COGGING TORQUE REDUCTION IN PERMANENT MAGNET SYNCHRONOUS MOTOR USED IN E VEHICLE

NANDHA KUMAR A, KEERTHANA R, ABISHEK R, FAKIR J
DEPARTMENT OF MECHATRONICS
BANNARI AMMAN INSTITUTE OF TECHNOLOGY
KEERTHANA R [1],ABISHEK R [2],FAKIR J [3], NANDHA KUMAR A [4]

Abstract

Increasing global warming and pollution forced the world to walk in the path of electric vehicles. Electric vehicles to sustain this new change should give better performance and range in comparison with IC vehicles. A vehicles performance greatly depends on its prime mover and in case of EVs it the motors used to power the vehicles. Among various motor types available across the globe, PMSM has a remarkable power and torque density which would make it more suitable for automotive applications. But Vibrations caused due to Cogging effect of PM machines limits the application of PMSM in the automotive domain. Cogging torque is a negative effect which causes the output torque of the machine to fluctuate. This effect is the outcome of interaction of flux between the rotor permanent magnet and stator slots. This not only results in cogging torque but is also indirectly responsible for torque ripple and vibrations. This vibration may crucially affect the structural strength of the vehicle and may also cause noise. This project attempts to reduce cogging torque of Interior permanent magnet synchronous machines by magnet shaping and orientational adjustments of the magnet. This will optimize the flux linkage with the slots and thereby reduces the cogging torque.

Key words: Cogging torque, e-vehicles, PMSM, Magnet shaping, torque density.

1.INTRODUCTION: With the ever-increasing concern over environmental sustainability, electric vehicles (EVs) have emerged as a promising solution to reduce greenhouse gas emissions and combat the challenges of climate change. As the propulsion system forms the heart of any EV, Permanent Magnet Synchronous Motors (PMSMs) have gained widespread popularity due to their inherent advantages, including high efficiency, compact size, and an excellent power-to-weight ratio. However, like any technology, PMSMs are not without their challenges, and one significant issue that has garnered attention is the phenomenon of "Cogging Torque."

1.1COGGING TORQUE IN PMSM : Cogging torque in Permanent Magnet Synchronous Motors is caused due to the magnetic attraction forces between the permanent magnets mounted on the rotor and the stator slot. These magnetic interactions lead to periodic variations in the electromagnetic torque output as the rotor rotates. The origin of cogging torque can be traced to the geometric arrangement of the motor components. The stator of a PMSM typically has a toothed structure to create an alternating magnetic field when current flows through the stator windings. The permanent magnets on the rotor, with their fixed magnetic polarity, exhibit a tendency to align with the stator teeth, resulting in the cogging effect during motor rotation. Cogging torque also leads to torque ripple, which in turn causes vibrations. These vibrations may lead to reduced fatigue and structural strength of the vehicles in case of being used as a propulsion motor.

1.2 METHODS OF REDUCING COGGING TORQUE: Cogging torque reduction includes two types of approach which are designed based reduction approach and control-based approach. The design-based approach includes skewing, magnet shaping, introducing dummy slots, notching etc.

Whereas the control-based method works by selecting the mathematically optimal parameter using control algorithms for cogging torque reduction Skewing of Stator Slots: Skewing involves displacing the stator slots from their conventional positions. By introducing an angular offset between adjacent stator slots, the cogging torque waveform is modified, leading to a smoother torque output. However, skewing may result in increased end-winding losses and may require design modifications for proper implementation. [4] Magnet Shaping and Placement: Modifying the shape and positioning of the permanent magnets on the rotor can alter the cogging torque characteristics. Non-uniform magnet shapes, such as trapezoidal or skewed magnets, can reduce the cogging effect. However, this approach demands careful design considerations to maintain magnetic flux density and rotor structural integrity. [15] Notching of Stator Teeth: Introducing specific notches or indentations on the stator teeth disrupts the magnetic alignment between the rotor magnets and stator teeth. This method effectively reduces cogging torque, but the increased complexity in manufacturing may add to the motor's cost. [15] Slot and Pole Combination Techniques: By carefully selecting specific combinations of stator slot and rotor pole numbers, it is possible to achieve partial or complete cancellation of cogging torque. This method requires meticulous design analysis and optimization to find suitable combinations. Advanced Control Strategies: Implementing advanced control algorithms, such as sensor less control techniques, model-based control, and adaptive control, can compensate for cogging torque during motor operation. These strategies require accurate motor parameter estimation and computational resources. Optimization Algorithms: Utilizing optimization algorithms can assist in finding the best combination of motor parameters and design features to minimize cogging torque while maintaining other desired performance characteristics. This approach necessitates comprehensive simulations and iterative design refinement. [6]

