DISCRETE – TIME RECURRENT NEURAL NETWORKS WITH TIME - VARYING DELAYS AND MARKOVIAN PARAMETER:EXPONENTIAL STABILITY ANALYSIS

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ABSTRACT

This paper is concerned with the analysis of exponential stability for a class of discrete time recurrent neural network with time varying delay and time delay.For the former the activation function is more general than the recently commonly used Lipschitz conditions. Under such mild conditions we prove the existence of the equilibrium point.Then by employing a Lyapunov – Krasovskii functional, a unfied Linear Matrix Inequality(LMI) is developed to establish sufficient conditions for DRNN to be globally exponentially stable. For the latter the stability is analysed with time delays and Markovian jumping parameters. The purpose of the problem addressed is to derive some easy-to-test conditions such that the dynamics of the neural network is exponentially stable independent of the time delays.Then by employing Lyapunov –Krasovskii, LMI approach is developed to verify the exponential stability

Keyword:- *Discrete recurrent neural networks; Exponential stability; Time-varying delays; Lyapunov-Krasovskii functional; Markovian jumping parameters; Linear matrix inequality.*

1.1INTRODUCTION:

The last few decades have seen successful applications of recurrent neural networks (RNNs) to a variety of information processing systems such as signal processing, pattern recognition, optimization, model identification and associative memories, where the rich dynamical behaviours of RNNs have played a key role. It has recently been revealed that signal transmission delays may cause oscillation and instability of the neural networks. Therefore, various analysis aspects for RNNs with delays have drawn much attention, and many results have been reported. In particular, the existence of equilibrium point, global asymptotic stability, global exponential stability, and the existence of periodic solutions have been intensively investigated. Note that, up to now, most recurrent neural networks have been assumed to act in a continuous-time manner. However, when it comes to the implementation of continuous-time networks for the sake of computer-based simulation, experimentation or computation, it is usual to discretize the continuous-time networks. In fact, discrete-time neural networks have already been applied in a wide range of areas, such as image processing, time series analysis, quadratic optimization problems and system identification, etc. In an ideal case, the discrete – time analogues should be produced in a way to reflect the dynamics of their continuous-time counterparts. Specifically, the discrete-time analogue should inherit the dynamical characteristics of the continuous-time networks under mild or no restriction on the discretization step-size and also maintain functional similarity to the continuous-time system and any physical or biological reality that the continuous-time networks has. Unfortunately, the discretization cannot preserve the dynamics of the continuous-time counterpart even for a small sampling period. In the other words, the RNNs may have finite modes, and the modes may switch from one to another at different times. It has been shown that the switching between RNN modes can be governed by a Markovian Chain. Hence, an RNN with a jumping character may be modelled as a hybrid one; that is the state space of the RNN contains both discrete and continuous states. The dynamics of the RNN is continuous but the parameter jumps among different modes may be seen as discrete events.

On the other hand, some global exponential stability criteria for the equilibrium point of discrete-time recurrent neural networks with variable delay have been presented with specific performances such as decay rate and trajectory bounds. Based on the linear matrix inequality(LMI), the uniqueness and global exponential stability of the equilibrium point have been investigated for discrete-time bi-directional associative memory(BAM) neural networks with variable delays. Very recently, a class of discrete-time neural networks involving variable delays have been dealt with, and sufficient conditions on existence, uniqueness and globally exponential stability of the equilibrium point have been derived by applying M-matrix theory. The activation functions of the discrete-time neural networks with time delays are assumed to satisfy Lipschitz conditions and the derived stability criteria are mostly delay-independent which tend to be conservative. The main purpose is to investigate the stability analysis problem of the exponential stability for a class of delayed discrete-time recurrent neural networks under more general description on the activation functions, and obtain less conservative stability criteria by using a unified linear matrix inequality(LMI) approach. It is shown that the delayed discrete-time recurrent neural networks are globally exponentially stable if a certain LMI is solvable.

