CFD ANALYSIS FOR SUPERSONIC FLOW OVER A WEDGE

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ABSTRACT

A wedge is triangular shaped geometrical structure. It is mostly used as supersonic airfoils and nose of supersonic airplanes. It is typically inclined plane which is at an angle with the horizontal. This paper helps in explaining most of the concepts related to wedge. In this paper the principle of working of wedges with the variation of half-wedge angle and also with variation of Mach number has been done. Theoretical analysis of supersonic flow over wedge is also done. The variation of flow parameters like Pressure, Temperature and Mach number is visualised using computational fluid Dynamics and it's contours and graphs has been plotted. The simulation of shockwave (attached and detached) through CFD is also analyzed.

Key words: *Wedge, Supersonic flow, Wave angle, Wedge-angle Variation, Mach number Variation, Oblique Shock, Detached Shock Computational Fluid Dynamics, Ansys-Fluent.*

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1. INTRODUCTION

Nomenclature:

β = Wave angle (degree)
V = Velocity (m/s^2)
M = Mach number
θ = Half-wedge angle (degree)
w = Tangential component of velocity (m/s^2)
u = Normal component of velocity (m/s^2)
M_t = Tangential Mach number
M_n = Normal Mach number
h = Enthalpy (J)
ρ = Density (kg/m^3)
p = pressure (Pa)
c_p = Specific heat at constant pressure (J/kg K)
γ = Specific heat ratio
Subscripts
1 = Upstream flow
2 = Downstream flow

2. WEDGE

A wedge is a triangular shaped or in other words a plane inclined with an angle to the horizontal.

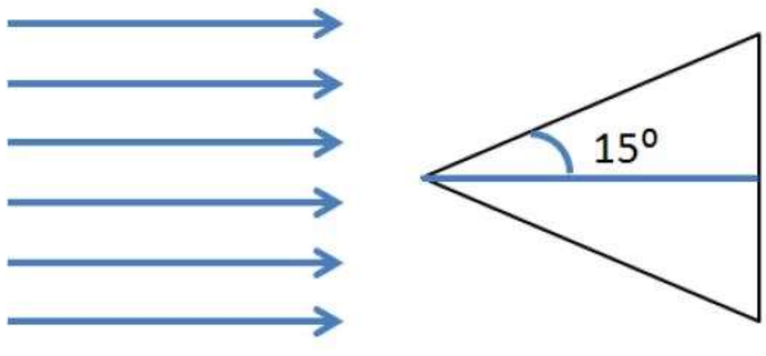

Fig. 1: Wedge

2.1 Shockwave

A shockwave is an extremely thin region, typically on the order of 10^{-5} cm across which the flow properties can change drastically.

2.1.1 Oblique Shockwave

When a shockwave makes an oblique angle with respect to upstream flow, it is termed as oblique shockwave. Oblique shockwave occurs when a supersonic flow is encountered at the wedge that effectively turns the flow into itself. Oblique shock is generally created at the nose of the wedge. Downstream of the oblique shock the properties change drastically.

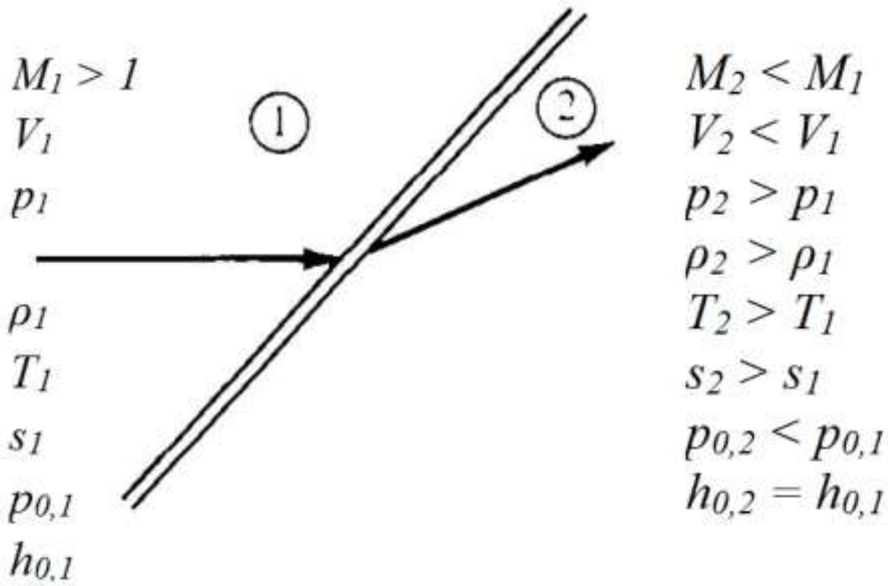

Fig. 2: Oblique Shockwave Properties Variation

2.2 Oblique Shock Equations

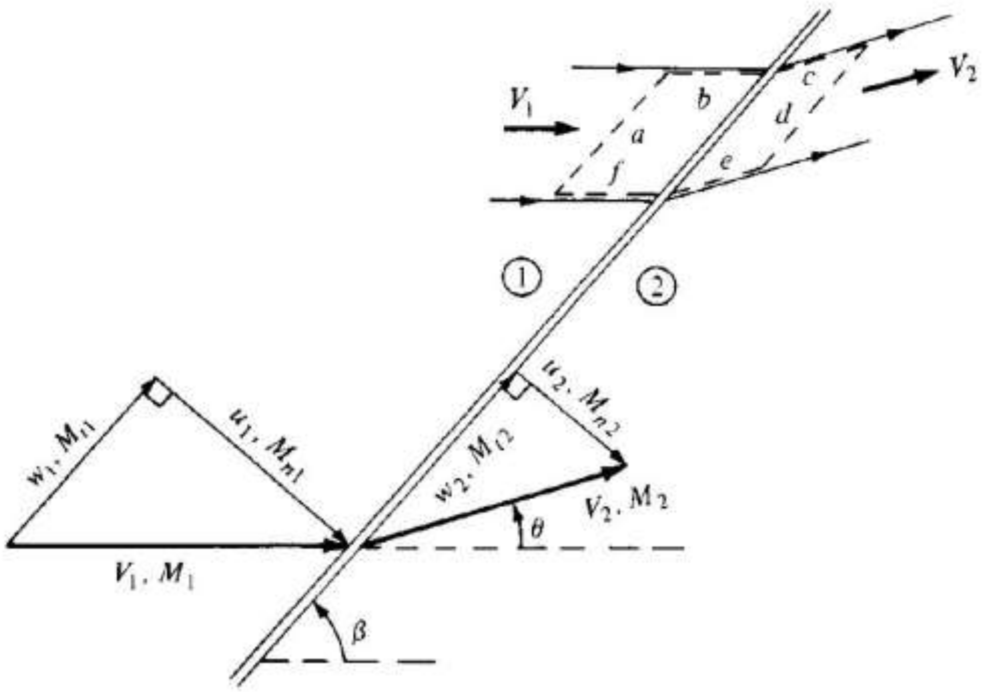

Fig. 3: Oblique shock properties

The continuity equation for an oblique shockwave is

$$\rho_1 u_1 = \rho_2 u_2 \tag{1}$$

The normal momentum equation for an oblique shockwave is

$$p_1 + \rho_1 u_1^2 = p_2 + \rho_2 u_2^2 \tag{2}$$

The energy equation for an oblique shockwave is

$$h_1 + \frac{u_1^2}{2} = h_2 + \frac{u_2^2}{2} \tag{3}$$

Expressions for changes across an oblique shock in terms of the normal component of the upstream Mach number $M_{n,1}$

$$M_{n,1} = M_1 \sin\beta \tag{4}$$

Thermodynamic properties ρ_2/ρ_1, p_2/p_1 and T_2/T_1 across a normal shockwave.

$$M_{n,2}^2 = \frac{1 + \left[\frac{\gamma - 1}{2}\right] M_{n,1}^2}{\gamma M_{n,1}^2 - (\gamma - 1)/2} \tag{5}$$

$$\frac{\rho_2}{\rho_1} = \frac{(\gamma + 1) M_{n,1}^2}{2 + (\gamma - 1) M_{n,1}^2} \tag{6}$$

$$\frac{p_2}{p_1} = 1 + \frac{2\gamma}{\gamma + 1}\left(M_{n,1}^2 - 1\right) \tag{7}$$

$$\frac{T_2}{T_1} = \frac{p_2}{p_1}\frac{\rho_1}{\rho_1} = \left[1 + \frac{2\gamma}{\gamma+1}\left(M_{n,1}^2 - 1\right)\right]\frac{2 + (\gamma-1)M_{n,1}^2}{(\gamma+1)M_{n,1}^2} \quad (8)$$

$M_{n,2}$ is the normal Mach number behind the shock wave. The downstream Mach number M_2 can be found from $M_{n,2}$ and the geometry of Fig 3 as

$$M_2 = \frac{M_{n,2}}{\sin(\beta - \theta)} \quad (9)$$

The changes across an oblique shock depend on two parameters- M_1 and β

$$\tan\theta = 2\cot\beta\ \frac{M_1^2\ sin^2\ \beta - 1}{M_1^2\ (\gamma +\ cos2\beta) +\ 2} \quad (10)$$

This is called *θ-β-M* relation and it specifies θ as a unique function of M_1 and β. This relation is vital to the analysis of oblique shock waves and results from it are plotted in Fig 4.

