degree of non-membership (namely $\gamma_A(x)$) of each element $x \in X$ to the set A, and satisfies the condition, $0 \leq \mu_A(x) + \sigma_A(x) + \gamma_A(x)) \leq 3 \ \forall \ x \in X$.

Definition 2.2: [10] Let A and B be NS of the form $A = \{\langle x, \mu_A(x), \sigma_A(x), \gamma_A(x)\rangle : x \in X\}$ and $B = \{\langle x, \mu_A(x), \sigma_A(x), \gamma_A(x)\rangle : x \in X\}$. Then

1. $A \subseteq B$ if and only if $\mu_A(x) \le \mu_B(x), \sigma_A(x) \ge \sigma_B(x)$ and $\gamma_A(x) \ge \gamma_B(x)$.
2. $A^c = \{\langle x, \sigma_A(x), \mu_A(x), \gamma_A(x)\rangle : x \in X\}$.
3. $A \cup B = \{\langle x, \mu_A(x) \vee \mu_B(x), \sigma_A(x) \wedge \sigma_B(x), \gamma_A(x) \wedge \gamma_{(x)}(x)\rangle : x \in X\}$.
4. $A \cap B = \{\langle x, \mu_A(x) \wedge \mu_B(x), \sigma_A(x) \vee \sigma_B(x), \gamma_A(x) \vee \gamma_B(x)\rangle : x \in X\}$
5. $0 = \{\langle x, 0,1,1\rangle : x \in X\}$, and $1 = \{\langle x, 1,0,0\rangle : x \in X\}$.

Definition 2.3: [12] A Pythagorean fuzzy subset A of a non-empty set X is a pair $A = (\mu_A, \vartheta_A)$ of a membership function $\mu_A : X \to [0,1]$ and a non-membership function $\vartheta_A : X \to [0,1]$, $0 \le \mu_A^2(x) + \vartheta_A^2(x) \le 1$ for each element $x \in X$. Supposing $\mu_A^2(x) + \vartheta_A^2 \le 1$, then there is a degree of indeterminacy of $x \in X$ to A defined by $\pi_A(x) = \sqrt{1 - [\mu_A^2(x) + \vartheta_A^2(x)]}$ and $\pi_A(x) \in [0,1]$. We denote the set of all Pythagorean fuzzy subsets over X by PFS(X).

Definition 2.4: [8] Let X be a non-empty set. Then A is said to be a Pythagorean Neutrosophic Fuzzy set (PNFS) of X there is a $A = \{\langle x, \mu_A, \sigma_A, \gamma_A\rangle : x \in X\}$ where the function $\mu_A : X \to [0,1], \sigma_A : X \to [0,1]$ and $\gamma_A : X \to [0,1]$ denote the degree of membership (namely $\mu_A(x)$), degree of indeterminacy (namely $\sigma_A(x)$) and degree of non-membership (namely $\gamma_A(x)$ of each element $x \in X$ to the set A, and satisfies the condition

$$0 \le \mu_A(x)^2 + \sigma_A(x)^2 + \gamma_A(x)^2 \le 1.$$

Definition 2.5: [8] A Pythagorean Neutrosophic Fuzzy Topology (PNFT) on a non-empty fixed set X is a family of τ of Pythagorean Neutrosophic Fuzzy Sets in X satisfying the following conditions:

i. $0,1 \in \tau$.
ii. For any $A_1, A_2 \in \tau$, we have $A_1 \cap A_2 \epsilon \tau$.
iii. $\cup_{i \epsilon I} A_i \in \tau$ for any arbitrary family $\{A_i; i \in I\} \subseteq \tau$.

In this case the pair (X, τ) is called a Pythagorean Neutrosophic Fuzzy Topological space (PNFTS) and each Pythagorean Neutrosophic Fuzzy subsets in τ is known as a Pythagorean Neutrosophic Fuzzy open set (PNFOS) in X. The complement of an open Pythagorean Neutrosophic Fuzzy subsets is called a closed Pythagorean Neutrosophic Fuzzy subset (PNFCS).

