Design of Sliding Mode Control for Nonlinear Uncertain System

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Abstract

This paper extends a recent result on sliding mode control with mismatched uncertainties. Sliding Mode Control has been recognized as one of the key approaches for the systematic design of robust controllers. Mostly it is found for controlling of complex nonlinear dynamic systems operating under uncertainty conditions. Sliding mode control is a nonlinear control technique that have properties of accuracy, robustness, and easy tuning and implementation. The major advantage of sliding mode controllers is inherent insensitivity to parameter variations and disturbances in the system, thereby eliminating the necessity of exact modeling of the system. The practical stability of the overall system is proved and the results are verified by simulation of an illustrative example.

Key word — Uncertain system, chatter reduction, sliding mode control (SMC).

1. Introduction

Controlling of process with uncertainties, non-linearities and disturbances is an interesting and challenging problem in control engineering. Sliding mode control (SMC) is an effective and popular control strategy for controlling systems affected by uncertainties and external unmeasurable disturbances and has found several applications in diverse fields and have properties of accuracy, robustness, and easy tuning and implementation. In control systems, sliding mode control, or SMC, is a nonlinear control method that alters the dynamics of a nonlinear system by application of a discontinuous control signal that forces the system to slide along a cross-section of the system's normal behavior. The state-feedback control law is not a continuous function of time. Instead, it can switch from one continuous structure to another based on the current position in the state space. Hence, sliding mode control is a variable structure control method. SMC is robust with respect to matched internal and external disturbances. The multiple control structures are designed so that trajectories always move toward an adjacent region with a different control structure, and so the ultimate trajectory will not exist entirely within one control structure. Instead, it will slide along the boundaries of the control structures. The motion of the system as it slides along these boundaries is called a sliding mode and the geometrical locus consisting of the boundaries is called the sliding surface. The sliding mode control approach is recognized as one of the efficient tools to design robust controllers for complex high-order nonlinear dynamic plant operating under uncertainty conditions. Sliding controller design provides a systematic approach to the problem of maintaining stability and consistent performance. The conventional SMC shows insensitivity to matched uncertainties and disturbances.

A variety of control strategies used to address the problem of mismatched uncertainties. Combining SMC or other control strategies with methods that give estimates of uncertainties and disturbances is an attractive proposition. Such a combination enables a reduction in the magnitude of the discontinuous component in control and thereby offers the possibility of mitigating the chatter in control. Sliding mode control must be applied with more care than other forms of nonlinear control that have more moderate control action. In particular, because actuators have delays and other imperfections, the hard sliding-mode-control action can lead to chatter, energy loss, plant damage, and excitation of unmodeled dynamics. Continuous control design methods are not as susceptible to these problems and can be made to mimic sliding-mode controllers. In this paper, the main theory of sliding mode control and sliding mode controller design will be introduced in an understanding way, requiring only a fundamental knowledge of control systems.
2. Design of Controller

The design of a sliding mode controller consists of three main steps:
1. Design of a sliding surface.
2. Selection of the control law, which holds the system trajectory on the sliding surface.
3. Key step is the chattering free implementation.

1. Design of Sliding Surface

Sliding mode control is a type of the variable structure control system, which is characterized by a discontinuous feedback control structure that switches as the system crosses certain manifold in the state space to force the system state to reach, and subsequently to remain on a specified surface within the state space called sliding surface. The multiple control structures are designed so that trajectories always move toward an adjacent region with a different control structure, and so the ultimate trajectory will not exist entirely within one control structure. Instead, it will slide along the boundaries of the control structures. The motion of the system as it slides along these boundaries is called a sliding mode and the geometrical locus consisting of the boundaries is called the sliding surface. The system is said to be in sliding motion when \( \sigma = S \dot{x} = 0 \). During sliding, the motion of the system in its state space is restricted to the surface defined by \( \sigma = 0 \), so the system demonstrates the behaviour of a linear system characterized by the selection of \( S \). This design is called the sliding surface design. Throughout the sliding motion the system is supposed to show desired dynamics. This is achieved by the appropriate design of the sliding surface that is through surface design. For linear systems, the sliding surface design determines the eigenvalues of the system during sliding motion.

![Figure 1: The behaviour of the system states in sliding mode control](image)

2. Selection of Control Law

The control law is chosen in order to enforce a sliding mode. First, a control law \( u \) is selected for justifying the sliding condition. Also for the review of the presence of modeling exactness and disturbances, the control law has to be discontinuous across \( S(t) \). The motivation for choosing these control laws are their varying combinations of behavior for both small and large values of \( S \).

3. Chattering Effect

In the implementation of sliding mode control theory in real systems, the main obstacle is an undesirable phenomenon of oscillation with finite frequency and amplitude, which is known as chattering. The chattering is harmful because it leads to low control accuracy, high wear of moving mechanical parts, and high heat losses in electrical power circuits. Two main causes have been identified. First is fast dynamics in the control loop, which were neglected in the system model, are often excited by the fast switching of sliding mode controllers. Second is digital implementations in microcontrollers. In theory, the trajectories slide along a switching function. In practice, there is high frequency switching. A high-frequency motion called chattering, so that no ideal sliding mode can occur in practice. Yet, solutions have been developed to reduce the chattering and so that the trajectories remain in a small neighborhood (boundary) of the surface. Called as chattering because of the sound made by old mechanical switches.
Chattering makes the system state oscillate around the sliding surface, as showed in Fig. 2, and introduces two problems in control systems:

1. the non-modeled high frequency dynamics can be excited, resulting in unforeseen unstable behavior;
2. the actuators are placed under undue stress, so their breakages can take place or their lifetime are reduced.