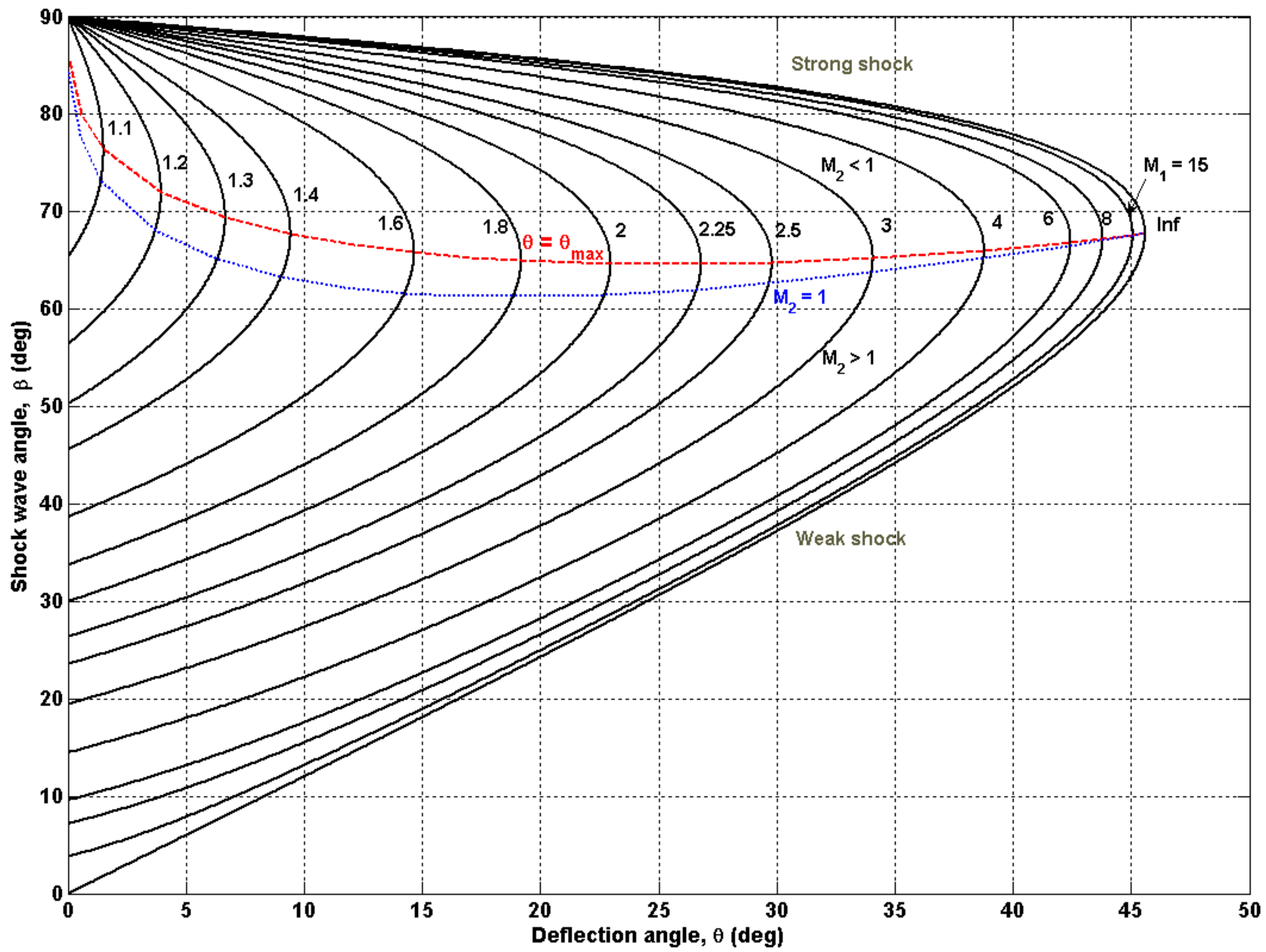

Fig. 4: The *θ-β-M diagram*

The *θ-β-M* diagram illustrates three physical phenomena associated with oblique shock waves.

1. For any given upstream Mach number M_1, there is a maximum deflection angle θ_{max}. If the physical geometry is such that $\theta > \theta_{max}$, then no solution exists for a straight oblique shock. Instead, nature establishes a curved shock, detached from the corner or the nose of the body. The value of θ_{max} increases with increasing M_1,

hence, at higher Mach numbers, the straight oblique shock solution can exist at higher deflection angles. However there is a limit; a M_1 approaches infinity, θ_{max} approaches 45.5^o (for γ = 1.4).

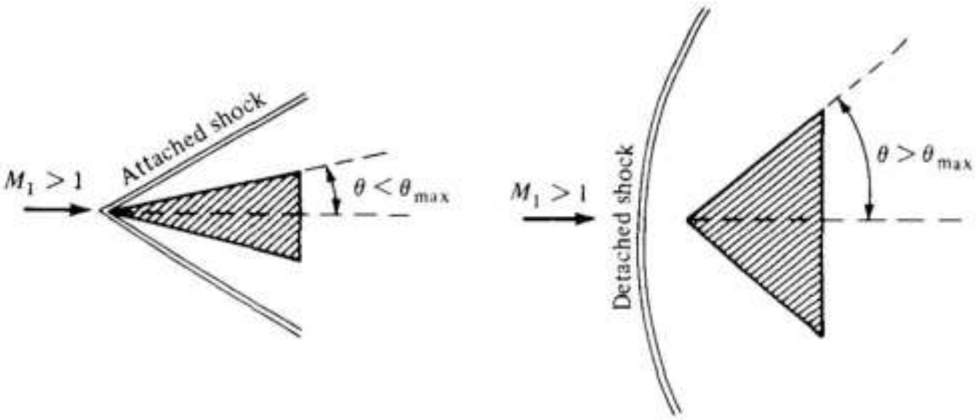

Fig. 5: Attached and detached shocks

2. If θ = 0, then β equals either 90^o or μ. The case of β = 90^o corresponds to a normal shock wave.

3. For attached shocks with a fixed deflection angle, as the upstream Mach number M_1 increases, the wave angle β decreases, and the shock wave becomes stronger. Going in the other direction as M_1 decreases, the wave angle increases, and the shock becomes weaker. Finally, if M_1 is decreased enough, the shock wave will detached.

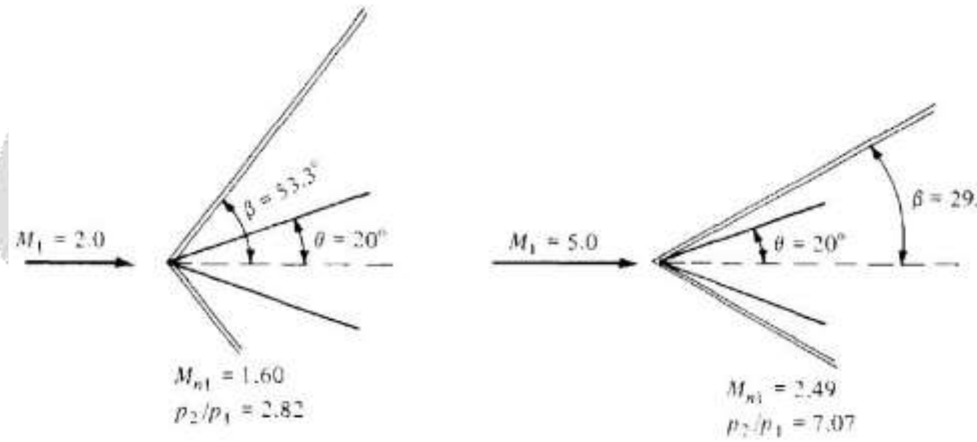

Fig. 6: Effect of increasing upstream Mach number

4. For attached shocks with a fixed upstream Mach number, as the deflection angle increases, the wave angle β increases, and the shock becomes stronger. However, once θ exceeds θ_{max} , the shock wave will become detached.

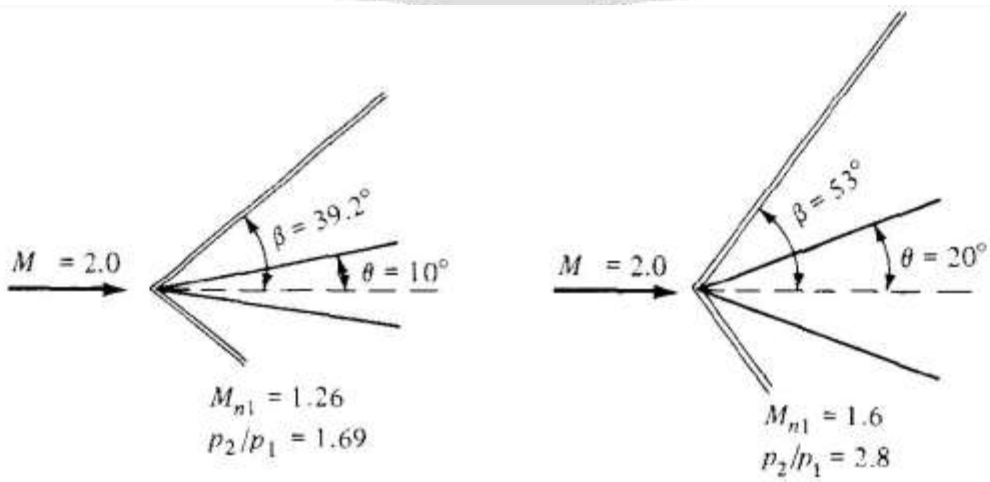

Fig. 7: Effect of increasing deflection angle.

3. SIMULATION OF WEDGE

Computer simulation of wedge was done using computational fluid dynamics (CFD). CFD is method to solve complex problems involving fluid flow.
The above conditions were taken as mentioned in the previous section and the computer simulation that is the analysis was done using Ansys-Fluent. When we performed this analysis, we found out the variation of parameters as we did in the case of theoretical treatment.

The CFD Analysis was done in the following steps:

- Modelling
- Meshing
- Pre processing
- Solver/Processing
- Post processing

3.1. Modelling

The modelling of the wedge was done in Ansys Design modeller. The geometry was created using *Line* tool under *Draw* tab in the *Sketching Toolboxes*. To add the dimensions for the geometry, click *Dimensions* under *sketching toolbox*, then select *General* for horizontal and vertical lines and select *Angle* to give angle to the wedge. To create surface form sketch, under Menu tool bar select Concept, select Surfaces from sketches, then select the sketch, apply, then click Generate.

	Dimensions
H2	0.5 m
H3	1.5 m
V1	1.259 m
A5	10, 20 & 30 degree

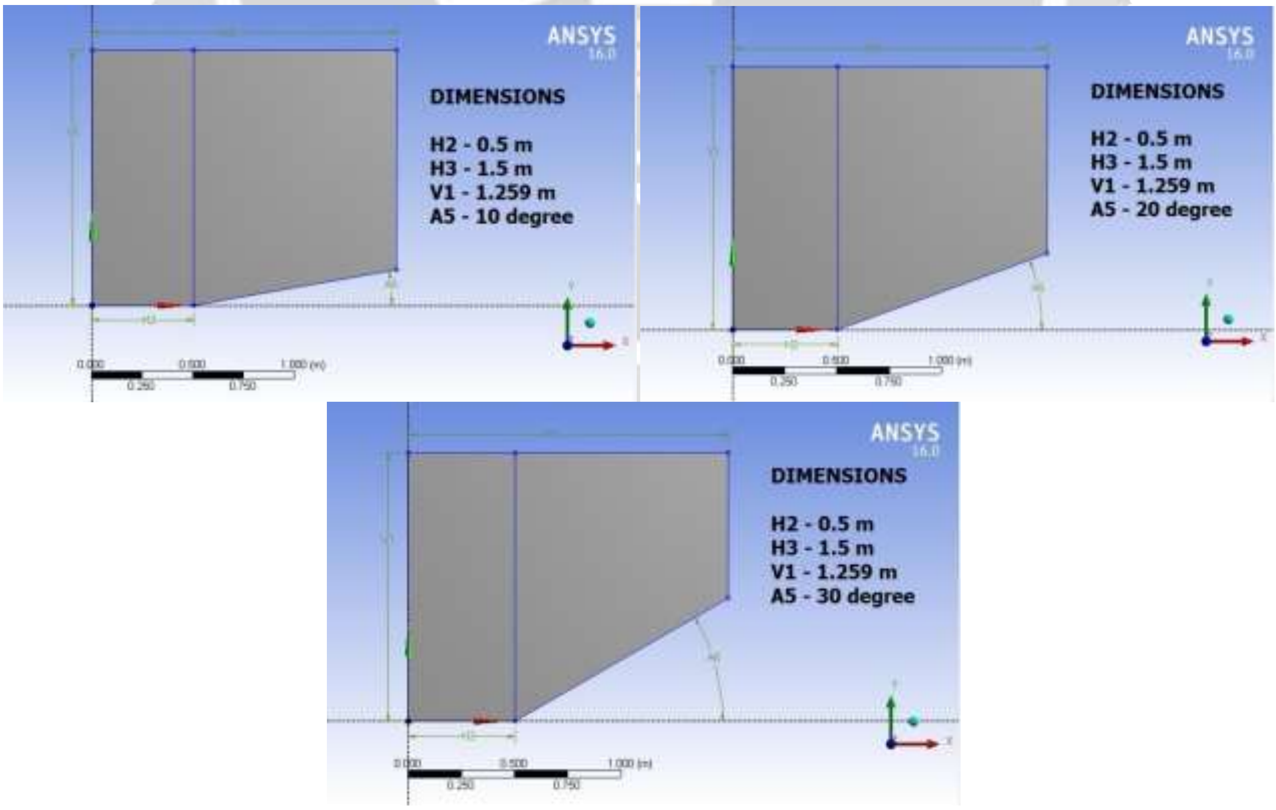

Fig. 8: CAD model of wedge at 10, 20 and 30 degree.