Definition 2.6: [8] Let (X, τ) be a PNFTS and A be a PNFS over X

1) Pythagorean Neutrosophic Fuzzy interior of A (PNFint (A)) is the union of all Pythagorean Neutrosophic Fuzzy open sets of X contained in A. That is,
$$PNFint(A) = \cup \{G : G \text{ is a } PNFO \text{ set in } X \text{ and } G \subseteq A\}.$$
2) Pythagorean Neutrosophic closure of A (PNFcl(A)) is the intersection of all Pythagorean neutrosophic Fuzzy closed sets of X containing A. That of all Pythagorean neutrosophic Fuzzy closed sets of X containing A. That is
$$PNFcl(A) = \cap \{H : H \text{ is a } PNFC \text{ set in } X \text{ and } A \subseteq H\}.$$

2. b Boundary in Pythagorean Neutrosophic Fuzzy:

Definition 3.1: [6] For a Pythagorean Neutrosophic Fuzzy set A in a Pythagorean Neutrosophic Fuzzy Topological Space (X, τ) is said to be Pythagorean Neutrosophic Fuzzy b- open set (PNFbOS) if $A \subseteq PNFint((PNFcl(A) \cup PNFcl(PNFint(A))$.

The complement of a Pythagorean Neutrosophic Fuzzy b - open set is called Pythagorean Neutrosophic Fuzzy b – closed set.

Definition 3.2:[6] Let (X, τ) be a PNFTS and A be a PNFS over X

1. Pythagorean Neutorsophic b Interior of A (PNFb int(A)) is the union of all Pythagorean Neutrosophic Fuzzy b open sets of X contained in A. That is
$$PNFbint(A) = \cup \{G : G \text{ is a } PNFbO \text{ set in } X \text{ and } G \subseteq A\}.$$
2. Pythagorean Neutrosophic Fuzzy b closure of A (PNFb cl(A)) is the intersection of all Pythagorean Neutrosophic Fuzzy b closed sets of X containing A. That is
$PNFbcl(A) = \cap \{H : H \text{ is a } PNFbC \text{ set in } X \text{ and } A \subseteq H\}$.

Theorem 3.5: [6] For a Pythagorean Neutrosophic Fuzzy Topological Space (X, τ) and $A, B \subseteq X$. Then following statements hold:

1) Every Pythagorean Neutrosophic Fuzzy open set is Pythagorean Neutrosophic Fuzzy b – open set.
2) $PNFb\ int(PNFb\ int(A)) = PNFb\ int(A)$.
3) $PNFcl(PNFb\ cL(A)) = PNFb\ cl(A)$.
4) Let A, B be two Pythagorean Neutrosophic Fuzzy b – open sets, then $PNFbOS(A) \cup PNFbOS(B) = PNFbOS(A \cup B)$.

5) Let A, B be two Pythagorean Neutrosophic Fuzzy b – closed sets, then $PNFbCS(A) \cap PNbCS(B) = PNbCS(A \cap B)$.
6) For any two sets $A, B, PNFb\ int(A) \cap PNFbint(B) = PNFbint(A \cap B)$.
7) For any two sets $A, B, PNFbcl(A) \cup PNFbcl(B) = PNFbcl(A \cup B)$.
8) If A is $PNFbOS\ in\ (X, \tau)$, then $A = PNFbint(A)$.
9) If $A \subseteq B, then\ PNFbint(A) \subseteq PNFbint(B)$.
10) For any two sets $A, B, PNFbint(A) \cup PNFbint(B) \subseteq PNFbint(A \cup B)$.
11) If A is $PNFbCS(X, \tau), then\ A = PNFbcl(A)$.
12) If $A \subseteq B, then\ PNFbcl(A) \subseteq PNFbcl(B)$.
13) For any two sets $A, B, PNFbcl(A \cap B) \subseteq PNFbcl(A) \cap PNFbcl(B)$.

3. Main Results:

Definition 4.1: Let A be PNFS in a PNFTS(X, τ). Then the Pythagorean Neutrosophic Fuzzy boundary of A is defined as $PNFbd(A) = cl(A) \cap cl(A^c)$. $PNFbd(A)$ is a Pythagorean Neutrosophic Fuzzy closed set.

Definition 4.2: Let A be PNFS in a PNFT(X, τ). Then the Pythagorean Neutrosophic Fuzzy b – boundary of A is defined as $PNFb(bd(A)) = bcl(A) \cap bcl(A^c)$. $PNFb(bd(A))$ is a Pythagorean Neutrosophic Fuzzy b – closed set.

