DESIGN AND ANALYSIS OF THICK WALLED CYLINDER WITH HOLES

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ABSTRACT

It is proposed to conduct stress analysis of a thick walled cylinder near the radial hole on the surface. The literature indicated that there will be a ductile fracture occurring in such cases. The radial holes cannot be avoided due to various piping attachments . Hence the stress analysis of cylinder and its ultimate failure under internal pressure beyond elastic limit is an appropriate scenario. The plastic zone appearing in vicinity of internal surface of cylinder propagates more fastly along hole side. When cylinder is unloaded it will cause reverse plasticity. Therefore it is proposed to obtain numerical solution using Finite Element analysis of cylindrical segment to obtain the radial & hoop stress distribution by including elastoplastic conditions.

In the present work the stress analysis of thick walled cylinders with variable internal pressure states is conducted Elastic analysis of uniform cylinder & cylinder with holes is predicted both from theory (lame's formulae) under & Finite element method. Also elasto plastic analysis with bilinear kinematic hardening material is performed to know the effect of hole sizes. It is observed that there are several factors which influence stress intensity factors . The Finite element analysis is conducted using commercial solvers ANSYS & CATIA. Theoretical formulae based results are obtained from MATLAB programs. The results are presented in form of graphs and tables.

Keyword: - *Stress analysis , equivalent stress, stresses & internal pressure and elastic & elastic- plastic deformation etc….*

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1. INTRODUCTON

1.1 Problem Statement

Thick walled cylinders are widely used in chemical, petroleum, military industries as well as in nuclear power plants .They are usually subjected to high pressures & temperatures which may be constant or cycling. Industrial problems often witness ductile fracture of materials due to some discontinuity in geometry or material characteristics The conventional elastic analysis of thick walled cylinders to final radial & hoop stresses is applicable for the internal pressures upto yield strength of material. But the industrial cylinders often undergo pressure about yield strength of material. Hence a precise elastic-plastic analysis accounting all the properties of material is needed in order to make a full use of load carrying capacity of the material & ensure safety w.r.t strength of cylinders.

The stress is directly proportional to strain upto yield point. Beyond elastic point , particularly in thick walled cylinders, there comes a phase in which partly material is elastic and partly it is plastic as shown in **FIG 1.1** . Perfect plasticity is a property of materials to undergo irreversible deformation without any increase in stresses or loads. Plastic materials with hardening necessitate increasingly higher stresses to result in further plastic deformation. There exists a junction point where the two phases meet. This phase exists till whole material becomes plastic with increase in pressure. This intermittent phase is Elastic- Plastic phase. In cylinders subjected to high internal pressures, often the plastic state shown as 2 in **FIG1.1** is represented as a power law:

$$\sigma = E_T \epsilon^n$$

Where $\mathbf{E_T}$ is strain hardening modulus, n is index (from 0 to 1).

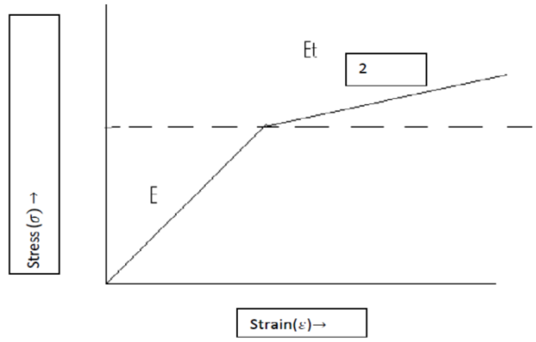

Fig. 1.1 Stress Strain Curve

Autofrettage is a phenomenon in which thick cylinders are subjected to enormous pressure building in compressive residual stresses. This increases ductile metal's resistance to stress cracking. In case of Autofrettage, material attains state of elastic-plastic state. At particular radius (critical radius) there exists a junction of elasticity and plasticity and is of great importance in designing.

In summery Autofrettage process the cylinder is subjected to a certain amount of pre internal pressure so that its wall becomes plastic. The pressure then released & the residual stresses lead to a decrease in maximum Von mises stresses in the working load range. This means an increase in pressure capacity of the cylinder of the cylinder . The main problem in analysis of Autofrettage process is to determine the optimum Autofrettage pressure & corresponding radius in elastic-plastic boundary.

The analysis of uniform cylinders can be conducted based on axi-symmetric conditions. However most of industrial cylinders incorporate openings in the main shell for variety of reasons such as

1. Instrumentation,
2. Burst in caports
3. Transfer of fluids.

Presence of opening in the shell causes a local stress concentration in the opening . The associated stress concentration factors depends on size, shape, location of opening.