1.3 SCOPE OF THE PROJECT : This is the era where e-vehicles are gaining momentum in the market. This gaining momentum has also increased the demand for highly efficient and high torque density motors. PM motors are one such electrical motor which can find its application in the automotive domain. But Cogging torque limits its application in this area. Successful implementation of this project can promote usage of PMSM motors in E-vehicles. Also, the scope of the project is not only limited for PMSM machines. This project can be implemented in all reluctance motors like, SRL motor, BLDC motors, PMDC motors, etc. Thus, this can promote e-vehicles by reducing the cost of it. It also indirectly helps in reduction of fossil fuel usage

2 LITERATURE SURVEY: Analyzing existing solutions for the problem statement and their viability can greatly account in saving time. In this chapter, detailed works of various authors in their publication has been discussed along with the pros and cons faced. Arindam Das et al., (2015) investigated the pros and cons of the Toyota Pirus 2004 motor and concluded that cogging torque reduction using combined approach of magnet shaping and rotor profiling can improve the efficiency and performance. For this investigation, they've kept the volume of the magnet constant and tried different topology including unit magnet topology, v configuration and tub configuration. Out of these the cogging torque was the least for the tub topology. Their investigation is not just limited to magnet configuration. They've also performed rotor shaping and concluded that shaping at magnet pole ends reduces the mean torque and also increases cogging torque whereas mid-pole surfacing reduces cogging torque without affecting mean torque of the machine. Similarly, Hahlbeck S, et al., (2008) in his works on "Design considerations for rotors with embedded V-shape permanent magnets" analyzed various methods to reduce cogging torque in PM machine without compromising the mechanical integrity of the machine. One such remarkable method to restore mechanical integrity in rotor is using Solid bridge which has been proposed in this paper. In this paper FEM is use to detect the cogging torque, back emf and ripple. For effective reduction of cogging torque and for easier analysis only reluctance flux is taken and the PM flux is neglected. Wang K. et al., (2014) in "Average Torque Improvement of Interior Permanent- Magnet Machine Using Third Harmonic in Rotor Shape" attempts to maintain the average torque of the machine while trying to reduce the cogging torque. To achieve this III harmonic was implemented in rotor magnet shape along with pole arc shaping. It is to be noted that without third harmonic though the cogging torque is reduced it also reduces the mean torque. Using III harmonic maintains the optimum air gap in both d and q axis. Ki-Chan Kim et al, in "Analysis on Correlation Between Cogging Torque and Torque Ripple by Considering Magnetic Saturation" explains the relationship between the cogging torque and torque ripple. The cogging torque which is caused by the interaction between rotor magnet and stator slot, is responsible for the generation of torque ripple but, cogging torque and torque ripple are not peak to peak directly proportional. Hong seok kim et al, proposed a solution for the reduction of cogging torque using magnetic field direction. To achieve this, NdFeB magnet is replaced with anisotropic ferrite material. This increased back emf and reduced cogging torque using dome shaped topology whose parameters has been chosen using genetic algorithm. In “Issues in reducing the cogging torque of mass-produced permanent magnet brushless DC motor”, Islam M S et al, explained the manufacturing feasibility of each design-based solutions for cogging torque reduction. This study enhances the most feasible solution to be produced. N. Bianchi et al, (2002) proposed a combined design approach of using magnet skewing, magnet arc width method and stator notches can effectively reduce cogging torque in SPM motors. T.Liand et al, 1988 “Reduction of cogging torque in permanent magnet motors” compared various reduction methods and concluded that the cogging torque is better optimized in IPM machines in comparison with SPM machine. Yang X-S et al, (2010) analytically approached the reduction of cogging torque in PM machines with various algorithm and concluded that the cuckoo algorithm was best suited for his model. In this paper applications and advantages of other algorithm including PSO and genetic algorithm was explained in detail. Łukasz Knypinski et al, (2021) in his works on “Sizing by optimization of line- start synchronous motor” discussed the use of various types of genetic algorithm and the cases where the should be used. He also concluded that for discrete variable equations, Particle Swarm Algorithm serves better. As this project also concerns with design of PMSM machine, some design related publications had been reviewed. Wen L. Soong et al,(2015) in his works on the “PM Machine Modelling and Design” discussed the critical dependent parameters of electrical machine design and the effects of the independent parameters in them. “Optimal Design of Line-Start Permanent Magnet Synchronous Motor Based on Magnetic Equivalent Parameters” by Abdul Waheed, Byongkuk Kim, Yun-Hyun Cho et al, in (2020) discusses about the rotor and permanent magnet design in context with the center of gravity.