3.1. Meshing

The model created by above dimensions was meshed in mesh mode of Ansys component systems. Meshing is all about converting an infinite number of particles model of finite number of particles. The details of mesh are Physics preference : CFD, Solver preference: Fluent. Mapped face Meshing was used with the face. Then Body Sizing, the element size was chosen 0.05m. The number of nodes were calculated as 775 and number of elements was 720.

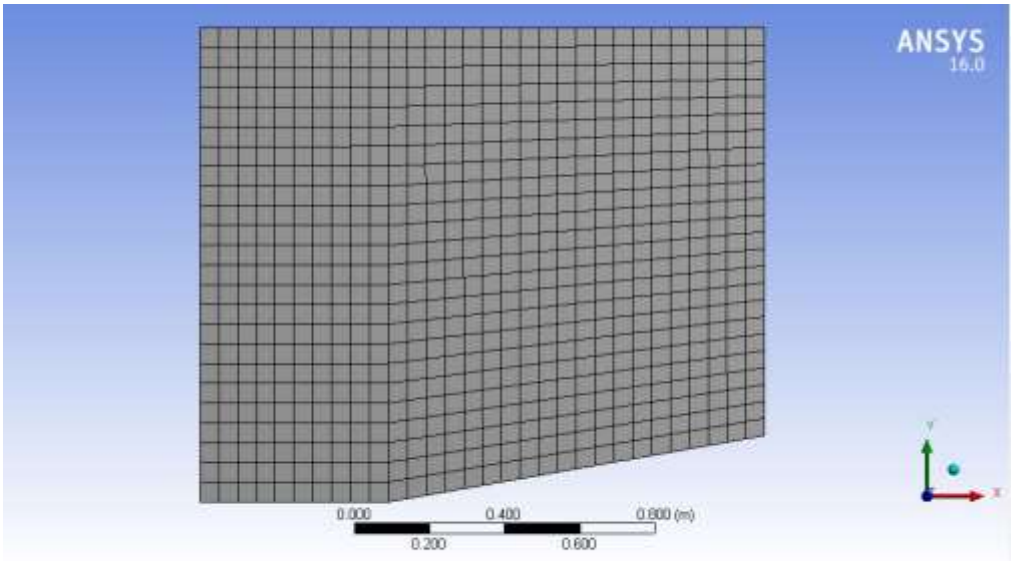

Fig. 9: Meshed Model

Then boundaries of the model was defined by Create Named Selection as Pressure Far Field, Symmetry, and wedge.

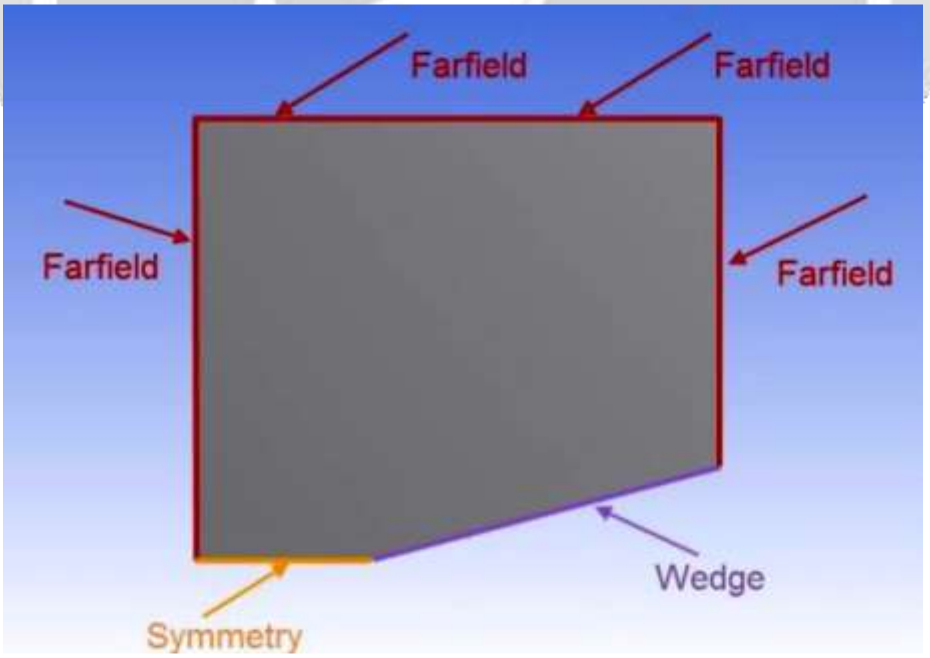

Fig.10: Boundaries of Model

3.3. Pre Processing

The next step of the CFD after meshing is pre processing. In pre processing appropriate boundary conditions are applied to the meshed model. The pre processing was done in Ansys Fluent.

Table 1: Pre Processing Details

<table>
<tr><td>General</td><td colspan="4">Solver Type : Density
2D space : Planer
Time : Steady</td></tr>
<tr><td>Models</td><td colspan="4">Energy : On
Viscous : Inviscid</td></tr>
<tr><td>Materials</td><td colspan="4">Density : Ideal Gas
c_p : 1006.43 J/kgK
γ : 1.4
Molecular Weight : 28.966 kg/kgmol</td></tr>
<tr><td rowspan="10">Boundary Conditions

Pressure-Far-Field</td><td>Half-Wedge angle θ (degree)</td><td>Mach Number</td><td>Gauge Pressure (Pa)</td><td>Temperature (k)</td></tr>
<tr><td>10</td><td>2</td><td>101325</td><td>300</td></tr>
<tr><td>10</td><td>3</td><td>101325</td><td>300</td></tr>
<tr><td>10</td><td>5</td><td>101325</td><td>300</td></tr>
<tr><td>20</td><td>2</td><td>101325</td><td>300</td></tr>
<tr><td>20</td><td>3</td><td>101325</td><td>300</td></tr>
<tr><td>20</td><td>5</td><td>101325</td><td>300</td></tr>
<tr><td>30</td><td>2</td><td>101325</td><td>300</td></tr>
<tr><td>30</td><td>3</td><td>101325</td><td>300</td></tr>
<tr><td>30</td><td>5</td><td>101325</td><td>300</td></tr>
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3.4. Solver

The next step is solver. In solver the solution is initialized and calculation is preceded with the desired number of iterations. It is the most important step of CFD analysis. Using Ansys-Fluent, it is possible to solve the governing equation related to the flow physical properties.

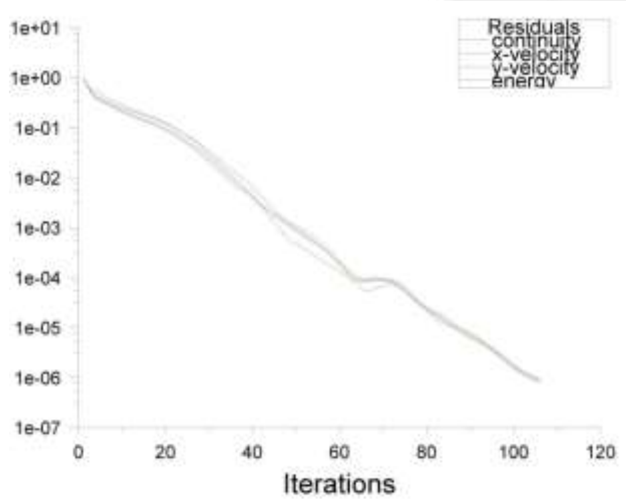
Table 2: Solver Details

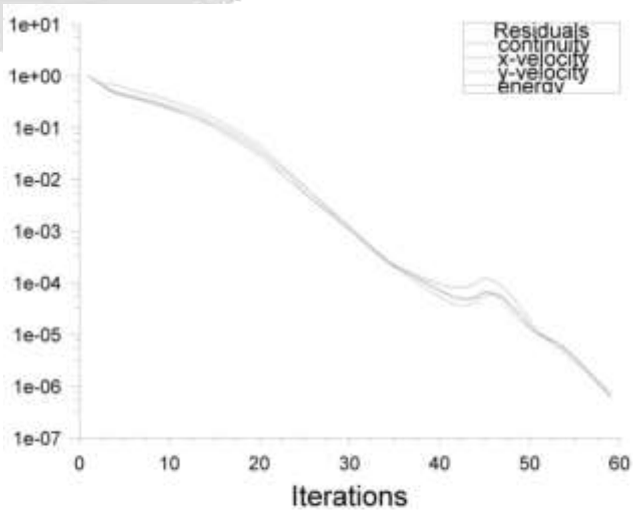
Solver Details	
Solution Controls	Courant number = 5
Solution Initialization	Compute from = pressure-far-field
Run Calculation	Number of iterations = 1000