It is important to minimize the stress raising effect in the opening . To analyze cylinders with such a radial openings (here after called as cross holes) subjected to internal pressures, 3 dimensional solid models are needed . Even the geometry maintains axi-symmetry. One cannot adapt axi-symmetry analysis approaches because of these holes on side of axis.

In vicinity of radial holes the initiation of plastic effects occur at lower pressures, than that of plain cylinder. This is especially dangerous during fatigue loads. The imitation of plasticity in cylinder with a hole takes place at the internal edges of the hole. The first plastic point appears at intersection of edges with cylinder generated by hole axis. The point at which the generator is tangent to the hole edge becomes partly unloaded & stress in vicinity are far from yield point. Therefore it is generally sufficient to analyze only one cylinder section going through cylinder & hole axis. The plastic zone rapidly propagates along hole side & reaches external edge.

General applications of Thick-walled cylinders include, high pressure reactor vessels used in metallurgical operations, process plants, air compressor units, hot water storage tanks, pneumatic reservoir, hydraulic tanks, storage for gasses like butane, LPG etc. The radial holes cannot be avoided because of various piping or measuring gauge attachments. Hence investigating stress distributions around hole area is an appropriate criteria for suitable design purpose.The reactor vessels are often subjected to extreme conditions of high pressure and temperature of working fluids. Sometimes fluids can be corrosive in nature due to reaction with vessel materials.

The operating pressures can be as high as 10000 psi(69.2 Mpa). The radial holes embedded in thick- walled cylinders create a problem in designing. The operating pressures are reduced or the material properties are strengthened. There is no such existing theory for the stress distributions around radial holes under impact of varying internal pressure. Present work puts thrust on this area and relation between pressure and stress distribution is plotted graphically based on observations. Here focus is on pure mechanical analysis & hence thermal, effects are not considered.

This section deals with the related work done in the area of thick walled cylinders with and without holes subjected to varying internal pressure amplitudes.

Xu & Yu [1] Carried down shakedown analysis of an internally pressurized thick walled cylinders, with material strength differences. Through elasto-plastic analysis, the solutions for loading stresses, residual stresses , elastic limit , plastic limit & shakedown limit of cylinder are derived.
Hojjati & Hossaini[2] studied the optimum auto frottage pressure & optimum radius of the elastic-plastic boundry of strain-hardening cylinders in plane strain & plane strain conditions
Ayub et al.[3] presented use of ABAQUS FE code to predict effects of residual stresses on load carrying capacity of thick walled cylinders.
Zheng & Xuan [4] carried out autofrettage & shake down analysis of power-law strain hardening cylinders S.T thermo mechanical loads.
Lavit & Tung [5] solved the thermoelastic plastic fracture mechanics problem of thick walled cylinder subjected to internal pressure and non uniform temperature field using FEM.
Makulsawatdom et al.[8] presented elastic stress concentration factors for internally pressurized thick walled cylindrical vessels with radial & offset circular & elliptical cross holes. Three forms of intersection between the cross hole & main bore are considered viz., plain, chamfered & blend radius. Makulsawatundom et al.[9] shown the shakedown behavior of thick cylindrical pressure vessels with cross holes under cyclic internal pressures, using FEA.
Nihons et al.[11] reported elastic stress concentration factors for internally pressurized thick walled cylinder with oblique circular to cross holes. Results of FEA for two wall ratios (2.25 & 4.5) and a range of cross-hole ratios (0.1-0.5) have been presented and shown that stress concentration factors sharply increase with inclination & cross hole axis.

1.2 Objective Of Work

1.2.1 Finding residual stresses :

Stresses that remain in material even after removing applied loads are known as residual stresses. These stresses occur only when material begins to yield plastically. Residual stresses can be present in any mechanical structure because of many causes. Residual stresses may be due to the technological process used to make the component. Manufacturing processes lead to plastic deformation.

1.2.2 Finding relations between various parameters in analysis of cylinders with holes :

With respect to the literature review, work has been not done to find fundamental equations depicting relationship between various parameters(pressure vs stress) for thick-walled cylinders with radial holes. Here attempt has been made to find a graphical relationship of the same based on results and observations obtained.

1.2.3 Co-ordination with finite element model :

The finite element method (FEM**)** (its practical application often known as finite element Analysis (FEA)) is a numerical technique for finding approximate solutions of partial differential equations (PDE) as well as of integral equations. The solution approach is based either on eliminating the differential equation completely (steady state problems), or rendering the PDE into an approximating system of ordinary differential equations, which are then numerically integrated using standard techniques such as Euler's method, Runge-Kutta, etc.

1.2.4 Objectives Of The Work

The following are the principal objectives of the work.