1.2 PROBLEM FORMULATION:

In this paper, the recurrent neural network with time delays is described as follows:

$$\dot{u}(t) = -Au(t) + W0g0(u(t)) + W1g1(u(t-h)) + V$$

(1) where u(t) = [u1(t), u2(t), … , un(t)]T $\in \mathbb{R}$n is the state vector associated with the n neurons, the diagonal matrix A = diag(a1,a2, … ,an) has positive entries ai > 0. The matrices W0 = (w_{ij}^{0})nxn and W1 = (w_{ij}^{1})nxn are the connection weight matrix and the delayed connection weight matrix, respectively. gk(u(t)) = [gk1(u1), gk2(u2),…, $g$kn(un)]T(k = 0, 1) denotes the neuron activation function with gk(0) = 0, and V = [V1, V2,… ,Vn]T is a constant external input vector. The scalar h > 0, which may be unknown, denotes the time delay. ASSUMPTION1:

The neuron activation functions in (1), gi (・), are bounded and satisfy the following Lipschitz condition

$$|g k(x)-g k(y)| \leq |Gk(x-y)|, \forall x,y \in \mathbb{R} (k=0,1) \qquad (2)$$

where Gk $\in \mathbb{R}$nxn is a known constant matrix. In the past, the activation functions have been required to be continuous, differentiable and monotonically increasing, such as the sigmoid-type of function. In this paper, these restrictions are removed, and only Lipschitz conditions and boundedness are needed in Assumption 1. For the purpose of simplicity, we can shift the intended equilibrium u* to the origin by letting x = u − u*, and then the system (1) can be transformed into:

$$\dot{x}(t) = -Ax(t) + W0l0(x(t)) + W1l1(x(t-h)) \qquad (3)$$

where x(t) = [x1(t), x2(t),…, xn(t)]T $\in \mathbb{R}$n is the state vector of the transformed system. It follows from (2) that the transformed neuron activation functions lk(x) = gk (x + u*) − gk (u*) (k = 0, 1) satisfy

$$|lk(x)| \leq |Gkx|, \qquad (4)$$

where Gk $\in \mathbb{R}$nxn (k = 0, 1) are specified in (2).

Now, based on the model (3), we are in a position to introduce the delayed recurrent neural networks with Markovian jumping parameters.

Let {r(t), t ≥ 0} be a right-continuous Markov process on the probability space which takes values in the finite space S = {1, 2, . . . ,N} with generator Γ = (γij) (i, j $\in$ S) given by

$$P\{r(t+\Delta) = j | r(t) = i\} = \begin{cases} \gamma_{ij\Delta+o(\Delta)} & \text{if } i \neq j \\ 1 + \gamma_{ij\Delta+o(\Delta)} & \text{if } i = j \end{cases}$$

Where Δ > 0 and limΔ→0o(Δ)/ Δ = 0, γij≥ 0 is the transition rate from i to j if i $\neq$ j and γii = $-\sum_{j\neq i}\gamma_{ij}$

In this paper, we consider the following delayed recurrent neural network with Markovian jumping parameters, which is actually a modification of (3):

$$\dot{x}(t)=-A(r(t))x(t)+W0(r(t))l0(x(t))+W1(r(t))l1(x(t-h) \qquad (5)$$

Where x(t), l0(x(t)) and l1(x(t − h)) have the same meanings as those in (3), and for a fixed system mode, A(r(t)), W0(r(t)) and W1(r(t)) are known constant matrices with appropriate dimensions.
Recall that the Markov process {r(t), t ≥ 0} takes values in the finite space S = {1, 2, . . . ,N}. For the sake of simplicity, we write

$$A(i): = Ai, W0(i): = W0i, W1(i): = W1i. \qquad (6)$$

Now we shall work on the network mode

r(t) = i, ∀ i ∈ S. Observe the neural network (5) and let x(t;ξ) denote the state trajectory from the initial data x(θ) = ξ (θ) on −h ≤ θ ≤ 0 in L2$\mathcal{F}$0([−h, 0]; $\mathbb{R}$n). Clearly, the network (5) admits an equilibrium point (trivial solution) x(t; 0) ≡ 0 corresponding to the initial data ξ=0.

DEFINITION

For the delayed recurrent neural network (5) and every ξ ∈ L2F0([−h, 0]; $\mathbb{R}$n),the equilibrium point is asymptotically stable in the mean square if, for every network mode

$$\lim_{t\to\infty} \mathbb{E}|x(t;\ \xi\ )|^2 = 0; \qquad (7)$$

and is exponentially stable in the mean square if, for every network mode, there exist scalars α > 0 and β > 0 such that

$$\mathbb{E}\ |x(t;\ \xi\ )|2 \le \alpha e\beta t \sup\text{-}h\le\theta\le 0\ \mathbb{E}\ |\ \xi\ (\theta)|2. \qquad (8)$$

Our objective of this paper is to establish LMI-based stability criteria under which the network dynamics of (5) is exponentially stable in the mean square, independent of the time delay.