Remark 4.3: In classical topology, for any arbitrary set A in topological space X, we have: $A \cup bd(A) = cl(A)$. However, for an arbitrary Pythagorean Neutrosophic Fuzzy set A in a $PNFS(X, \tau)$, the equality may not hold. $A \cup PNFb\big(bd(A)\big) \subseteq cl(A)$,

Example 4.4: Let $X = \{a, b\}$, $A = \{x, (a, 0.2,0.4,0.3), (b, 0.3,0.6,0.5)\}$, and $B = \{x, (a, 0.2,0.3,0.2), (b, 0.1,0.6,0.6)\}$ then the family $\tau = \{0,1, A\}$ is PNFT on X,since $cl(B) = A \cup B$ but $PNb(bd(B)) = B$, then we have $B \cup PNFb\big(bd(B)\big) \neq A \cup B$.

Proposition 4.5: Let A and B be PNFSs in a $PNFTS(X, \tau)$. Then the following condition hold

1) $PNFb\big(bd(A)\big) \subseteq PNF\big(bd(A)\big)$.
2) $bcl(PNFb\big(bd(A)\big) \subseteq PNF\big(bd(A)\big)$.
3) $PNFb\big(bd(A)\big) = PNFb\big(bd(A^c)\big)$.
4) If A is $PNFbCS$, then $PNFb\big(bd(A)\big) \subseteq A$.
5) If A is $PNFbOS$, then $PNFb\big(bd(A)\big) \subseteq A^c$.
6) $(PNFb(bd(A))^c = bint(A) \cup bint(A^c)$.

Proof:

1) Since $bcl(A) \subseteq cl(A)$ and $bcl(A^c) \subseteq cl(A^c)$. Then $PNFb\big(bd(A)\big) = bcl(A) \cap bcl(A^c) \subseteq cl(A) \cap cl(A^c) = PNF(bd(A)$. Hence, $PNFb\big(bd(A)\big) \subseteq PNF\big(bd(A)\big)$.
2) $bcl(PNFb\big(bd(A)\big) = bcl(bcl(A) \cap bcl(A)^c) \subseteq bcl\big(bcl(A)\big) \cap bcl\big(bcl(A^c)\big) = bcl(A) \cap bcl(A^c) = PNFb(bd(A)) \subseteq PNF\big(bd(A)\big)$.
3) $PNFb(bd(A)) = bcl(A) \cap bcl(A^c) = bcl(A^c)^c \cap bcl(A^c) = bcl(A^c) \cap bcl(A^c)^c = PNFb(bd(A^c))$.
4) Let A be an $PNFbCS$. Then
$PNF\big(bd(A)\big) = bcl(A) \cap bcl(A)^c \subseteq bcl(A) = A \Longrightarrow PNFb\big(bd(A)\big) \subseteq A$.
5) If A is PNFbOS, then A^c is PNFbCS. By (3) and (4) we have $PNF(bd(A)) = PNFb(bd(A^c)\ and\ PNFb(bd(A^c)) \subseteq A^c \Longrightarrow PNFb(bd(A)) \subseteq A^c$.
6) $(PNFb(bd(A))^c = (bcl(A) \cap bcl(A^c))^c = (bcl(A))^c \cup (bcl(A^c))^c = bint(A^c) \cup bint(A)$. Then we have $(PNFb(bd(A))^c = bint(A) \cup bint(A^c)$.

Example 4.6: Let $X = \{a, b\}$, $A_1 = \{x, (a, 0.8,0.1,0.1), (b, 0.5,0.2,0.3)\}$, $A_2 = \{x, (a, 0.7,0.2,0,1), (b, 0.6,0.2,0.2)\}$, define $\tau = \{0,1, A_1, A_2\}$ is PNFT, if $A = \{x, (a, 0.7,0.2,0.1), (b, 0.4,0.3,0.3)\}$, note that
$PNF\big(bd(A)\big) = \{x, (a, 0.8,0.1,0.1), (b, 0.6,0.2,0.2)$, then we have
$PNFb\big(bd(A)\big) = \{x, (a, 0.7,0.2,0.1), (b, 0.5,0.3,0.2)\} \subseteq PNF\big(bd(A)\big)$, and
$bcl(PNFb\big(bd(A)\big) = \{x, (a, 0.75,0.1,0.1), (b, 0.55,0.2,0.2\} \subseteq PNF\big(bd(A)\big)$.

