"DEVELOPMENT OF MATHEMATICAL MODEL FOR OPTIMIZATION OF LUMBER TRACTION TREATMENT PROCESS"

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ABSTRACT

The evolution of multi-operational physiotherapy table comprises of various physiotherapy treatment devices, with traction as one of the complex and important phenomenon. There are many factors affecting the performance of Traction table, in this paper attempt is made to present the adopted design of experimentation in detail. Data collected during experimentation need to be analyzed to check the performance of the traction table designed, it is expected to develop logical relationship between various independent and dependent parameters. A mathematical model is a description of a system using mathematical concepts. The process of developing a mathematical model is termed mathematical modeling. The model helps to explain a system and to study the effects of different components and to make predictions about behavior of system components under various working conditions..

Keywords: *Multi operational, Traction, Mathematical modeling, Physiotherapy.*

INTRODUCTION

Qualitative and Quantitative analytical methods are generally used to analyze the research work. The primary aim of a Qualitative Research is to provide a complete, detailed description of the research topic. It is usually more exploratory in nature. Quantitative Research on the other hand focuses more in counting and classifying features and constructing statistical models and figures to explain what is observed. Qualitative Research is ideal for earlier phases of research projects while for the latter part of the research project, Quantitative Research is highly recommended. Quantitative Research provides the researcher a clearer picture of what to expect in his research compared to Qualitative Research.

BACKGROUND OF RESEARCH

The principle of homogeneity of dimensional equation is used to derive a relation between various physical quantities. To derive a physical relation, the dependent factors of a given physical quantity are found. Assuming its dimensions in terms of these factors, the final dimensional equation is written in terms of mass, length and time. Equating the powers of M, L and T on both sides of the dimensional equation, three equations are formed by which, value of unknown powers can be calculated. By substituting these values in the equation, the real form of relation is achieved. Hence to achieve dimensional homogeneity, Buckingham PI theorem will be used.

METHODOLOGY

Buckingham's Pi Theorem states that if there is a physically meaningful equation involving a certain number n of physical variables, then the original equation can be rewritten in terms of a set of p = n – k dimensionless parameters π1, π2, ..., πp constructed from the original variables. Where p, n and k is number of pi terms formed, number of basic fundamental dimensions of all parameters and number of repeating variables respectively. The theorem can be seen as a scheme for non-dimensionalization because it provides a method for computing sets of dimensionless parameters from the given variables, even if the form of the equation is still unknown.

To calculate dimensionless pi terms, all dependent and independent parameters are expressed in terms of fundamental physical quantities first. Following table expresses various parameters involved in this research work in terms of fundamental physical quantities.

Table 1 Fundamental dimensions of input and output parameters

Sr. No.	Description of Variables	Symbol	Unit	Dimensions	Type of Variable	Nature
1	Load applied	L	N	$[M^1L^1T^{-2}]$	Independent	Varying
2	Traction angle	θ	^{0}	$[M^0L^0T^0]$	Independent	Varying
3	No. of cycles	N	-	$[M^0L^0T^0]$	Independent	Varying
4	Lumber length	l	mm	$[M^0L^1T^0]$	Independent	Varying
5	Treatment time	T	min	$[M^0L^0T^1]$	Dependent	Reponse
6	Height of patient	H	cm	$[M^0L^1T^0]$	Independent	Varying
7	Weight of patient	w	N	$[M^1L^1T^{-2}]$	Independent	Varying
8	acceleration due to gravity	g	m/s^2	$[M^0L^1T^{-2}]$	Independent	Constant

Before commencing analysis of a problem it is required to choose the repeating variables. Hence next step of the Buckingham pi theorem deals with the selection of repeating variables. Repeating variables are those which have considerable influence on the process and which will appear in all the pi terms if required. There is considerable freedom allowed in the choice. Still it is expected that selection of repeating variable should follow some criteria. When combined, these repeating variables variable must contain all of dimensions (M, L, T). A combination of the repeating variable, height of patient, weight of patient and acceleration due to gravity are selected as variables must not form a dimensionless group. The repeating variables should be chosen to be measurable in an experimental investigation. According to the criteria required to be followed by repeating repeating variables