- **Half-Wedge angle (degree) = 10**

Table 3: Scaled Residuals foe Half-wedge angle =10^o

Mach Number	**No of Iterations (Convergence)**
2	106
3	59
5	46





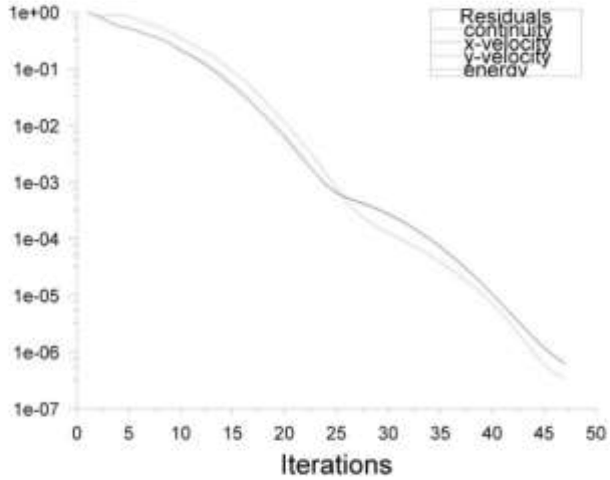


Fig- 11: Scaled Residuals

- **Half-Wedge angle (degree) = 20**

Table 4: Scaled Residuals foe Half-wedge angle =20^o

Mach Number	No of Iterations (Convergence)
2	435
3	80
5	61

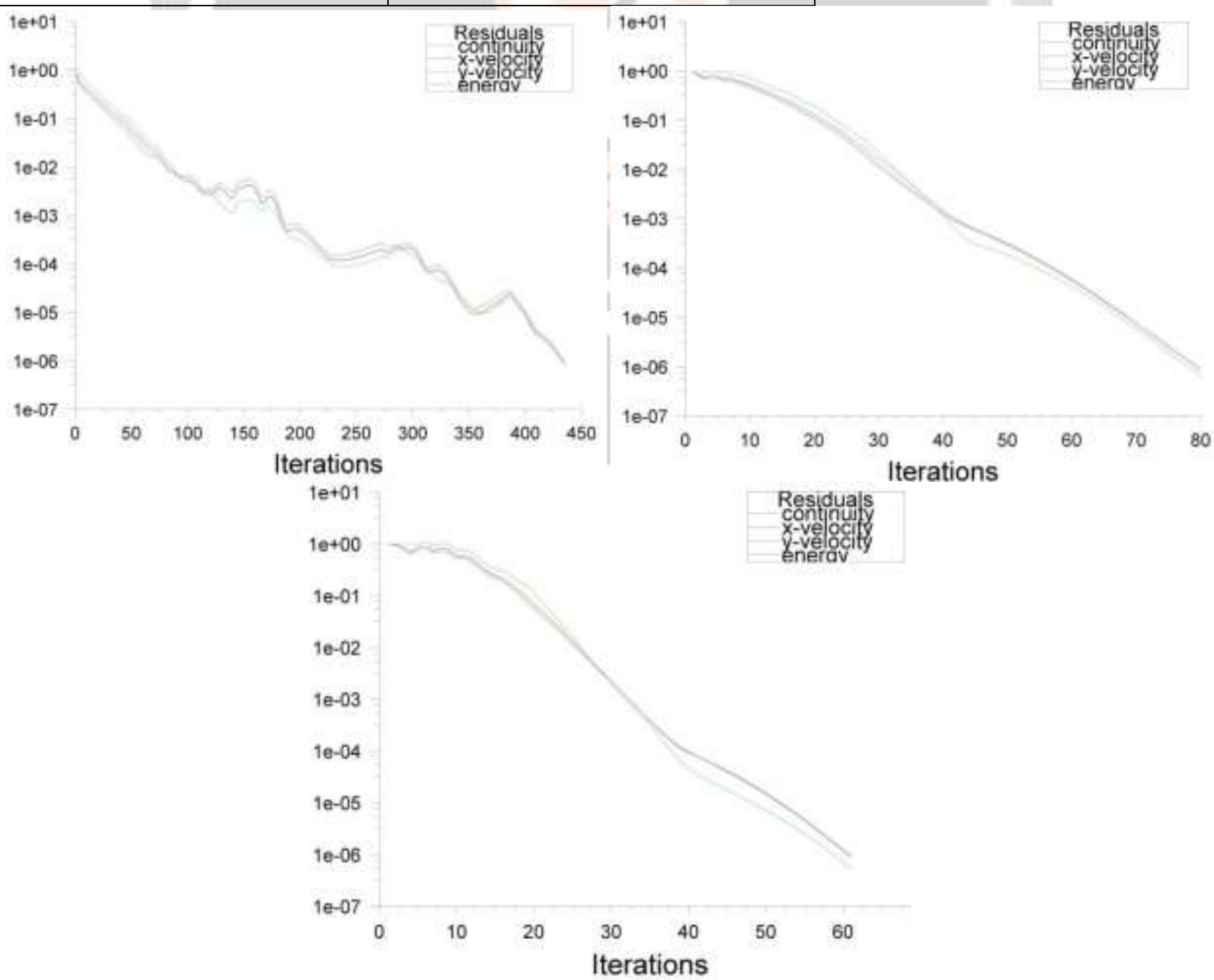


Fig- 12: Scaled Residuals

- **Half-Wedge angle (degree) = 30**

Table 5: Scaled Residuals foe Half-wedge angle =30°

Mach Number	No of Iterations (Convergence)
2	586
3	409
5	85

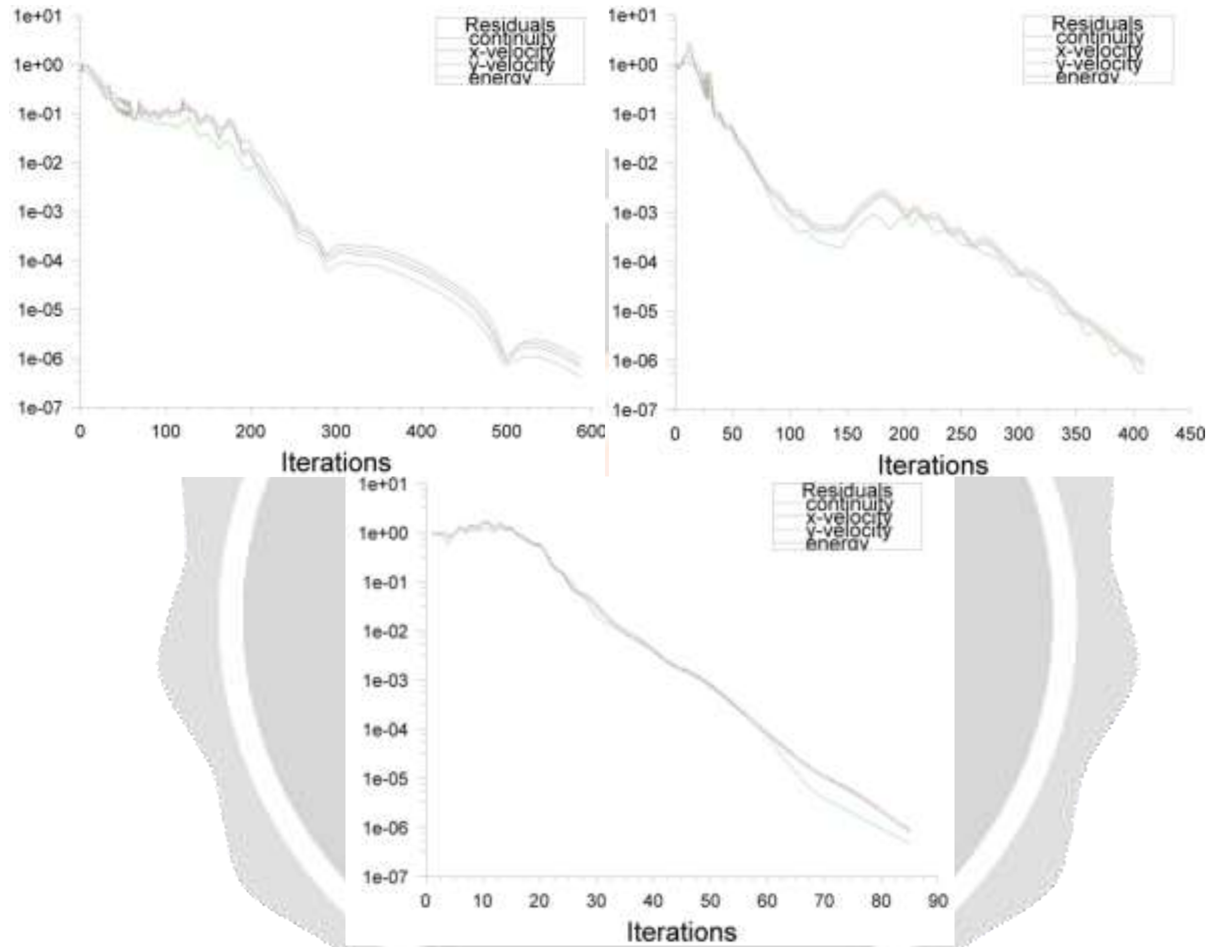


