Comparison of Identification Codes : Maxcode and Mincode

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1. Abstract

The numerical technique used for any application will be very useful in solving any type of problem very easily and fastly too. The objective being identification of the kinematic chain with an identification code either a maxcode or mincode which is known as canonical numbering being a unique technique of enumeration and identification of Kinematic chains. In the proposed paper the identification codes i.e. maxcode and mincode are explained along with an example of Stephenson's chain. Also the comparison has been made between them.

Keywords/Index words *: Kinematic Chain, Adjacency Matrix, UTAM(Upper Triangular Adjacency Matrix), Maxcode, Mincode, Decodability.*

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2. Introduction

Machine designers have synthesized kinematic chains unconsciously since time immemorial, Uicker and Raicu[1] proposed the characteristic polynomial, but the computations are rather tedious. A modification to the matrix notation was proposed by Mruthyunjaya and Raghavan, [2] with a view to permit derivation of all possible mechanisms from a kinematic chain and distinguishing the structurally distinct ones by, changing the concept of adjacency matrix. Rao and Raju [3], Rao[4] proposed the secondary Hamming Number Technique for the generation of planar kinematic chains which was accepted. A major problem is faced these years has been the absence of a reliable and computationally efficient technique to pick the non-isomorphic chains. The reason why designers have been plodding through so many new routes instead of sticking to what ought to have been a 'straight-as-an-arrow' path is easy to visualize with an implied requirement of decodability.

Read and Corneil [5] remark that a good solution to the coding problem provides a good solution to the isomorphism problem , though, the converse is not necessarily true. This goes to suggest that a successful solution to the isomorphism problem can be obtained through coding. The concept of canonical numbers provide identification codes which are unique for structurally distinct kinematic chains. One important feature of canonical numbers is that they are decodable, and also promises a potentially powerful method of identifying structurally equivalent links and pairs in a kinematic chain. While describing a method of storing the details of adjacency matrix in a binary sequence, maxcode and mincode are the tools for the identification of kinematic chains. The test of isomorphism then reduces to the problem of comparing max/mincodes of the two chains.

2.Concept of Identification code :

According to Ambekar and Agrawal [6] the concepts of maxcode and mincode were introduced as canonical number for the enumeration. For every kinematic chain of n-links, there are n! different ways of labeling the links and hence, n! different binary numbers are possible for the same chain. By arranging these n! binary numbers in an ascending order, two extreme binary numbers can be identified , as significant ones for the same chain. These two binary numbers are : the maximum number and minimum number designated , respectively as maxcode M(K) and mincode m(K) . Since M(K) and m(K) denotes two extreme values of binary numbers for a given kinematic chain , and each has a unique position in the hierarchical order and is easily recognized, they are called as canonical numbers. Again each binary number of a given kinematic chain corresponds to a particular adjacency matrix , and hence it also corresponds to a particular labeling scheme.

The unique labeling scheme for the links of a kinematic chain, for which the binary number is in some (either maximum or minimum) canonical form, is called **canonical numbering** (labeling) of the chain. The adjacency matrix, which corresponds to canonical numbering, is said to be in some canonical form.

3. Property of Canonical Numbering :

In a binary number an entry of '1' as an (i+1)th digit, counted from the right hand end , has a contribution to the decimal code equal to 2^i . Also, it follows from the basic property of binary numbers that,

$$2^i > 2^0 + 2^1 + 2^2 + 2^3 + \ldots\ldots + 2^{(i-1)}$$

This is obvious because the right hand side of the above inequality represents summation of terms in geometrical progression and hence, can be shown to be equal to $(2^i - 1)$. This goes to prove that a contribution of any '1', in a binary number , is more significant than even the joint contribution of all the subsequent 'ones' in that binary number. This is fundamental to a basic understanding of any algorithm on maxcode and mincode.

For the purpose of establishing a binary code one considers upper triangular adjacency matrix for the canonical labeling of the chain. Binary sequence is established by laying strings of zeros and ones in rows, '1' through (i – 1) row, one after the other in a sequence from top to bottom. This binary sequence may be regarded as a binary number illustrated by an example: Consider the **Stephenson's Chain** as shown in Fig. 1. with 6 links : 4 binary and 2 ternary links with single degree of freedom.

According to Bauchabaum and Freudenstein [7] who gave the graphical method to represent kinematic structure which consists of polygons and lines representing links of different degrees, connected by small circles representing pairs/joints. It is the powerful tool as it is well suited to computer implementation , by using adjacency matrices to represent the graph. The graphical representation of the same chain can be explained in Fig. 2.