3 DESIGN OF A PMSM MACHINE : In order to achieve the objective of reducing cogging torque of a PMSM motor used in electric vehicle, an appropriate motor is to be designed. Taking this stage one design as a benchmark, adaptation for reducing cogging torque is to be implemented. The result in terms of cogging torque is to be compared and the reduction percentage will be calculated.

The design of the motor has some pre-requisites which results in optimal design. Such pre-requisite parameters include the below mentioned.

Table I: Motor parameters

Parameters	**Value**
Rated power	4 kW
Rated torque	23 Nm
Rated speed	1800 rpm
Power factor	0.9
Efficiency	95%
Number of poles	6
Number of slots	36
Input voltage	48V

The listed parameters are chosen as this range has found its application in various automobiles in market. The design process starts with the stator design. For the design of stator, the stack length, stator outer diameter, inner diameter, slot dimensions have to be identified which is calculated as follows

3.1 STATOR CALCULATIONS : The stator of an electric motor is the part of the motor which is responsible for creating the rotating magnetic field. It consists of windings and slot whose dimensions play a crucial role in the performance of the machine.

3.1.1 Stator Inner diameter calculation: The Inner diameter of the stator is thebase dimension of the design based on which the outer diameter of the rotor is determined. The input voltage is a parameter on which the stator inner diameter is typically from a battery in our case. This has to be converted to AC input as PMSM is an AC machine and supplied to the motor. Thus, the rms Voltage has to be calculated which is given by [5],

$$V_{ac} = \frac{\sqrt{2}*\sqrt{3}}{\pi} * V_s \qquad \text{--------------------------} \ (1)$$

Majority of vehicle battery provide either 48V or 64V nominal output. In our case 48V is chosen and on substituting the same in the equation (1) we get the AC input voltage as 36.64 V.

Any electrical machine has its own losses and thus the rated power is not necessarily equivalent to working power. This power is based on the efficiency of the machine designed and it is given by [1]

$$Q = power / (\cos Ø * \eta)$$ ------------------------- (2)

Where η is efficiency and cos Ø is the power factor of the machine. Considering the requirement, we're aiming to design a machine with 95% efficiency and a power factor of 0.91. On substituting these values, we get the power input in kVA is 4.6783kVA.

Similar to Input power, the rated output is not as same as the working output of the machine due to some inevitable losses. Thus, it is given as a factor of C_o, Output co-efficient which is given by [6]

$$C_o = 11 * kw * B_{av} * A_c * 10^{-3}$$ -------------------------- (3)

$C_o = 55.10717$

Where the values of the B_{av} (specific magnet loading) and A_c (specific electric loading) of the synchronous machine is 0.75 T and 20000 A/m [1]. And the value of winding factor is 0.966 which is a constant value. Calculating for these input the value of output co-efficient is estimated to be 55.10717

The inner diameter of the stator is a function of input voltage (AC), output co-efficient and the synchronous speed of the machine which is given by [14]

$$\mathbf{D_i^2 L = Q / Co * Ns}$$ -------------------------- (4)