Proposition 4.7: Let A be a Pythagorean Neutrosophic Fuzzy Topology set in a $PNFTS(X, \tau)$. Then,

1) $PNFb\big(bd(A)\big) = bcl(A) \cap (bint(A))^c$.
2) $PNFb(bd\big(bint(A)\big) \subseteq PNFb\big(bd(A)\big)$.
3) $bint(A) \subseteq A \cap \big(PNFb\big(bd(A)\big)^c\big)$.

Proof:

1) Since $PNFb(bd(A)) = bcl(A) \cap bcl(A^c)$. But $bcl(A^c) = (bint(A))^c$. Then $PNFb(bd(A)) = bcl(A) \cap (bint(A))^c$.
2) $PNFb\left(bd\big(bint(A)\big)\right) = bcl\big(bint(A)\big) \cap bcl(bint(A))^c = bcl\big(bint(A)\big) \cap bcl(bcl(A)^c) = bcl\big(bint(A)\big) \cap bcl(A)^c \subseteq bcl(A) \cap bcl(A)^c = PNFb\big(bd(A)\big)$.
3) Consider
$$A \cap \left(PNFb\big(bd(A)\big)^c\right) = A \cap [bcl(A) \cap bcl(A)^c]^c = A \cap [bint(A)^c \cup bint(A)]$$
$$= (A \cap bint(A)^c) \cup \big(A \cap bint(A)\big) \subseteq bint(A).$$

Example 4.8: Let (X,τ) be PNFTS defined in example (4.6), and
$B = \{x,(a,0.2,0.4,0.3),(b,0.5,0.2,0.1)\}$, $b\big(int(B)\big) = 0 \subseteq B \cup b(bd(B)^C) = \{x,(a,0,0.4,0.3),(b,0,0.2,0.1)\}$.

Theorem 4.9: Let A and B be $PNFSs$ in a $PNFTS(X,\tau)$. Then $PNFb\big(bd(A \cup B)\big) \subseteq PNFb\big(bd(A)\big) \cup PNFb\big(bd(B)\big)$.

Proof:

From Theorem (3.5), we have
$$PNFb\big(bd(A \cup B)\big) = bcl(A \cup B) \cap bcl(A \cup B)^c = bcl(A \cup B) \cap bcl\big((A^c) \cap (B^c)\big)$$
$$\subseteq \big(bcl(A) \cup bcl(B)\big) \cap \big(bcl(A^c) \cap bcl(B^c)\big)$$
$$= bcl(A) \cap (bcl(A^c) \cap bcl(B^c)) \cup (bcl(B) \cap \big(bcl(A^c) \cap bcl(B^c)\big)$$
$$= (PNFb(bd(A)) \cap (bcl(A^c)) \cup (PNFb(bd(B)) \cap (bcl(A^c))$$
$$\subseteq PNFb(bd(A)) \cup PNFb\big(bd(B)\big).$$
$$\Rightarrow PNFb\big(bd(A \cup B)\big) \subseteq PNFb\big(bd(A)\big) \cup PNFb\big(bd(B)\big).$$

Example 4.10: Let $X = \{a,b\}$, $A = \{x,(a,0.6,0.3,0.1),(b,0.3,0.4,0.2)\}$,$\tau = \{0,1,A\}$,
and then we have
$$B = \{x,(0.3,0.2,0.3),(b,0.5,0.2,0.2)\}, C = \{x,(a,0.2,0.4,0.3),(b,0.4,0.3,0.2)\}$$
$$PNFb\big(bd(C \cup B)\big) = \{x,(0.3,0.4,0.2),(b,0.4,0.2,0.3)\} \subseteq \{x,(a,0.4,0.4,0.3),(b,0.5,0.3,0.4)\}$$
$$= PNFb\big(bd(B)\big) \cup PNFb\big(bd(C)\big).$$

Theorem 4.11: Let A and B be PNFSs in a $PNFTS(X,\tau)$. Then
$PNFb(bd(A \cap B)) \subseteq \Big(PNFb\big(bd(A)\big) \cap bcl(B)\Big) \cup \Big(PNFb\big(bd(B)\big) \cap bcl(A)\Big)$.