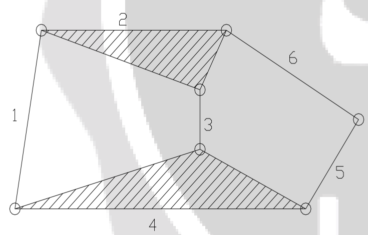


Fig. 1. Stephenson's Chain

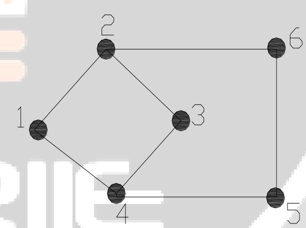


Fig.2.Graph representation of Stephenson Chain(Arbitrary Labeling)

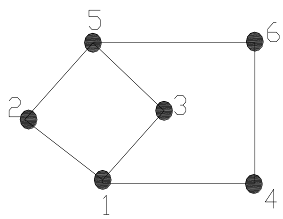
For the arbitrarily labeled graph as shown in Fig.2, the adjacency matrix and the corresponding UTAM (Upper Triangular Adjacency Matrix)is as under

$$A = \begin{vmatrix} 0 & 1 & 0 & 1 & 0 & 0 \\ 1 & 0 & 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 1 & 0 & 0 \\ 1 & 0 & 1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 & 0 & 1 \\ 0 & 1 & 0 & 0 & 1 & 0 \end{vmatrix} \qquad UTAM = \begin{vmatrix} 1 & 0 & 1 & 0 & 0 \\ & 1 & 0 & 0 & 1 \\ & & 1 & 0 & 0 \\ & & & 1 & 0 \\ & & & & 1 \end{vmatrix}$$

There are fifteen entries in the UTAM which, if written consecutively by rows as 10100 ; 1001;100; 10; 1,; results in a binary sequence = 101001001100101.

4. Application of canonical numbering with Decodability :

Intuitively, the labeling of the same graph can be as at Fig.3, giving the max code M(G) and the corresponding UTAM will be



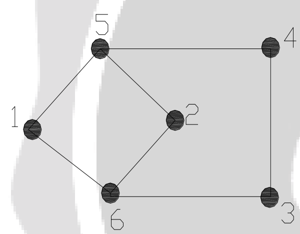
$$\text{UTAM} = \left|\begin{array}{ccccc} 1 & 1 & 1 & 0 & 0 \\ & 0 & 0 & 1 & 0 \\ & & 0 & 1 & 0 \\ & & & 0 & 1 \\ & & & & 1 \end{array}\right|$$

Fig.3.Graph with maxcode labeling

Corresponding binary sequence is: M (G) = 1110000 100 100 11. One can look at the resulting strings of 'ones' and 'zeros' as representing digits a binary code. And then it is more convenient to express maxcode in corresponding decimal form as –

$M(G) = 1.2^{14}) + 1.(2^{13}) + 1.(2^{12}) + 0.(2^{11}) + 0.(2^{10}) + 0.(2^{9}) + 0.(2^{8}) + 1.(2^{7}) +$
$0.(2^{6}) + 0.(2^{5}) + 1.(2^{4}) + 0.(2^{3}) + 0.(2^{2}) + 1.(2^{1}) + 1.(2^{0}) = 28{,}819$

Intuitively, the labeling of the same graph can be as at Fig.4. giving the min code m(G) and the corresponding UTAM will be



$$\text{UTAM} = \left|\begin{array}{ccccc} 0 & 0 & 0 & 1 & 1 \\ & 0 & 0 & 1 & 1 \\ & & 1 & 0 & 1 \\ & & & 1 & 0 \\ & & & & 0 \end{array}\right|$$

Fig.4.Graph with mincode labeling

At Fig. 4, for min code m(G) corresponding binary sequence is

m(G) = 000110011101100

And the corresponding decimal min code is

$m(G) = 0.(2^{14}) + 0.(2^{13}) + 0.(2^{12}) + 1.(2^{11}) + 1.(2^{10}) + 0.(2^{9}) + 0.(2^{8}) +$
$1.(2^{7}) + 1.(2^{6}) + 1.(2^{5}) + 0.(2^{4}) + 1.(2^{3}) + 1.(2^{2}) + 0.(2^{1}) + 0.(2^{0}) = 3{,}308$

Decodability : For the given identification code, it is possible to reconstruct the linkage topology on the basis of these identification codes alone. This is made possible by the division of the identification code by 2.The reminders are arranged sequentially to get again the binary number with which the linkage topology can be reconstructed.

5.Conclusion :

This paper demonstrates the power and potential of identification codes i.e.canonical number, in identifying kinematic chains and mechanisms. Both maxcode and mincode will be giving a unique code for the given kinematic chains .Thus, the canonical numbering (either maxcode or mincode) : being unique and decodable holds great promise in cataloguing (storage and retrieval) of kinematic chains and mechanisms.The only difference is that when all the labeling are arranged in hierarchial order , the maxcode represents the highest value and the mincode represents the lowest value.

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