Where Ns, the synchronous speed which is

$N_s = N/60$
Therefore, $N_s = 30$
The equation (4) that relates the diameter and power has another unknown parameter, the stack length. For the seamless working of the machine, the stack length and the diameter of the machine should be in a proportion which is given by
$L = 0.75 * \tau_p$
Where, $\tau_p = \pi D_i / p$
On substitution of the values of p (number of poles) and π we can arrive at the relation given below
$\tau_p = 0.5235 D_i$
On substitution of equation (6) in (5) we get,

-------------------------- (5)
$L = 0.3925 D_i$ -------------------------- (7)
-------------------------- (6)

On substituting the equation (7) in (4), we get the following relation,

$0.0008054 D_i^3 = Q / (C_o * N_s)$
On substitution of constants in the above equation the values of the stack length and inner diameter of the stator can be determined as 77mm and 196.7mm respectively.
3.1.2 Stator Outer diameter calculation: The outer diameter of the stator is dependent on the shape and size of the slot. Pear shaped stator slots has been chosen for our design. This is because pear shaped stator slots offer the following advantages;

- Reduced eddy current loss due to smoother curves
- Reduced harmonic content
- High torque density

After referring some PMSM data sheets, optimal slot dimensions for our motor specification are identified as follows [8]

Slot dimensions

Table II: Slot dimension

Parameter	Value
b_0	4 mm
b_1	6 mm
r_s	3.4 mm
h_g	28.23 mm
h_0	1.2 mm
h_s	24 mm
b_2	10 mm

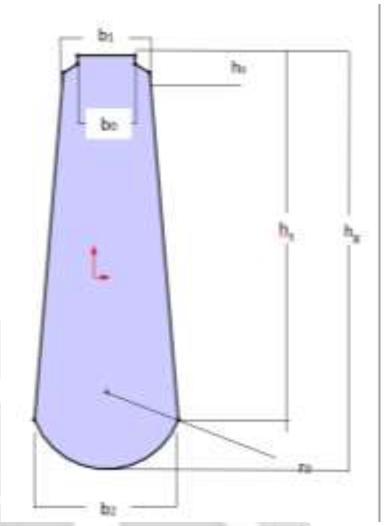


Fig I: slot

Thus, mathematically the outer diameter of the stator is given by,

$$D_o = S_i + 2[\, h_s + h_{sy}] \qquad (8)$$

On substitution of the appropriate values to equation (8), we get the value of the outer diameter of the stator as 291 mm.

3.1.3 Stator conductor turn & area calculation

Winding of the stator is responsible for creating the rotating magnetic field in the motor. These windings are the source of excitation which repels the rotor permanent magnet and results in conversion of electrical energy to mechanical energy.

The rotating magnetic field in turn causes a voltage as per the lenz law which is known as back EMF. PMSM is a reluctance type motor, which means it reluctance caused by the back EMF is also contribute in the average output torque of the machine. This is also the major contributor of cogging torque because, reluctance is not uniform in every position throughout the cycle. Thus, back emf determines the output torque of the motor and is given by [14]

Back emf $E_{ph} = V_{rms} * E/V_{ph}$ (9)

The ratio of the input voltage to the phase voltage is considered to be 0.91 and with that the back emf is 33.3424 V.

In order to reduce eddy current loss, the field windings are usually given as cluster small of wires and not as a single wide wire. These are referred to as turns and given by

$$T_{ph} = \frac{Eph}{4.44 * f * \text{Ø} * kw} \qquad (10)$$

Clearly from the equation, the number of turns of the winding is a function of frequency of the rotation of the rotating magnetic field and it is given by[16]

$f = P * N_s / (120)$

In which substitution of the value of synchronous speed in RPM is estimated to give a frequency value of 90 Hz.

The flux linkage determines the torque output of the machine and is given by[17]

$\text{Ø} = B_{av} * \pi * D_i L / P$

On substitution of the values in this equation (11) we get, flux as 0.00248715. (11)

Substituting the values of flux, frequency and winding factor to equation (), it is calculated that the number of turns per phase is approximately 54 turns.

With this the number of conductors per phase and number of conductors per slot can be calculated.