Fig- 13: Scaled Residuals

3.5. Post Processing/ Result

3.5.1 Half-Wedge Angle = 10

- *Mach number Variation*

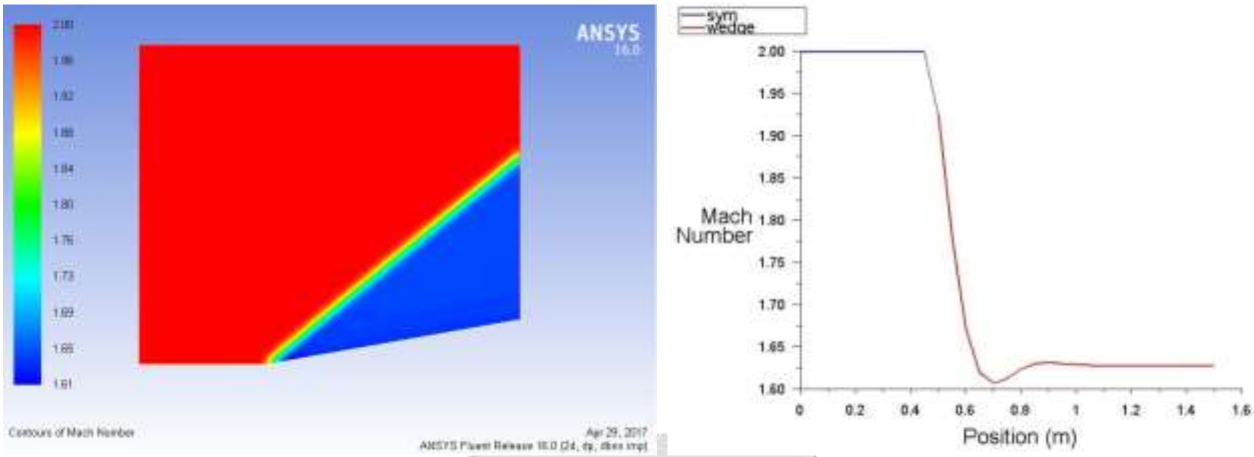


Fig-14: Contours and graph for Mach number 2

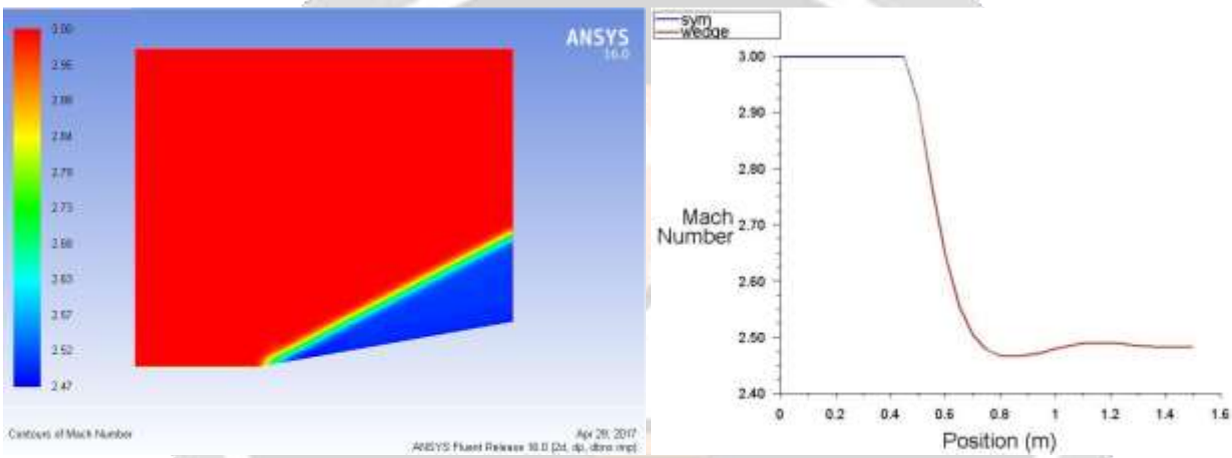


Fig-15: Contours and graph for Mach number 3

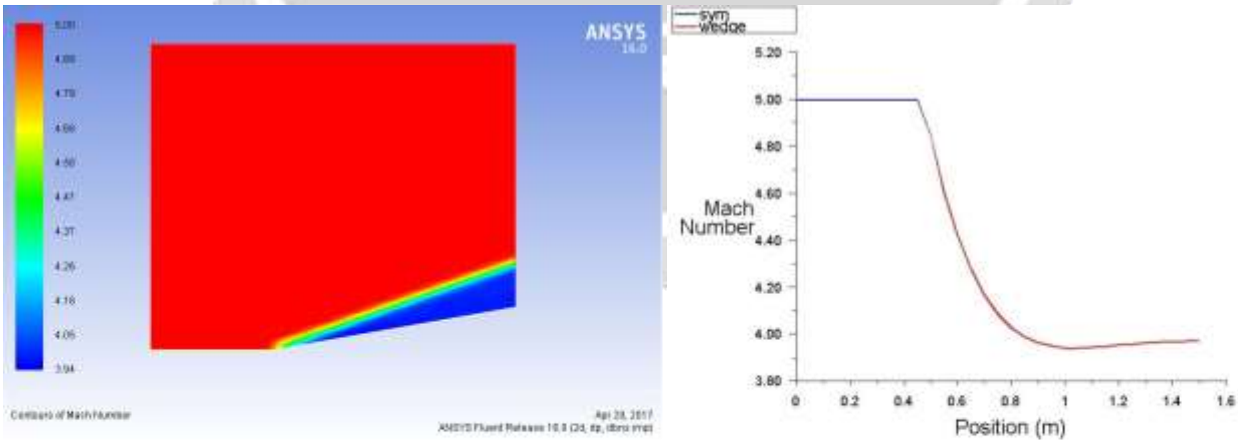


Fig-16: Contours and graph for Mach number 5

- *Pressure Variation*

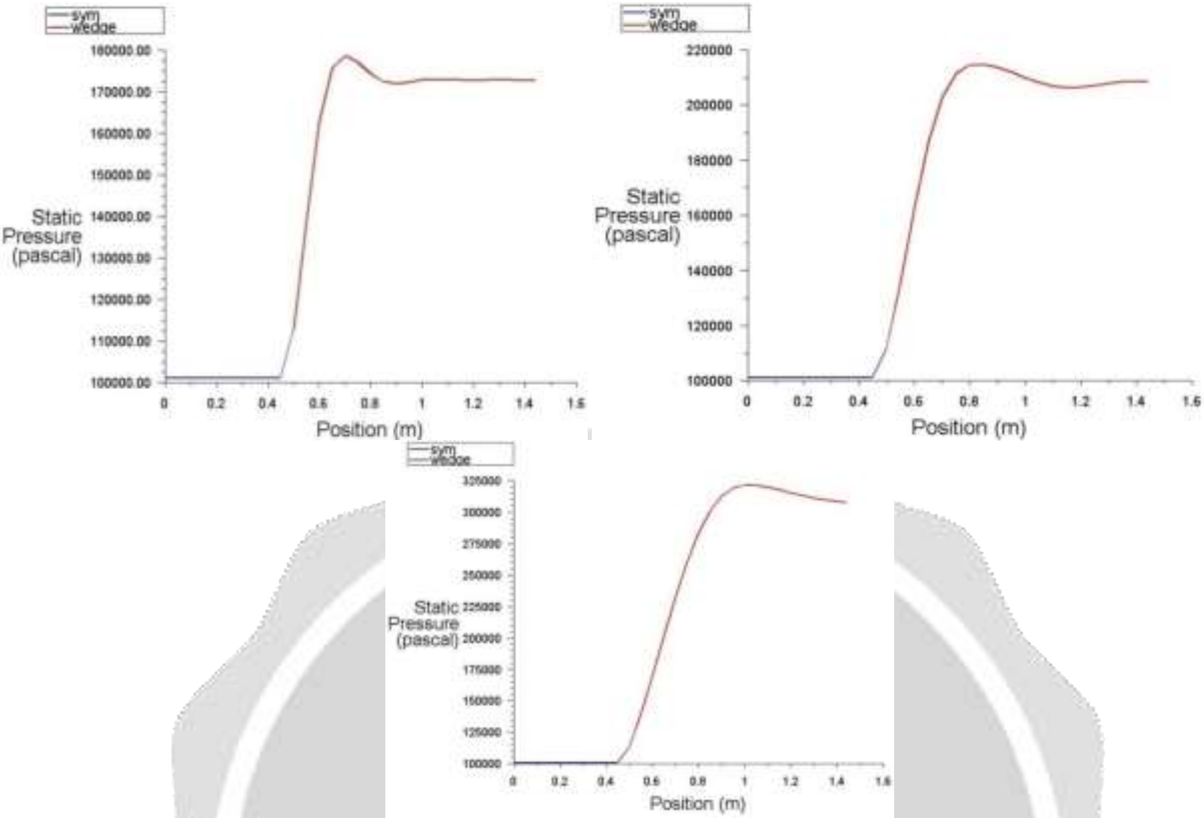


Fig-17: Pressure Variation Graph for Mach number 2, 3 and 5 respectively.

- *Temperature Variation*

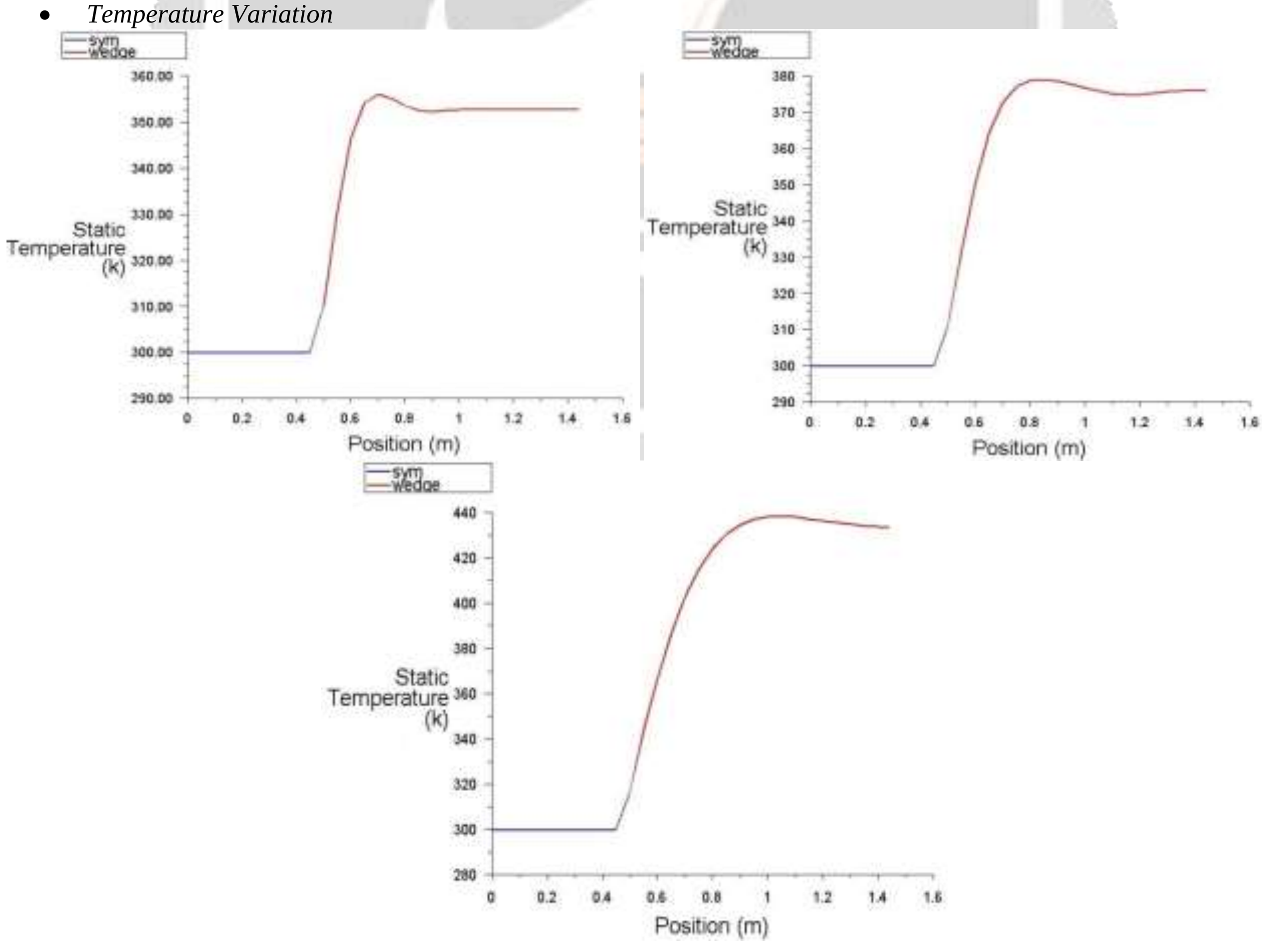


Fig-18: Temperature Variation Graph for Mach number 2, 3 and 5 respectively.