Formation of pi terms

$$\pi_1 = f(L, H, w, g)$$

$$\pi_1 = L \times H^a \times w^b \times g^c$$

$$[M^0L^0T^0] = [M^1L^1T^{-2}] \times [M^0L^1T^0]^a \times [M^1L^1T^{-2}]^b \times [M^0L^1T^{-2}]^c$$

$$[M^0L^0T^0] = [M^1L^1T^{-2}] \times [M^0L^aT^0] \times [M^bL^bT^{-2b}] \times [M^0L^cT^{-2c}]$$

$$[M^0L^0T^0] = [M^{b+1}\, L^{a+b+c+1}\, T^{-2b-2c-2}]$$

Comparing the indices of M, L and T gives

$$b + 1 = 0$$
$$a + b + c + 1 = 0$$
$$-2b - 2c - 2 = 0$$

Solving above equations gives,

$$a = 0, b = -1 and\ c = 0$$

Similarly,

$$\pi_2 = f(\theta, H, w, g) \qquad \pi_3 = f(N, H, w, g) \qquad \pi_4 = f(l, H, w, g)$$

By solving above relationship, following table is obtained

π_1	π_2	π_3	π_4
$a = 0$ $b = -1$ $c = 0$	$a = 0$ $b = 0$ $c = 0$	$a = 0$ $b = 0$ $c = 0$	$a = -\frac{1}{2}$ $b = 0$ $c = \frac{1}{2}$

In terms of π it can be written as

$\boldsymbol{\pi_1 = \frac{L}{w}}$	$\boldsymbol{\pi_2 = \theta}$	$\boldsymbol{\pi_3 = N}$	$\boldsymbol{\pi_4 = \frac{L}{H}}$

Same method can be used to form the pi terms of the dependent variable also.

$$\boldsymbol{\pi_{01} = T\sqrt{\frac{g}{H}}}$$

These 4 independent pi terms and 1 dependent pi term will be used to form mathematical model.

$$\pi_{01} = f(\pi_1, \pi_2, \pi_3, \pi_4)$$

Initial observation about trend between dependent and independent variable is having exponential relationship. Hence

$$\pi_{01} = k_1 \times {\pi_1}^{a_1} \times {\pi_2}^{a_2} \times {\pi_3}^{a_3} \times {\pi_4}^{a_4}$$

Now forming the mathematical model means to find the value of unknowns in the above equation.
Taking log of the both the sides of above equation gives

$$\log \pi_{01} = \log k_1 + \log {\pi_1}^{a_1} + \log {\pi_2}^{a_2} + \log {\pi_3}^{a_3} + \log {\pi_4}^{a_4}$$
$$\log \pi_{01} = \log k_1 + a_1 \log \pi_1 + a_2 \log \pi_2 + a_3 \log \pi_3 + a_4 \log \pi_4$$

Above equation is valid for all the readings collected during experimentation. Hence putting summation on both the sides

$$\sum_{i=1}^{i=n} \log \pi_{01} = \sum_{i=1}^{i=n} \left(\log k_1 + a_1 \log \pi_{1_i} + a_2 \log \pi_{2_i} + a_3 \log \pi_{3_i} + a_4 \log \pi_{4_i}\right)$$

Where n is number of readings

$$\sum_{i=1}^{i=n} \log \pi_{02} = \log k_2 + a_1 \sum_{i=1}^{i=n} \log \pi_1 + a_2 \sum_{i=1}^{i=n} \log \pi_2 + a_3 \sum_{i=1}^{i=n} \log \pi_3 + a_4 \sum_{i=1}^{i=n} \log \pi_4$$

All these mathematical equations are solved in software package MATLAB, code for which is provided in annexure I.

Table 2 Index Values of Mathematical Model of Pi Terms

K	**a_1**	**a_2**	**a_3**	**a_4**
51879	**1.2500**	**-1.7000**	**0.3500**	**0.3500**

The same can be written as follows

$$\pi_{01} = 51879 \times {\pi_1}^{1.25} \times {\pi_2}^{-1.7} \times {\pi_3}^{0.35} \times {\pi_4}^{-0.35}$$

All above equations are in form of dimensionless pi terms. It is required to express them in terms of variable for the purpose of analysis of the impact of variable.
Hence dissolving the pi terms now

$$T\sqrt{\frac{g}{H}} = k_1 \times \left[\frac{L}{w}\right]^{a_1} \times [\theta]^{a_2} \times N^{a_3} \times \left[\frac{L}{H}\right]^{a_4}$$

$$T = K \times L^a \times \theta^b \times N^c \times l^d \times H^{\frac{1}{2}-d} \times w^{-a} \times g^{-\frac{1}{2}}$$

Table 3 Index Values of Mathematical Model of Variables

	K	**L**	**Ɵ**	**N**	**l**	**H**	**w**	**g**
T	51879	1.25	-1.7	0.35	0.35	0.15	-1.25	-0.50

The same can be written as follows

$$T = 51879 \times L^{1.25} \times \theta^{-1.7} \times N^{0.35} \times l^{0.35} \times H^{0.15} \times w^{-1.25} \times g^{-0.5}$$

This mathematical model which will be used for analysis and performance of the lumber traction treatment process.

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