Number of conductors per phase is given by the formula

$N_{ph} = 2 * T_{ph}$, which is 108 turns. And Total number of conductors is given by the formula $Z = 6 * T_{ph,}$ which is 648 turns.

From the above values Number of conductors per slot is given by,

$Z_s = Z/S = 648/36$

Thus, the number of conductors per slot is 18

3.1.4 Area of the wire: The current density of the wire material determines the cross-sectional area of the conductor. Circular conductor of 4 A/mm^2 current density is planned to be used [15].

$$A_s = I_{ph}/J_c \quad \text{-------------------------- (12)}$$

On substitution we get, the area of the wire as 0.8231 mm^2

The diameter of the conductor is given by the simple mathematical relation between diameter and area as mentioned below.

$$D_s = \sqrt{\frac{4\,As}{\pi}}$$

Thus, from the above equation the diameter of the wire is 1.024 mm which falls under the ISO gauge size 18 and the particular is to be used. The winding is planned in double layer as this offers almost uniform flux linkage. Thus, there is planned to be 2 conductors each consist of 9 strands is to be used.

3.1.4 Stator 3D Modelling

The stator for the calculated value of parameters is designed in SOLIDWORKS. This model was the used to export the design parameters to Ansys MAXWELL.

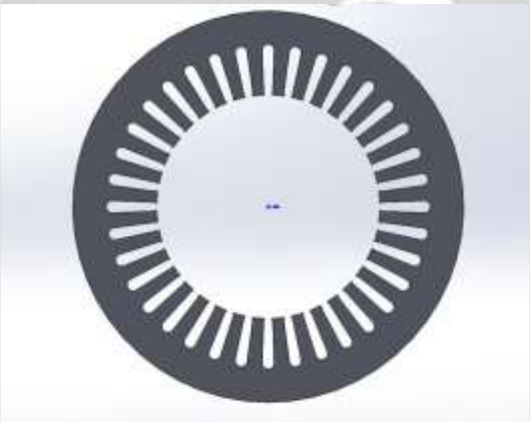


Fig II: Stator stack

The stator stamp modelled, ensures the mechanical durability of the stator and SS1010 grade steel is chosen as the core material. This material is selected because it is easy to manufacture, durable and its thermal properties are promising for our application.

In order to avoid core losses, stamping thickness are limited to the minimal possible value. In our case 0.4 mm is chosen as the thickness and for this value we would require about 192 stampings.

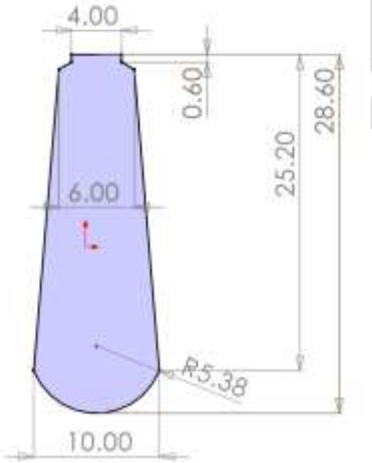


Fig III: Stator slot

The design of slot modelled in SOLIDWORKS was used to analyze the package efficiency of the slots within the

stator stamping and also used to evaluate the feasibility of windings within the slot area.
The stator dimensional parameters are summarized below.

Table III: Stator parameters

Parameter	**Value**
D_i	0.7 mm
D_o	196.7 mm
L	196 mm
T_{ph}	54
Z	9
D_s	1.024 mm

3.2 ROTOR CALCULATION: Rotor of an electrical machine in our model is a Interior permanent magnet motor which means that the magnet in embedded inside the rotor core. The magnet volume determines the magnetic intensity, which contributes to the torque.

3.2.1 ROTOR OUTER DIAMETER CALCULATION: The inner diameter of the rotor is calculated form the inner stator diameter and air gap length. For an optimal flux linkage, the airgap length required for our specification is 0.7mm [15].

The rotor parameters are

Table IV: Rotor parameter

Parameter	**Value**
Air gap length, l_g	0.7 mm
Rotor OD, R_o	196 mm
Rotor ID, R_i	45 mm
Number of poles, p	6
Number of magnets	12

The inner diameter of the rotor is nothing but the shaft's OD and hence it is 45 mm which has been chosen based on our load conditions [15].