3.5.2 Half-Wedge Angle = 20

- *Mach number Variation*

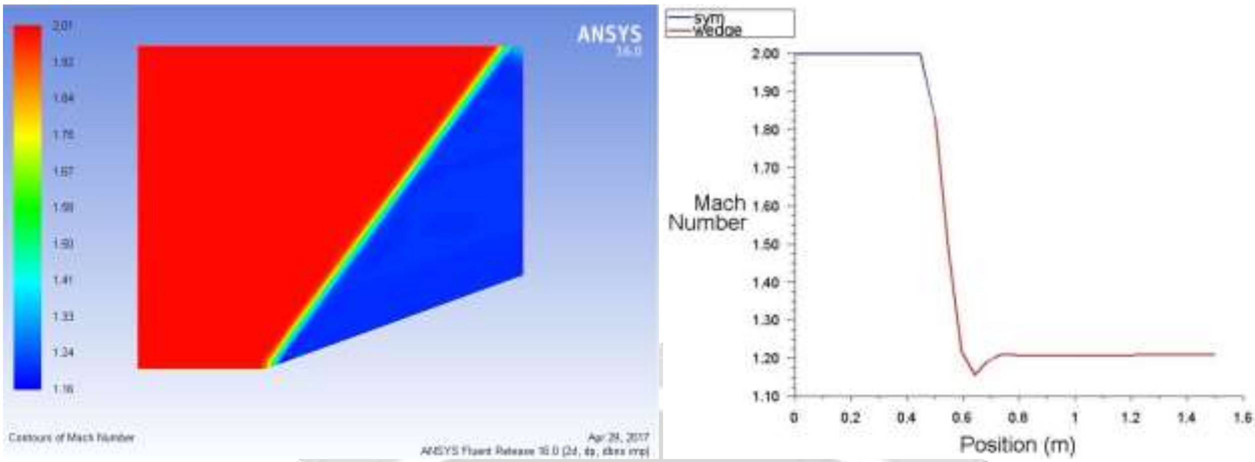


Fig-19: Contours and graph for Mach number 2

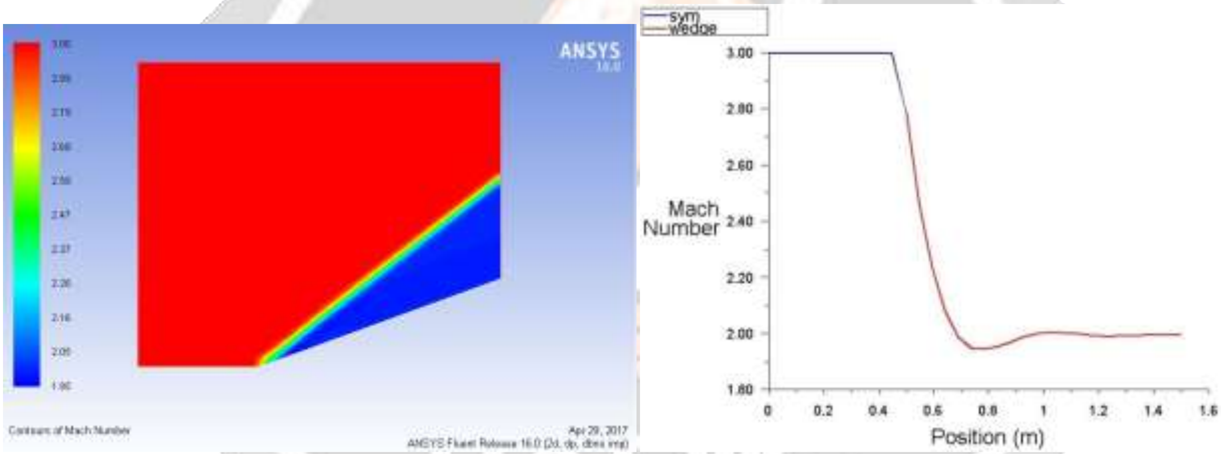


Fig-20: Contours and graph for Mach number 3

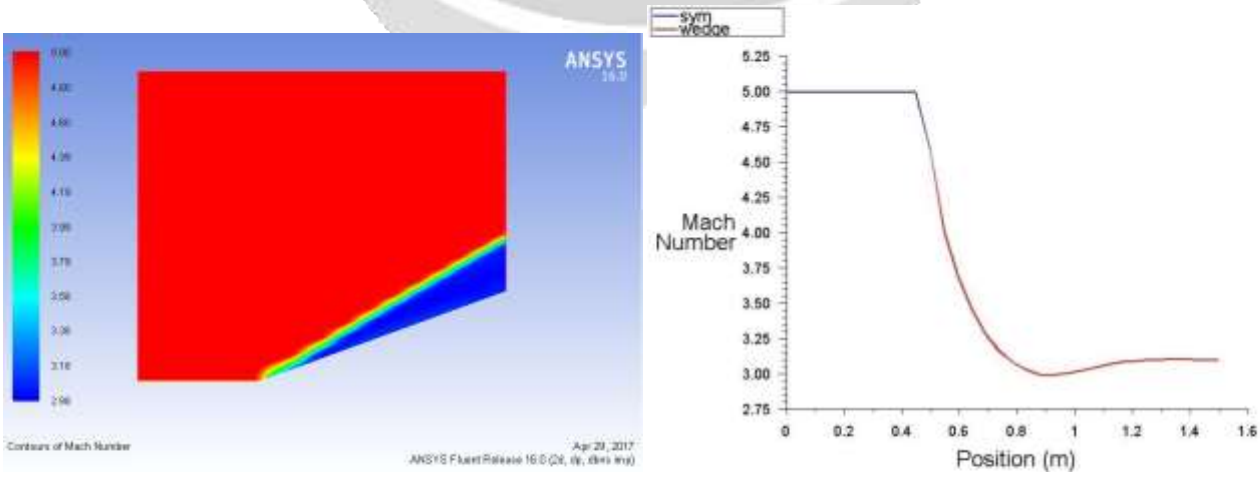


Fig-21: Contours and graph for Mach number 5

- *Pressure Variation*

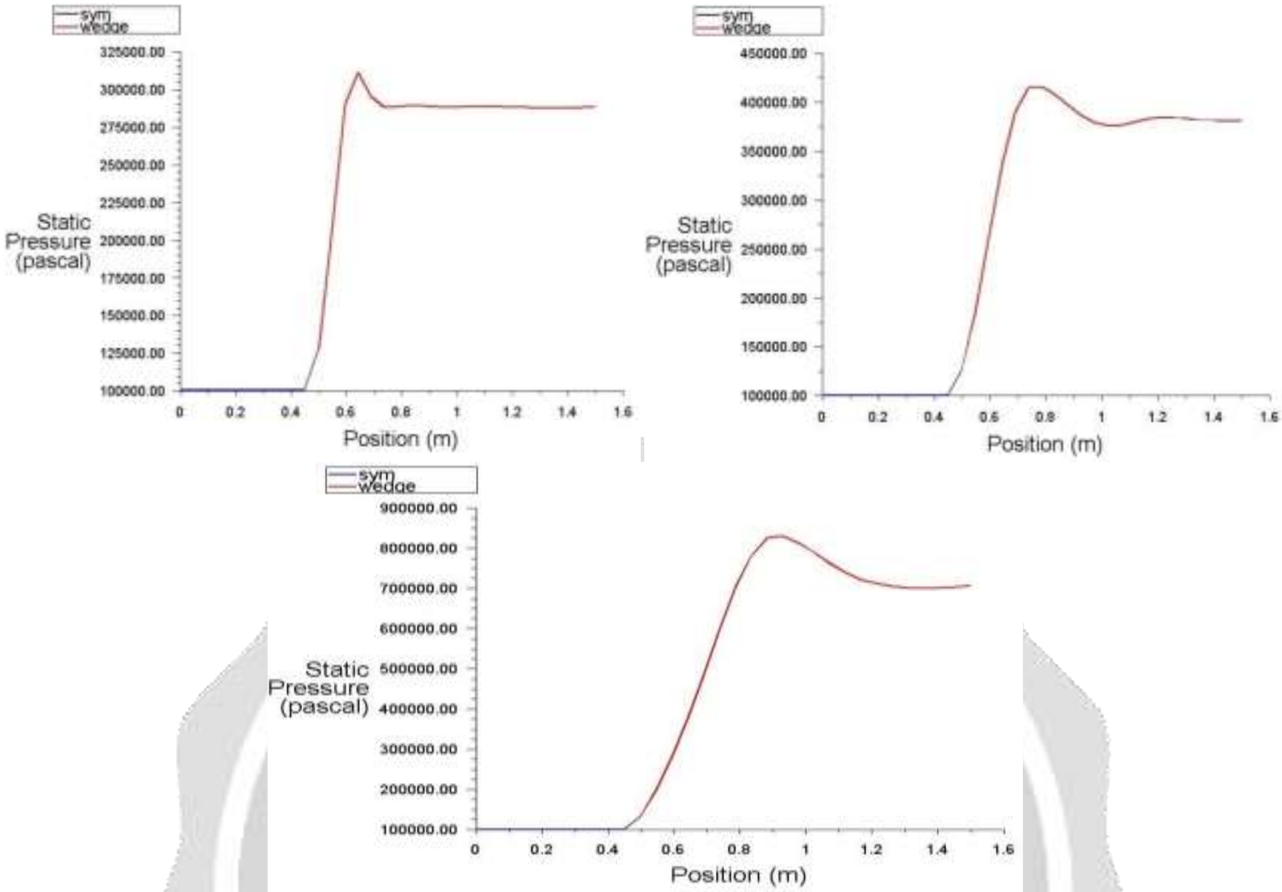


Fig-22: Pressure Variation Graph for Mach number 2, 3 and 5 respectively

- *Temperature Variation*

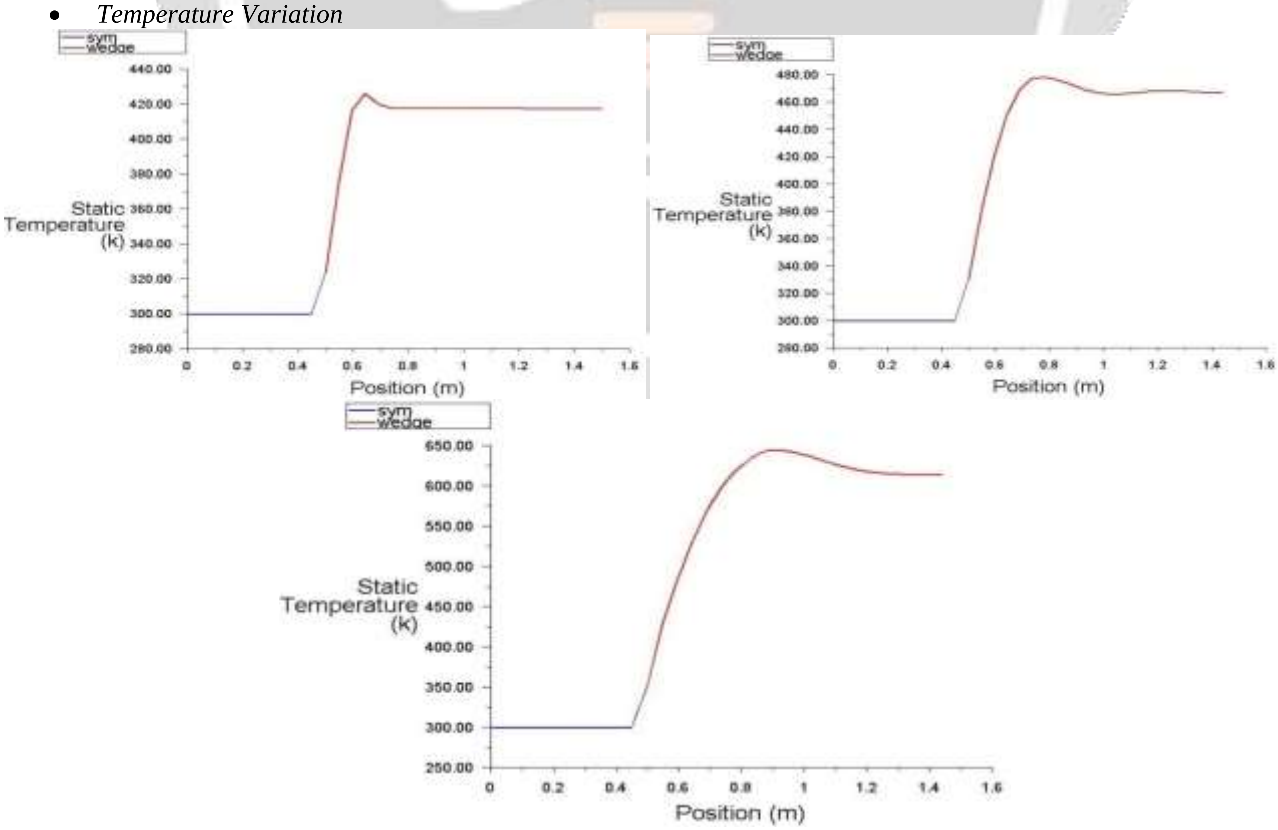


Fig-23: Temperature Variation Graph for Mach number 2, 3 and 5 respectively

3.5.3 Half-Wedge Angle = 30

- *Mach number Variation*

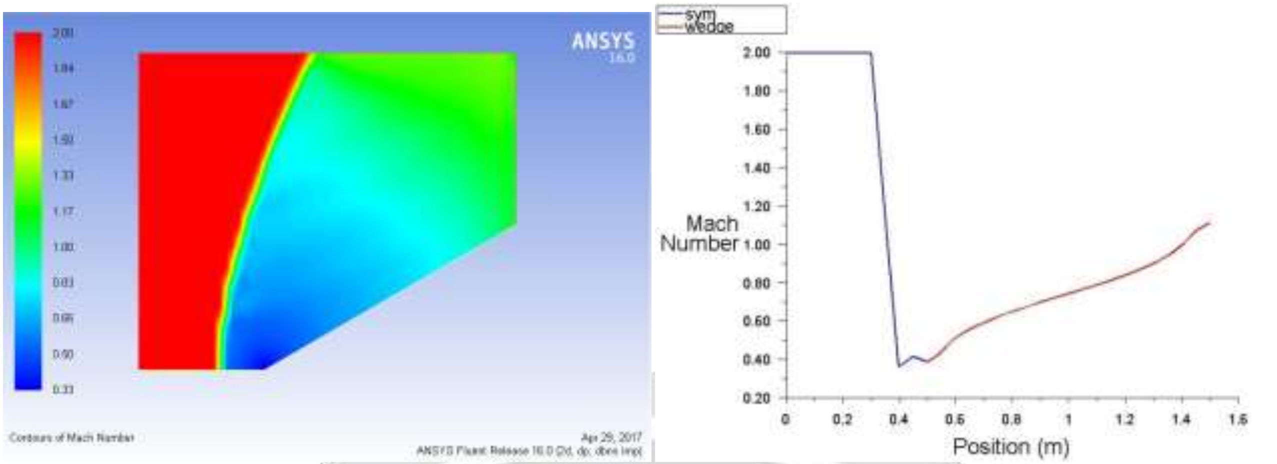


Fig-24: Contours and graph for Mach number 2

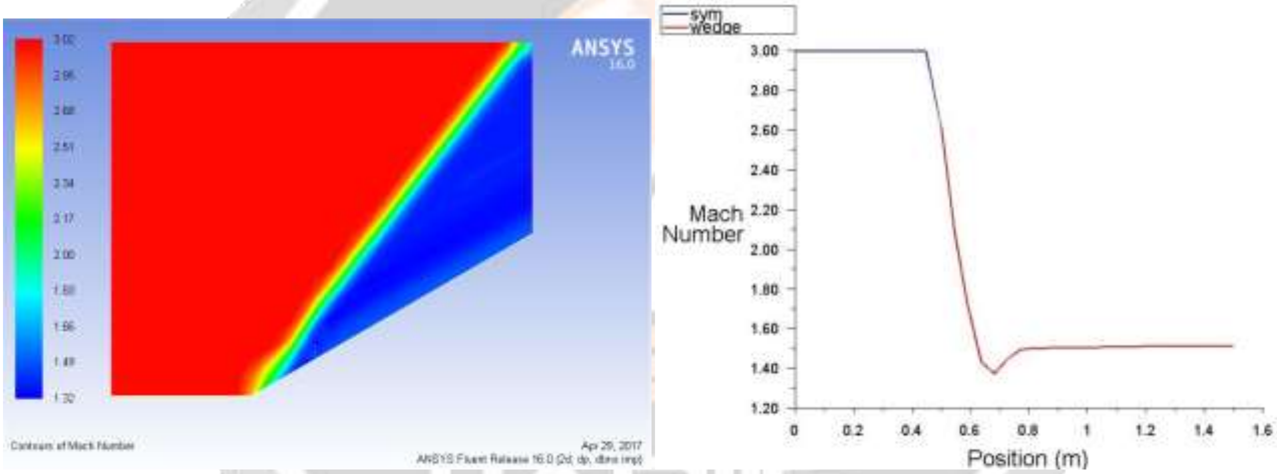


Fig-25: Contours and graph for Mach number 3

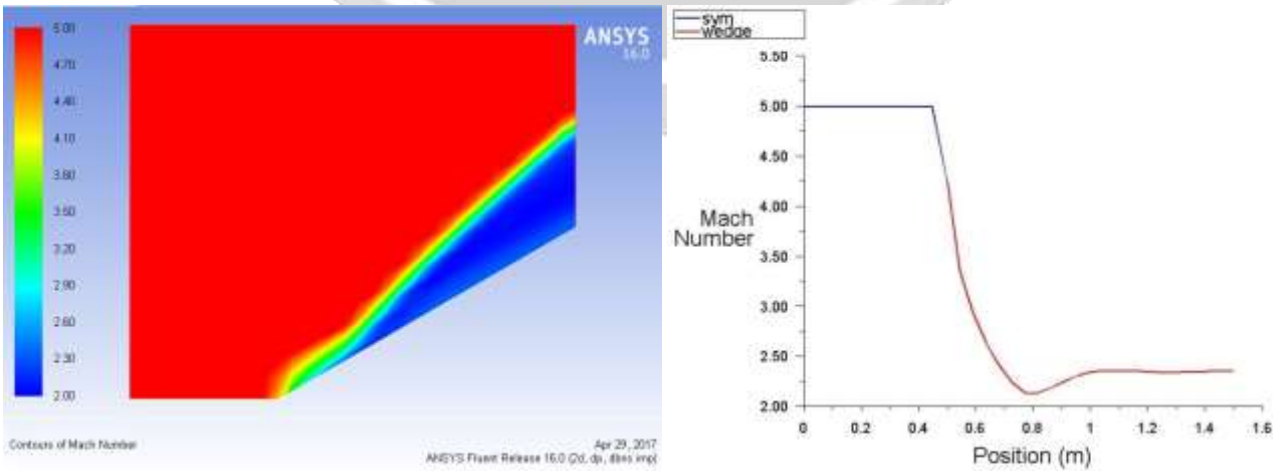


Fig-26: Contours and graph for Mach number 5

- *Pressure Variation*

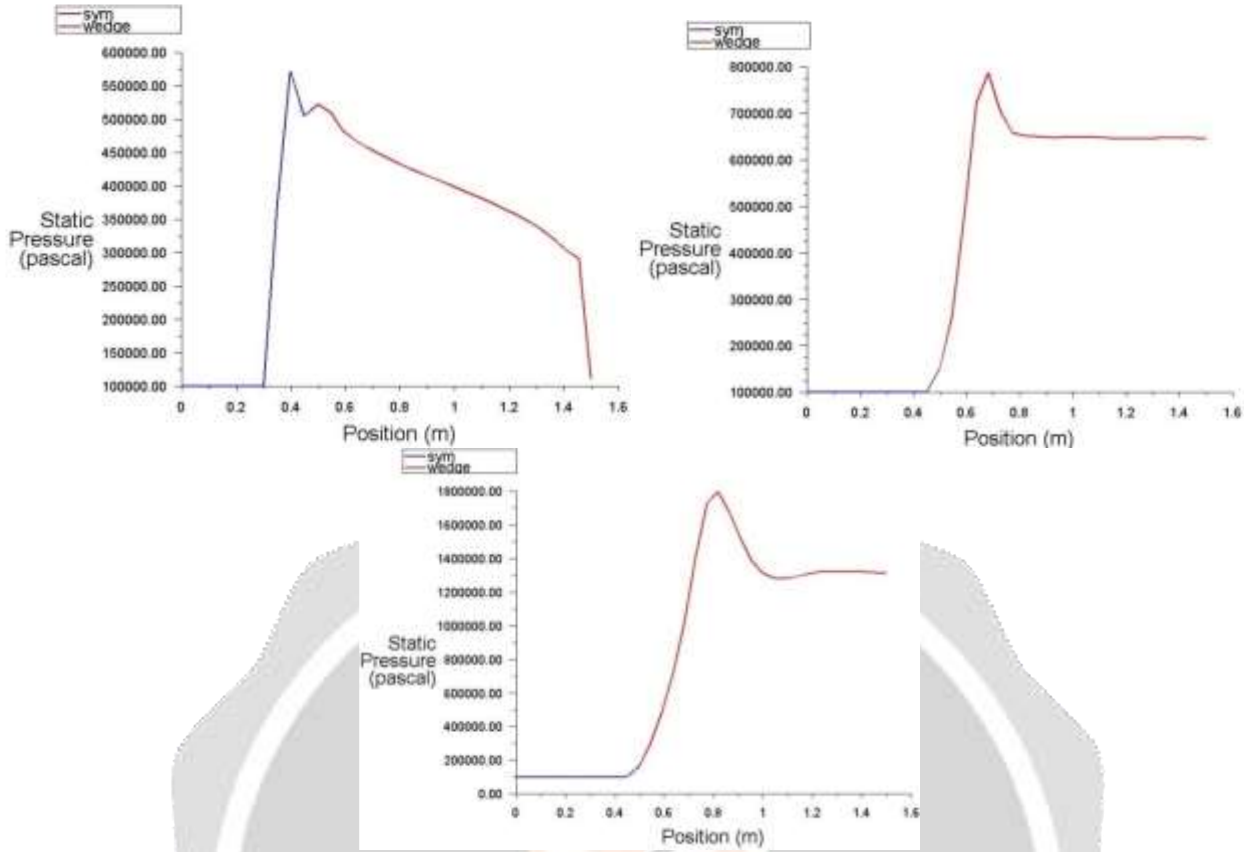


Fig-27: Pressure Variation Graph for Mach number 2, 3 and 5 respectively

- *Temperature Variation*

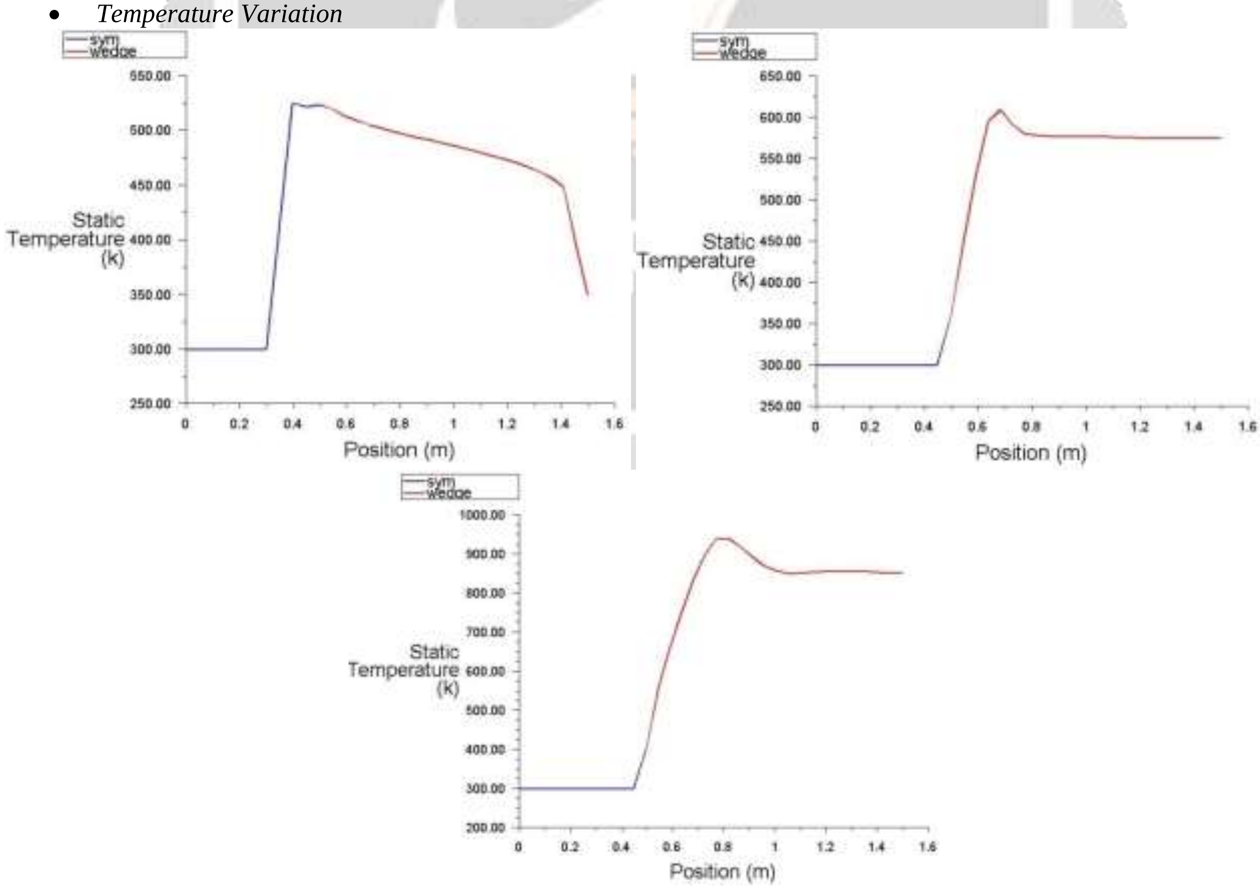


Fig-28: Temperature Variation Graph for Mach number 2, 3 and 5 respectively

4. CONCLUSION

The results from *θ-β-M* relation are analyzed with the help of CFD.

- For any given upstream Mach number M_1, there is a maximum deflection angle θ_{max}. If the physical geometry is such that $\theta > \theta_{max}$, then no solution exists for a straight oblique shock. Instead, nature establishes a curved shock, detached from the corner or the nose of the body. Here two figures are shown