1. Stress analysis of thick walled cylinders with radial holes & understand the effect of relative dimensions/parameters of hole on equivalent stress developed due to internal pressure.
2. Study of Autofrettage process & find out the residual stresses theoretically & using FEM Method by considering bi linear kinematic hardening state(elasto-plastic state), for uniform cylinder as well as cylinder with radial hole.
3. Depicting relationship between internal pressure applied and equivalent stress graphically for elastic-plastic cases of uniform cylinder as well as cylinder with radial holes.

2. MATHEMATICAL MODELLING

This chapter gives mathematical relations for stresses & internal pressure during elastic & elastic- plastic deformation.

2.1 Pressure Limits Of Thick Walled Cylinders

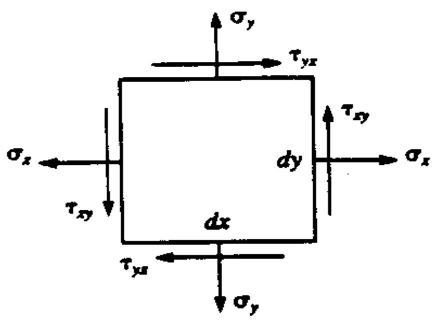

Fig 2.1 Two dimensional stresses and strains

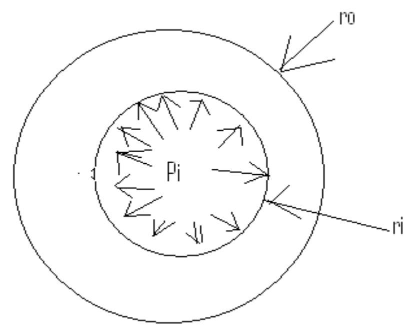

Fig 2.2 Cylinder under internal pressure

Plane stress state of any material is the case where the stresses are two dimensional . It can be defined as state of stress in which normal stress (σz), shear stresses τxz and τyz , directed perpendicular to assumed X-Y plane are zero. The plane stress case is one of the simplest methods to study continuum structures. Plane strain is defined as state of strain in which strain normal to X-Y plane and shear strain τxz, τyz are zero. In plain strain case one deals with a situation in which dimension of the structure in one direction is very large as compared to other two directions. The applied forces act in X-Y plane and does not effectively act in Z direction. Our present work is of same case. For any thick walled axially symmetric, having plain stress state has the following equations for stress distributions across the thickness derived from lame's equations:

$$\frac{\partial \sigma_r}{\partial r} + \frac{\sigma_r - \sigma_\theta}{r} = 0 \qquad (1)$$

$$\varepsilon_r = \frac{\partial u_r}{\partial r} = \left(\frac{1}{E}\right)[\sigma_r - \nu\sigma_\theta] \qquad (2)$$

$$\epsilon_\theta = \frac{1}{E}[\sigma_\theta - \sigma_r] \qquad (3)$$

Where σr is the radial stress; σθ is the hoop stress ; E is the young's modulus ν is the poission ratio. Ur is the deformation (change in directions).

In general thick walled cylinders are subjected to internal pressure, as shown in **Fig 2,** which cause radial and hoop stress distributions across the thickness.(Assuming geometric linearity in material). There exist a set of equations which give us relationship between Internal pressure and stresses developed, derived from above mentioned equations (1),(2),(3), which are in turn derived from lame's equations of thick cylinder.

Consider a plain strain cylinder internal radius $\mathbf{r_i}$ & outer radius $\mathbf{r_o}$.
When pressure Pi is large enough the cylinder begins to yield from surface r = $\mathbf{r_i}$. There exists a radius rc at the elastic and elastoplastic boundary interface. The associated pressure is Pc.So the material can be analyzed as [14] region between $\mathbf{r_i}$ <r< $\mathbf{r_c}$ and $\mathbf{r_c}$ <r< $\mathbf{r_o}$. The first one is in plastic state and second being in elastic state.

2.1.1 Elastic State :

$\sigma_a = (p_i r_i^2 - p_o r_o^2)/(r_o^2 - r_i^2)$
where

σ_a = stress in axial direction (MPa, psi)
p_i = internal pressure in the tube or cylinder (MPa, psi)
p_o = external pressure in the tube or cylinder (MPa, psi)
r_i = internal radius of tube or cylinder (mm, in)
r_o = external radius of tube or cylinder (mm, in)
The stress in circumferential direction at a point in the tube or cylinder wall can be expressed as:
= [(pi ri2 - po ro2) / (ro2 - ri2)] - [ri2 ro2 (po - pi) / r2 (ro2 - ri2)] (5)

where = stress in circumferential direction (MPa, psi)
The stress in tangential direction at a point in the tube or cylinder wall can be expressed as:

σr = [(pi ri2 - po ro2) / (ro2 - ri2)] + [ri2 ro2 (po - pi) / r2 (ro2 - ri2)] (6)

where Radial stress in tangential direction.