3.2.2 Rotor Permanent magnet calculations: The rotors magnet's volume determines the magnetic field which is going to oppose the stator's rotating magnetic field of the machine. In other words, the magnet volume is a function of Input power and is given by [15]

$$\text{Input Power} = \frac{P}{coscos\,\emptyset * \cap} \quad \text{-------------------------- (13)}$$

On substitution of the values of rated power, power factor and efficiency, we get input power as 4678.363 W

From the calculated power the magnet volume is related as mentioned below

$$V_m = C_v * P_i / (f * B_r * H_c) \quad \text{-------------------------- (12)}$$

Where C_v is a co – efficient expanded as

$$C_v = \frac{0.2 * \sigma * Km * Kad}{\in}$$

Where δ the phasor angle difference between Phase current and Back Emf ,
where Km maximum armature current per phase, where Kad is armature reaction co-efficient, where σ is flux leakage, where ∈ is magnet use ratio, where f is frequency, where B_r is remanence flux density , where H_c is magnet coercivity

The values of magnet coercivity and remanence flux density depends on the magnet material. In our case the magnet material used is NdFeB grade 35 magnets which has 0.75 T flux density and 907 kA/m coercivity.

$V_m = 0.00144\ m^3$

This is the total magnet volume. We've chosen V shaped magnet topology with 6 poles. Which means there are 12 magnets in the motor. Thus, the individual magnet volume is given by

$V_{im} = 0.00144/12\ m^3$

Which on simplification gives the volume of individual magnet as 120.401 mm. A typical magnet length to produce uniform magnetic field throughout the axis of the machine should posses the length equal to the stack length of the machine. Thus, height of the magnet is

h_m = 77 mm

By basic mathematic we know that the volume of the magnet is nothing but the algebraic product of the length, height and width of the magnet. Magnetic materials are usually brittle and this makes is hard to manufacture in custom sizes. Thus, in order to attain the expected mechanical strength, the magnet width should be within the range of 10mm to 20 mm [14]. Hence the length od the magnet is

L_m = 20 mm

W_m = 10 mm

The magnet parameters are given in the table below

Table V: Magnet parameters

Parameters	**Value**
Height of the magnet, h_m	77 mm
Length of the magnet, L_m	20 mm
Width of the magnet, W_m	10 mm
Angle between the 2 magnets,α	32 deg
Coil span	5 slots
Distance from the magnet tip to the rotor outer diameter	2 mm

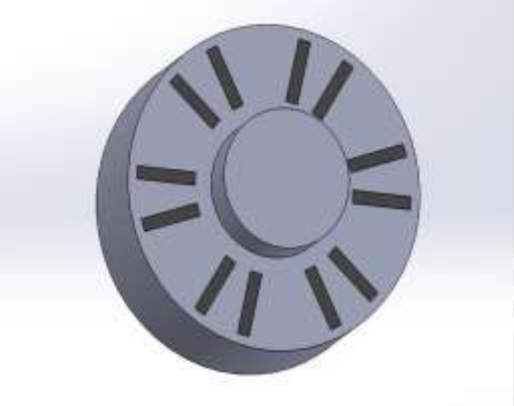
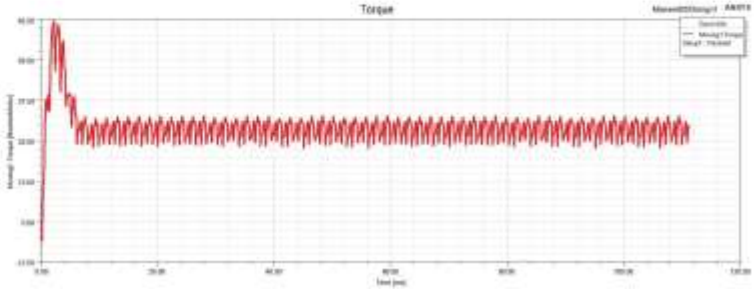


Fig IV: rotor

For the calculated dimensions, A 3D model of the motor is modelled in SOLIDWORKS software. With this the alignment of the rotor magnets are iterated and then exported to Ansys maxwell for further analysis. The screen shots of the models are attached below