  - When $\theta < \theta_{max}$, the shock is attached, here $\theta = 10^o$
  - When $\theta > \theta_{max}$, the shock is detached, here $\theta = 30^o$

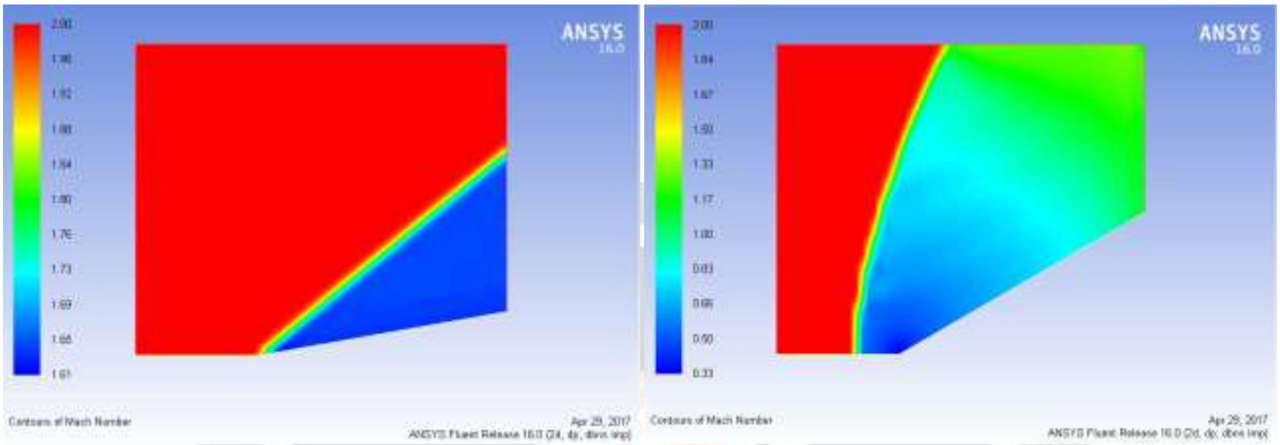


Fig-29: Attached and detached shock.

- For attached shocks with a fixed deflection angle, as the upstream Mach number M_1 increases, the wave angle β decreases, and the shock wave becomes stronger. Here two figures are shown
  - When $M_1 = 2$ the wave angle β is 54^o
  - When $M_1 = 5$ the wave angle β is 30^o

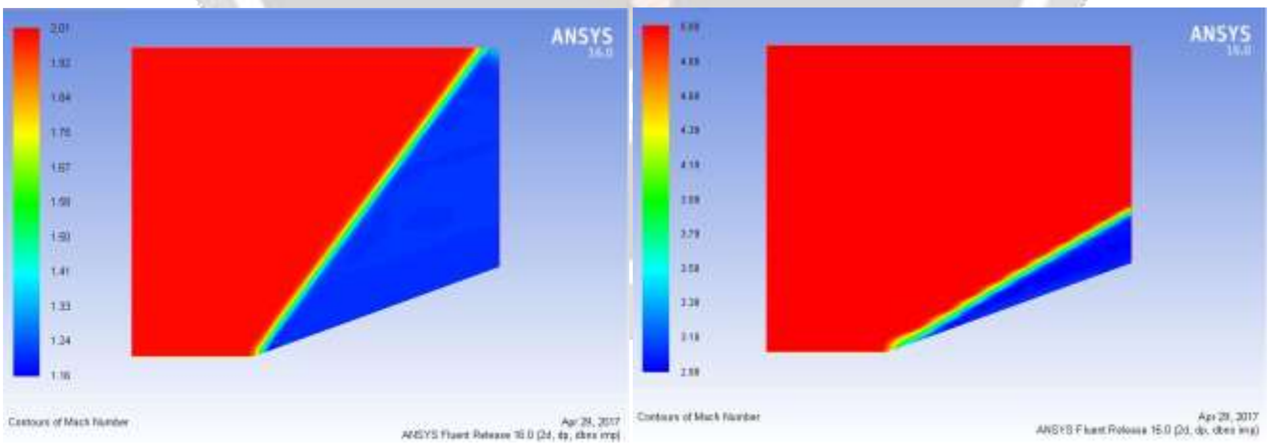


Fig-30: Effect of increasing upstream Mach number at constant wedge angle

- For attached shocks with a fixed upstream Mach number, as the deflection angle increases, the wave angle β increases, and the shock becomes stronger. Here two figures are shown
  - When $M_1 = 2$ and $\theta = 10^o$ the wave angle β is 40^o
  - When $M_1 = 2$ and $\theta = 20^o$ the wave angle β is 54^o

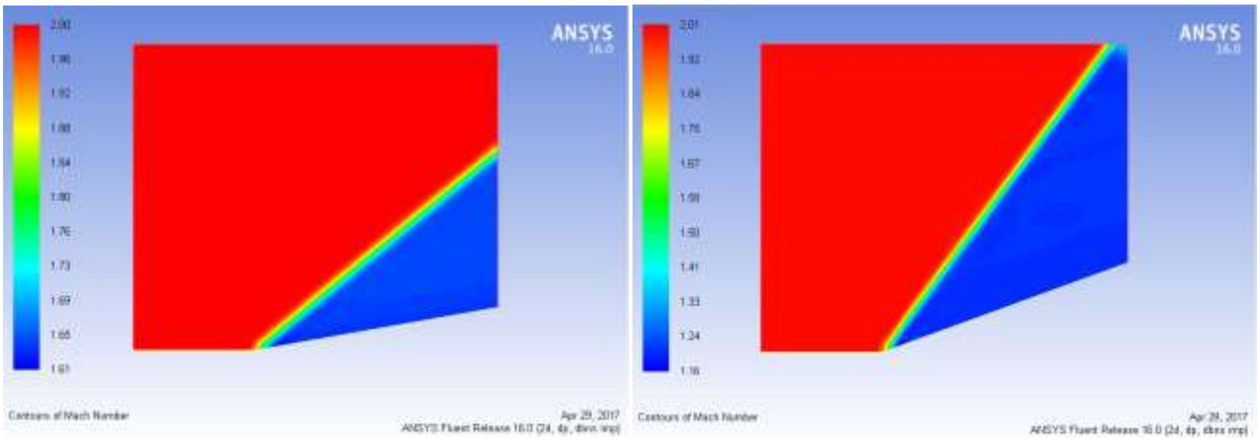


Fig-31: Effect of increasing wedge angle at constant upstream Mach number.

4.1 Half-Wedge Angle = 10 ^{o}

Table 6: Parameters Variation for half wedge angle = 10^o

Parameter	Mach Number = 2		Mach Number = 3		Mach Number = 5	
	β = 40^o		**β = 28^o**		**β = 20^o**	
	Theoretical	**CFD**	**Theoretical**	**CFD**	**Theoretical**	**CFD**
M_2	1.587	1.6	2.383	2.46	3.673	3.94
p_2 / p_1	1.759	1.764	2.14	2.12	3.24	3.14