3. Sliding Mode Controller Design

Consider the system,

\[ \dot{x}_1 = x_2 + d_1(t) \]  
\[ \dot{x}_2 = f(x) + g(x) u \]  
\[ y = x_1 \]  

controller design: The switching sliding surface is then defined by

\[ \sigma = c x_1 + x_2 = 0 \]  

To determine a control law, that keeps the system on \( \sigma(x) = 0 \), we introduce the Lapnov function,

\[ v(x) = \frac{1}{2} \sigma^2 > 0 \]  

Positive definite

\[ \dot{v} = \sigma, \dot{\sigma} < 0 \]  

Negative definite

\[ \dot{\sigma} = c \dot{x}_1 + \dot{x}_2 \]  
\[ \dot{\sigma} = c x_2 + f(x) + g(x) u \]  
\[ \sigma.\dot{\sigma} = \sigma(c x_2 + f(x) + g(x) u) < 0 \]  
\[ \sigma.\dot{\sigma} = \sigma(c x_2 + f(x) + g(x) u) = -\rho \, |\sigma| \]  
\[ = (c x_2 + f(x) + g(x) u) = -\rho \frac{|\sigma|}{\sigma} \]  
\[ = (c x_2 + f(x) + g(x) u) = -\rho \, \text{sgn}(\sigma) \]
\[ g(x)u = -\rho \text{sgn}(\sigma) - cx_2 - f(x) \]
\[ u = \frac{1}{g(x)}[cx_2 + f(x) + \rho \text{sgn}(\sigma)] \quad (10) \]
\[ \sigma . \dot{\sigma} = -\rho |\sigma| \quad (11) \]

Reachability condition

4. Design example and simulations

Example 1: Consider the following illustrative example.

\[ \dot{x}_1 = x_2 + d_1 \]
\[ \dot{x}_2 = -2x_1 - x_2 + e^{x_1} + u \]
\[ y = x_1 \]

Controller design: SMC

\[ \sigma = cx_1 + x_2 \quad (12) \]
\[ \sigma = ce_1 + e_2 \]
\[ e_1 = x_1 - x_d \]
\[ \dot{e}_1 = e_2 = \dot{x}_1 - \dot{x}_d \]
\[ e_2 = x_1 + d_1 - \dot{x}d \]

If \( \sigma > 0 \) then \( \sigma . \dot{\sigma} < 0 \). If \( \sigma < 0 \) then \( \sigma . \dot{\sigma} > 0 \)

\[ \dot{\sigma} = c\dot{e}_1 + \dot{e}_2 \]
\[ = ce_2 + (\dot{x}_2 + d_1 - \dot{x}d) \]
\[ = c(x_2 + d_1 - \dot{x}d) + (2x_1 - x_2 + e^{x_1} + u) + d_1 - \dot{x}d \]
\[ = (cx_2 + cd_1 - cxd) - 2x_1 - x_2 + e^{x_1} + u + d_1 - \dot{x}d \]
\[ \sigma . \dot{\sigma} = -2x_1 - (1 - c)x_2 + e^{x_1} + cd_1 + \dot{d}_1 - \dot{x}d + u = -\rho |\sigma| \]

\[ \sigma . \dot{\sigma} = \sigma [2x_1 - (1 - c)x_2 + e^{x_1} + cd_1 + \dot{d}_1 - cxd - \dot{x}d + u] = -\rho |\sigma| \quad (14) \]
\[ -2x_1 - (1 - c)x_2 + e^{x_1} + 1 - cd_1 + \dot{d}_1 - cxd - \dot{x}d + u = -\rho |\sigma| \]
\[ u = 2x_1 + (1 - c)x_2 - e^{x_1} - cd_1 - \dot{d}_1 + cxd + \dot{d} - \rho \text{sgn}(\sigma) \quad (15) \]

Simulation results for example (1) is as shown in figure (3).
5. Advantages

There are a number of advantages to this approach:
1. The dynamic behaviour of the system may be tailored by the particular choice of switching function.
2. The closed-loop response becomes totally insensitive to a particular class of uncertainty in the system.
3. Controller design provides a systematic approach to the problem of maintaining stability and consistent performance in the face of modeling imprecision.
4. Possibility of stabilizing some nonlinear systems which are not stabilizable by continuous state feedback laws.
5. Robustness property that is once the system is on sliding surface then it produced bounded parameter variation and bounded disturbances.
6. The sliding mode control approach is assumed to be one of the efficient tool to design robust controllers for complex high order non linear plant operating under certain conditions.

6. Applications

1. It has been extensively used in motion control, power systems, robot arms and process control etc. The applications of second order sliding mode control to several types of mechanical systems including cranes, robot manipulators, train pantographs, etc.
2. Sliding mode control used in linear systems for eigen value placement, optimization, disturbance rejection and identification.
3. The real time control fan overhead crane prototype was addressed using sliding mode control techniques.
4. The motion control for jet propelled marine vehicles has been addressed by first and second order sliding mode control methods.
7. conclusion

In this paper, the basic properties and interests of sliding modes have been discussed. The main advantages of the sliding mode control approach are the simple in both design and implementation. sliding mode control scheme is a well known robust control scheme for dynamic uncertain systems. This paper extends the result for a larger class of unmatched disturbances. It is possible to eliminate the chatter completely or alleviate it greatly. The expected results were verified by simulations.

8. References