1.3 MAIN RESULTS AND PROOFS

Let us first give the following lemmas which will be frequently used in the proofs of our main results in this paper.

LEMMA 1:

Let x ∈ $\mathbb{R}$n, y ∈ $\mathbb{R}$n and ε > 0. Then we have xT y + yT x ≤ ε xT x + ε -1 yT y.

PROOF:

The proof follows from the inequality (ε1/2 x- ε-1/2 y)T(ε1/2 x- ε-1/2 y) ≥ 0 immediately. LEMMA 2:

Given constant matrices Ω1, Ω2, Ω3, where Ω|1= Ω1T, and 0 < Ω2=Ω2T, then Ω1 + Ω3T Ω2-1 Ω3 < 0 if and only if $\begin{bmatrix} \Omega_1 & \Omega_1^T \\ \Omega_3 & -\Omega_2 \end{bmatrix}$ < 0, or $\begin{bmatrix} -\Omega_2 & \Omega_3 \\ \Omega_3^T & \Omega_2 \end{bmatrix}$ < 0.

THEOREM 1:

If there exist two sequences of positive scalars {µ0i > 0, µ1i > 0, i ∈ S} and a sequence of positive definite matrices Pi = PiT > 0 (i ∈ S) such that the following linear matrix inequalities

$$\begin{bmatrix} -A_iP_i - P_iA_i + \sum_j^N = 1\,\gamma_{ij}P_j & \mu_{0i}G_0^T & P_iW_{0i} & \mu_{1i}G_1^T & P_iW_{1i} \\ \mu_{0i}G_0 & -\mu_{0i}I & 0 & 0 & 0 \\ W_{0i}^TP_i & 0 & -\mu_{0i}I & 0 & 0 \\ \mu_{1i}G_1 & 0 & 0 & -\mu_{1i}I & 0 \\ W_{1i}^TP_i & 0 & 0 & 0 & -\mu_{1i}I \end{bmatrix} < 0 \qquad (9)$$

hold, then the dynamics of the neural network (5) is globally exponentially stable in the mean square.

PROOF:

Let C2,1($\mathbb{R}^n \times \mathbb{R}_+$ ×S; $\mathbb{R}_+$)denote the family of all nonnegative functions Y (x, t, i) on $\mathbb{R}^n \times \mathbb{R}_+$ ×S which are continuously twice differentiable in x and differentiable in t. Denote

$\varepsilon_{oi} = \mu_{oi}^{-1}$, $\varepsilon_{1i} = \mu_{1i}^{-1}$ (10)

Fix $\varepsilon \in L^2_{\mathcal{F}_0}$([-h,0]; $\mathbb{R}^n$) arbitrarily and write x(t,ε)=x(t). Define a Lyapunov functional candidate Y(x, t, i) $\in$ C2,1(($\mathbb{R}^n \times \mathbb{R}_+$ ×S; $\mathbb{R}_+$) By

Y(x(t), r(t) = i) := Y (x(t), t, i)= xT (t)Pix(t)+ $\int_{t-h}^{t} x$T(s)Qx(s)ds (11)

where Q ≥ 0 is given as

Q = $\varepsilon_{1i}^{-1} G_1^T G_1$ (12)

It is known (see [13]) that {x(t), r(t)} (t ≥ 0) is a 1, 0]; $\mathbb{R}^n$)×S-valued Markov process.From (5), the weak infinitesimal operator L (see [10]) of the stochastic process {r(t), x(t)} (t ≥ 0) is given by:

$$LY(x(t), r(t)) = \lim_{\Delta \to 0^+} \frac{1}{\Delta}[\mathbb{E}\{Y(x(t+\Delta), r(t+\Delta))|x(t), r(t) = i\} - Y(x(t), r(t) = i)]] =$$
$$xT(t)\left[-AiPi - PiAi + \sum_{j=1}^{N} \gamma_{ij} P_j + Q\right]x(t) + xT(t)PiW0il0(x(t)) + l_0^T(x(t))W_{0i}^T Pix(t) +$$
$$xT(t)PiW1il1(x(t-h)) + l_1^T(x(t-h))\, W_{1i}^T Pix(t) - xT(t-h)Qx(t-h) +$$
$$\sum_{j=1}^{N} \gamma_{ij} \int_{t-h}^{t} x\,T(s)Qx(s)ds$$