Proof:
$$PNFb\big(bd(A \cap B)\big) = bcl(A \cap B) \cap bcl(A \cap B)^c \subseteq \big(bcl(A) \cap bcl(B)\big) \cap \big(bcl(A^c) \cup bcl(B^c)\big)$$
$$= \big[bcl(A) \cap \big(bcl(B)\big) \cap bcl(A^c)\big] \cup [\big(bcl(A)\big) \cap \big(bcl(B)\big) \cap bcl(B^c)\big]$$
$$= (PNFb(bd(A)) \cap (bcl(B)) \cup (PNFb(bd(B)) \cap (bcl(A))$$
$$\Rightarrow PNFb(bd(A \cap B)) \subseteq (PNFb\big(bd(A)\big) \cap bcl(B)) \cup (PNFb\big(bd(B)\big) \cap \big(bcl(A)\big).$$

Example 4.12: Let $X = \{a,b\}$, $A = \{x,(a,0.4,0.4,0.3),(b,0.6,0.3,0.5\}$ and $\tau = \{0,1,A\}$
be PNFT,
$B = \{x,(a,0.3,0.5,0.4),(b,0.7,0.2,0.3\}$,
$C = \{x,(a,0.4,0.3,0.5),(0.2,0.6,0.4)\}$,
then we have $PNFb(bd(B \cap C) = A \subseteq \Big(PNFb\big(bd(B)\big) \cap bcl(C)\Big) \cup \Big(PNFb\big(bd(C)\big) \cap bcl(B)\Big) = X$.

Conclusion:

In this paper, we explored the structure of Pythagorean Neutrosophic Fuzzy Topological Spaces and introduced the concept of b-boundary within this framework. Several theoretical properties were examined, and illustrative examples were provided to demonstrate the validity of the proposed concepts. The results confirm that the b-boundary operator in this generalized topological environment behaves consistently with known topological principles. These findings may open new directions for future research in the study of fuzzy, Neutrosophic, and intuitionistic topologies, particularly in application involving vague or incomplete information.

References:

1. Arockiarani, I., Dhavaseelan, R., Jafari, S. and Parimala, M.: On some notations and functions in neutrosophic topological spaces, Neutrosophic sets and Systems. Vol. 16(2017):16-19.
2. Atannasov, K.: Intuitionistic fuzzy sets, Fuzzy sets and Systems. Vol. 20(1) (1965): 87-96.
3. Atannasov, K.: Intuitionistic fuzzy sets, Springer Physica-Verlag, Heidelberg.(1999).
4. Banu, V. and Chandrasekar, S.: Neutrosophic α gs Continuity And Neutrosophic α gs Irresolute Maps, Neutrosophic sets and Systems. Vo. 27 (2019): 163 170.
5. Chang, C.: Fuzzy topological spaces, J. Math. Anal Appl. Vol. 24(1)(1968): 182 190.

6. Granado, C.: Pyhthagorean neutorsophic b-open sets in Pythagorean nutorsophic topological spaces, South Asian Journal of Mathematics, vol. 10(2) (2020): 62-70.
7. Jansi, R., Mohana, K. and Smarandache, F.: Correlation Measure for Pythagorean Neutrosophic Sets with T and F Dependent Neutrosophic components, Neutrosophic sets and Systems. Vol 30 (2019): 202 212.
8. Sneha, T. and Nirmala, F.: Pythagorean neutrosophic b-open and semi-open sets in Pythagorean neutrosophic topological space, Infokara Research. Vol. 9(1) (2020): 860 872.
9. Salama, A. and Albowi, S.: Neutrosophic set and Nutrosophic topological space, IOSR Jouranl of Mathematics. Vol. 3.35 31 (2012) (4).
10. Smarandache, F.: A unifying field in logics-neotrusophic: Neutrosophic probability, set and logic, Rehoboth: American Research Press. (1999).
11. Xu, Z. and Yagar, R.: Some geometric aggregation operations based on intuitionistic fuzzy sets, Int. J. Gen. Syst. Vol. 35 (2006): 417 433.
12. Yager, R. and Abbasov, A.: Pythagorean membership grades, complex numbers and decision making, Int. J . Intell Syst. Vol. 28(2013): 436 452.
13. Zadeh, L.: Fuzzy sets, Inform and control. Vol. 8 (1965).