The strain components are as follows

$$\varepsilon_r = \left(\frac{1+\upsilon}{E}\right)\left(\frac{p_c.r_c^2}{(r_o^2} - r_c^2\right)\left(1 - \frac{r_o^2}{r^2} - 2\nu\right) \tag{7}$$

$$\varepsilon_\theta = \left(\frac{1+\nu}{E}\right)\left(\frac{p_c r_c^2}{r_o^2 - r_c^2}\right)\left(1 - 2\nu + \frac{r_o^2}{r^2}\right) \tag{8}$$

$\varepsilon_z = 0$ Longitudinal strain As the case is a plain strain problem.

2.1.2 Elastic-Plastic State :

The governing equations in formulating stress for elastic-plastic region have been derived by considering power-law hardening model, strain gradient(modified von mises)theory[14] for axi-symmetric problem .

$$\sigma_\theta - \sigma_r = \frac{r\partial\sigma_r}{\partial r} \tag{9}$$

$$r\left(\frac{\partial\varepsilon_\theta}{\partial r}\right) = \varepsilon_r - \varepsilon_\theta \tag{10}$$

From above equations, employing classical plasticity solution, final useful equations we get is:

$$p_i = \left(\frac{\sigma_y}{\sqrt{3}}\right)[\left(1 - \frac{r_c^2}{r_o^2}\right) + 2\ln\frac{r_c}{r_i} \tag{11}$$

$$\sigma_r = \left(\frac{\sigma_y}{\sqrt{3}}\right)\left[-1 + \frac{r_c^2}{r_o^2} - 2\ln\frac{r_c}{r}\right] \tag{12}$$

$$\sigma_\theta = \left(\frac{\sigma_y}{\sqrt{3}}\right)\left[1 + \frac{r_c^2}{r_o^2} - 2\ln\frac{r_c}{r}\right] \tag{13}$$

Where σ_y is the yield strength of material. And p_i is the internal pressure applied. Here main assumption in that external applied pressure/load is zero.

2.2 ANALYSIS OF AUTOFRETTAGE PROCESS

Residual stresses induced(both tension as well as compression) in thick cylinders due to internal pressure application forcing the maximum equivalent stress to cross the yield point. This is autofrettage phenomenon. The fatigue

The pressure to initiate auto frottage is known as autofrettage pressure. Pa

$$p_A = \frac{\sigma_y}{2}\left[1 - \frac{m^2}{K^2}\right] + \sigma_y \ln m \qquad (14)$$

2.2.1 stress distribution under autofrettage pressure loading

$$\sigma_r = \sigma_y\left[\ln\left(\frac{r}{R_p}\right) - \left(\frac{1}{2}\right)\left(1 - \frac{R_P^2}{r_o^2}\right)\right] \qquad (15)$$

$$\sigma_\theta = \sigma_y[\ln\left(\frac{r}{R_p}\right) - \left(\frac{1}{2}\right)\left(1 - \frac{R_P^2}{r_o^2}\right)] \qquad (16)$$

Above equations give radial and hoop stresses for an autofrettage phenomenon.

2.2.1 Residual stress distributions

It is assumed that during unloading the material follows HOOKE's law & the pressure is considred to be reduced(applied in negative pressure) elastically across the whole cylinder. Residual stress after unloading can then be obtained by removing Autofrettage pressure load elastically across the whole cylinder. The unloading elastic stress distribution being given as

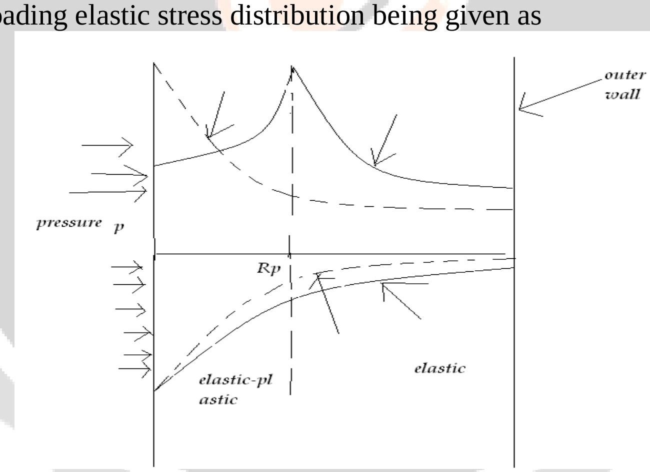

Fig 2.3 Residual stress distributions

The dotted lines show the unloading distribution curves and solid lines show the loading distribution curves

$$\sigma_r = p_a\left[\frac{1-\frac{r_o^2}{r^2}}{k^2-1}\right] \qquad (17)$$

$$\sigma_\theta = p_a[\frac{1+\frac{r_o^2}{r^2}}{k^2-1}] \qquad (18)$$

Where k $= r_o/r_i$, m $= R_p/r_i$, $R_P = \sqrt{(r_i * r_o)}$, Pa is the autofrettage pressure.