It follows from $\sum_{j=1}^{N} \gamma_{ij} = 0$ (13)

$$\sum_{j=1}^{N} \gamma_{ij} \int_{t-h}^{t} x^T(s)ds = \left(\sum_{j=1}^{N} \gamma_{ij}\right)\left(\int_{t-h}^{t} x^T(s)Qx(s)ds = 0\right)$$ (14)

From Lemma

Let x $\in \mathbb{R}$n, y $\in \mathbb{R}$n and ε > 0. Then we have xT y + yT x ≤ ε xT x + ε -1 yT y & |lk (x)| ≤ |Gkx|,(where Gk $\in \mathbb{R}$nxn (k = 0, 1), we have:

xT(t)PiW0il0(x(t)) + l_0^T(x (t)) W_{0i}^T Pix(t)≤ $x^T(t) P_i W_{0i} W_{0i}^T P_i x(t) + \varepsilon_{0i}^{-1} l_0^T(x(t)) l_0(x(t))$

≤ $x^T(t)(\varepsilon_{0i} P_i W_{0i} W_{0i}^T P_i + \varepsilon_{0i}^{-1} G_0^T G_0)x(t)$ (15)

and xT(t)PiW1il1(x(t-h)) + l_1^T (x (t-h)) W_{1i}^T Pix(t)

≤ $\varepsilon_{1i} x^T(t) P_i W_{1i} W_{1i}^T P_i x(t) + \varepsilon_{1i}^{-1} l_1^T(x(t-h) l_1(x(t-h)$

≤ $\varepsilon_{1i} x^T(t) P_i W_{1i} W_{1i}^T P_i x(t) + \varepsilon_{1i}^{-1} x^T(t-h) G_1^T G_1 x(t-h)$ (16)

Define

$\Pi := -$ AiPi - PiAi+$\sum_{j=1}^{N} \gamma_{ij} P_j$ +$\varepsilon_{0i}^{-1} G_0^T G_0 + \varepsilon_{0i} P_i W_{0i} W_{0i}^T P_i + \varepsilon_{1i}^{-1} G_1^T G_1 + \varepsilon_{1i}$ (17)

In view of (12) and (14)-(17), it follows from (13) that

LY (x(t), i) ≤ xT(t)IIx(t). (18)

Now, Pre- and post-multiplying the inequality (9)by the block-diagonal matrix diag{I, $\varepsilon_{0i}^{1/2}$ I, $\varepsilon_{0i}^{1/2}$ I, $\varepsilon_{1i}^{1/2}$ I, $\varepsilon_{1i}^{1/2}$ I }

$$\begin{bmatrix} -A_i P_i - P_i A_i + \sum_j^N = 1\,\gamma_{ij} P_j & \varepsilon_{0i}^{-1/2} G_0^T & \varepsilon_{0i}^{1/2} P_i W_{0i} & \varepsilon_{1i}^{-1/2} G_1^T & \varepsilon_{1i}^{1/2} P_i W_{1i} \\ \varepsilon_{0i}^{-1/2} G_0 & -I & 0 & 0 & 0 \\ \varepsilon_{0i}^{1/2} W_{0i}^T P_i & 0 & -I & 0 & 0 \\ \varepsilon_{1i}^{-1/2} G_1 & 0 & 0 & -I & 0 \\ \varepsilon_{1i}^{1/2} W_{1i}^T P_i & 0 & 0 & 0 & -I \end{bmatrix} < 0,$$

(19) Or $\begin{bmatrix} \Omega_1 & \Omega_3^T \\ \Omega_3 & -\Omega_2 \end{bmatrix} < 0$