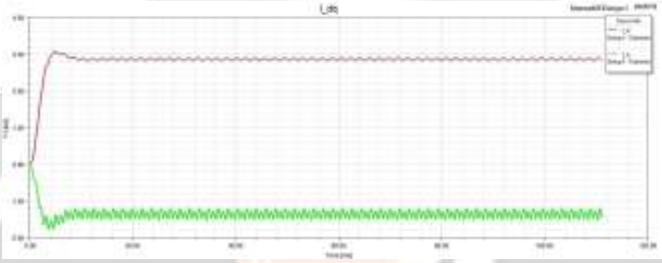
4 RESULT AND ANALYSIS : After successful modelling of the machine in SOLIDWORKS, its orientational position and manufacturing feasibility has been analyzed. Further the model is to be analyzed for cogging torque and appropriate reduction method has been chosen.

4.1 COGGING TORQUE GRAPH : Ansys Electronic desktop is platform to analyze the flux density, flux linkage, and torque performance of an electrical machine. RMxprt extension has been used to generate 2D model of the motor which is then converted to Maxwell 2D model. The model was given boundary conditions and meshing conditions as required and the EDDY Current is neglected. After the following adjustments a 2D sectional model is generated which is attached below



The fluctuation in the torque graph is the cogging torque of the machine which is significantly high. From the graph we can identify that the the rated torque of the machine is 23 Nm and the torque value fluctuates around 20 to 30 Nm throughout the cycle.

4.2 REDUCTION METHOD: Fluctuation in reluctance created by the q- axis (quadrant axis) flux is the major causative agent of cogging torque. This project aims to reduce cogging torque by optimizing the q axis flux linkage and making it almost uniform throughout the cycle. To achieve this, the shape of the flux guide and the magnet position is adjusted and cogging torque in every particular point is verified. After some iteration the most optimal solution was found when the flux guide is shaped for a 5 mm diagonal along its edge. This optimizes the flux linkage and reduces the cogging torque. The corresponding flux and torque graph is given below.



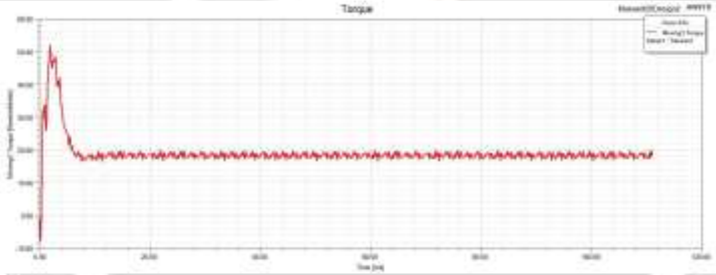


Fig X: Torque graph after optimization

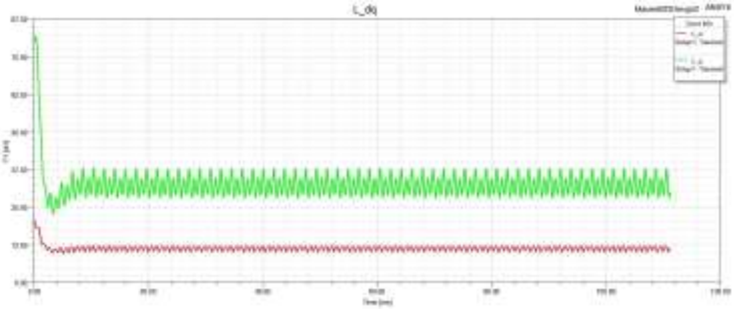


Fig XI: q-axis inductance graph after optimization

5 CONCLUSION AND FUTURE WORKS: Application of PMSM motor in automotive will greatly increase the acceleration performance especially in case of off-road vehicles. This is because the PMSM motor offers higher power and torque density. Reduction of cogging torque indirectly aids in reduction of torque ripple which is another parasitic outcome of cogging torque which may result in vibration. The vibration may affect the structural stability of the vehicle. This project with its proved outcome ensures no such hazards. It is to be noted that the cogging torque is reduced with no significant reduction in average toque output of the machine.

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