ρ_2 / ρ_1	1.488	1.494	1.703	1.678	2.21	2.174
T_2 / T_1	1.181	1.186	1.256	1.263	1.466	1.46

4.2 Half-Wedge Angle = 20 ^{o}

Table 7: Parameters Variation for half wedge angle = 20^o

Parameter	Mach Number = 2		Mach Number = 3		Mach Number = 5	
	β = 54^o		**β = 39^o**		**β = 30^o**	
	Theoretical	**CFD**	**Theoretical**	**CFD**	**Theoretical**	**CFD**
M_2	1.185	1.156	1.837	1.94	2.954	2.96
p_2 / p_1	2.887	3.09	3.987	4.1	7.125	8.20
ρ_2 / ρ_1	2.06	2.17	3.101	2.574	3.33	3.83
T_2 / T_1	1.4	1.422	1.285	1.59	2.139	2.17

4.3 Half-Wedge Angle = 30 °

Table 8: Parameters Variation for half wedge angle = 30°

Parameter	Mach Number = 2		Mach Number = 3		Mach Number = 5	
	Detached Shock		β = 53°		β = 43°	
	Theoretical	CFD	Theoretical	CFD	Theoretical	CFD
M_2	Detached Shock		1.34	1.32	2.02	1.99
p_2 / p_1			6.52	7.96	13.32	17.74
ρ_2 / ρ_1			3.20	3.84	4.188	5.69
T_2 / T_1			2.03	2.08	3.18	3.33

By this paper we have explained the basic concepts connected with wedges. The study of attached and detached shockwave has been done. The effect of increasing upstream Mach number at constant wedge angle and also the effect of increasing wedge angle at constant upstream Mach number has been analyzed through CFD. Through this study and analysis, we have found out the values of variation of parameters such as pressure, density, temperature and Mach number by theoretical method and also using CFD. Thus we can conclude that values of these methods are approximately similar to each other.

5. REFERENCE

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