(20) $\Omega_1 = -$ AiPi -PiAi+$\sum_{j=1}^{N} \gamma_{ij} P_j$, $\Omega_2 = I$,
$\Omega_3 = [\varepsilon_{0i}^{-1/2} G_0^T \quad \varepsilon_{0i}^{1/2} P_i W_{0i} \quad \varepsilon_{1i}^{-1/2} G_1^T \quad \varepsilon_{1i}^{1/2} P_i W_{1i}]_T$ It follows from the Schur Complement Lemma (Lemma 2) that (20) holds if and only if

$$\Omega_1 + \Omega_3{}^T \Omega_2{}^{-1} \Omega_3 < 0,$$

or

$$-A_i P_i - P_i A_i + \sum_{j=1}^{N} \gamma_{ij} P_j + \varepsilon_{0i}^{-1} G_0^T G_0 + \varepsilon_{0i} P_i W_{0i} W_{0i}^T P_i + \varepsilon_{1i}^{-1} G_1^T G_1 + \varepsilon_{1i} P_i W_{1i} W_{1i}^T P_i \quad (21)$$

Which means II<0 where II is defined in (17). We are now ready to prove the exponential stability in the mean square for the neural network (5). Letβ> 0 be the unique root of the equation λ_{min}(-II) – βλ_{max}(P_i) – βhλ_{max}(Q)xβh = 0, (22)

Where Q is defined in (12), P_i is the positive definite solution to (9) or (21), and h is the time delay. We can obtain from (11) that

$$\mathcal{L}[x^{\beta t} Y(x(t), r(t))] = x^{\beta t}[\beta Y(x(t), r(t)) + \mathcal{L}Y(x(t), r(t))] \le x^{\beta t}(-\lambda_{min}(-\mathrm{II})) - \beta \lambda_{max}(P_i) |x(t)|^2 + \beta \lambda_{max}(Q) \int_{t-h}^{t} |x(s)|^2 ds$$

Then, integrating both sides from 0 to T > 0 gives $x^{\beta t} \mathbb{E} Y(x(T), r(T) \le [\lambda_{max}(P_i) + h \lambda_{max}(Q) \sup_{-h \le \theta \le 0} \mathbb{E} |\xi(\theta)|^2 - [\lambda_{min}(-\mathrm{II})$

$$-\beta \lambda_{max}(P_i) \mathbb{E} \int_0^T x^{\beta t} |x(t)|^2 dt + \beta \lambda_{max}(Q) \mathbb{E} \int_0^T x^{\beta t} \int_{t-h}^{t} |x(s)|^2 ds dt$$

Notice that

$$\int_0^T x^{\beta T} \int_{t-h}^{t} |x(s)|^2 ds dt \le \int_{-h}^{T} \left(\int_{\max(s,0)}^{\min(s+h,T)} x^{\beta T} dt \right) |x(s)|^2 ds \le \int_{-h}^{T} h x^{\beta(s+h)} |x(s)|^2 ds \le h x^{\beta h} \int_0^T x^{\beta T} |x(t)|^2 dt \,| + h x^{\beta h} \int_{-h}^{0} |\xi(\theta)|^2 d\theta$$

Then, considering the definition of β in (22), we have

$$x^{\beta T} \mathbb{E} Y(x(T), r(T)) \le [\lambda_{max}(P_i) + h \lambda_{max}(Q)] \sup_{-h \le \theta \le 0} \mathbb{E} |\xi(\theta)|^2 + \beta \lambda_{max}(Q)\, h^2 x^{\beta h} \sup_{-h \le \theta \le 0} \mathbb{E} |\xi(\theta)|^2$$

&

$$\mathbb{E} |x(T)|^2 \le \lambda_{min}^{-1}(P_i) \left([\lambda_{max}(P_i) + h \lambda_{max}(Q)] \sup_{-h \le \theta \le 0} \mathbb{E} |\xi(\theta)|^2 + \beta \lambda_{max}(Q) h^2 x^{\beta h} \sup_{-h \le \theta \le 0} \mathbb{E} |\xi(\theta)|^2 \right) x^{-\beta h}$$

Since T > 0 is arbitrary, the definition of mean square exponential stability is then satisfied, hence the proof of Theorem 1 is completed. Hence the proof.

CONCLUSION:

This paper, we have dealt with the problem of global exponential stability analysis for a class of general recurrent neural networks, which both time delays and Markovian jumping parameters. We have removed the traditional monotonicity and smoothness assumptions on the activation function. A linear matrix inequality (LMI) approach has been developed to solve the problem addressed. The conditions for the global exponential stability have been derived in terms of the positive definite solution to the LMIs, and a simple example has been used to demonstrate the usefulness of the main results.

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