The elastic stresses developed during loading condition can be given as

$$\sigma_\theta = \sigma_y[1 + \ln\left(\frac{r}{Rp}\right) - \left(\frac{1}{2}\right)\left(1 - \left(\frac{R_P}{r_o}\right)^2\right] \quad for\ r_i \le r \le R_p \tag{19}$$

$$\sigma_\theta = \frac{\sigma_y R_P^2}{2r_o^2(r_o^2 - R_P^2)}\left[1 - \left(\frac{r_o^2}{r^2}\right)\right] \quad for\ R_p \le r \le r_0 \tag{20}$$

$$\sigma_r = \sigma_y\left[\ln\frac{r}{R_P} - \left(\frac{1}{2}\right)\left(1 - \frac{R_P^2}{r_o^2}\right)\right] \quad for\ r_i \le r \le R_P \tag{21}$$

$$\sigma_r = \frac{\sigma_y R_P^2}{2r_o^2(r_o^2 - R_P^2)}\left[1 + \frac{r_o^2}{r^2}\right] \quad for\ R_p \le r \le r_0 \tag{22}$$

$$\sigma_{res\ hoop} = \sigma_{\theta\ unloading} - \sigma_{\theta\ loading} \tag{23}$$

$$\sigma_{res\ radial} = \sigma_{r\ unloading} - \sigma_{r\ loading} \tag{24}$$

No yielding occurs due to residual stresses. Superimposing these distributions on the previus loading distributions allow the two curves to be subtracted both for the hoop and radial stress and produce residual stresses.

2.3 Cylinders With Radial Holes

The elastic hoop stress concentration factor is defined as the ratio of maximum principal stress & lame's hoop stress on the inside surface of the pressurized cylinder

$$\text{SCF} = \frac{\sigma_{max}}{\sigma_{lame}}. \tag{25}$$

For the cylinder with wall ratio k=1/β & internal pressure p, the refrence stress is

$$\sigma_{lame} = \left(\frac{k^2+1}{k^2-1}\right). \tag{26}$$

SCF is a measure of relative influence of cross hole & may be used to define the peak loads for cyclic loading. SCF= Actual stresses(with holes) / theoretical stresses(without holes).

2.4 Finite Element Model

In most cases of uniform cylinders theoretical stress relations are available that is uniform cylinders operated within elastic and plastic pressure regions. The verification can be done with Finite element Analysis. In the FE method, often symmetry is employed to avoid the analysis of whole vessel . The uniform cylinders having axis of symmetry are analyzed using axi-symmetric elements. These elements adapt a different stress strain matrix & stiffness matrix is derived a/c the following formula

$$k = \iint B^T . D . B\, dr . d\theta. \tag{20}$$

B= strain displacement matrix.

D = stress strain matrix.

K= stiffness matrix.

Generally all iso-parametric elements can be used as axi-symmetric elements. In the present work 4 node 2 degree of freedom iso-parametric element is employed to most the cylinder wall. It requires Young's modulus, Poisson's ratio as well as yield stress & strain hardening modulus to conduct the stress analysis. When there are holes on the surface of cylinder , the axi-symmetry is lost & the analysis has to be done using 3 dimensional solid elements. 8 node 3 degree of freedom solid elements are quite commonly used in commercial solid modeling software like CATIA , the tetrahedron elements are by default.

In present case for analysis of thick-walled cylinders with radial hole, a cylinder segment is considered. For a given cylinder thickness & hole radius Ri the pressure p is varied such that plasticity condition occurs.

3. RESULTS & DISCUSSION

This chapter presents stress analysis results of uniform cylinders & cylinders with radial hole subjected to internal pressures. Initially material & geometric data is described.

3.1 The Geometry And Material Properties Considered

In the thick-walled cylinder problem, generally ductile materials are used heavily for industrial purpose. The main reason being , ability to withstand higher internal pressures loads. Hence their ductile fracture study is an interesting work. In our present work, standard steel is chosen for analysis taking industrial application point of view.
The dimensions for the steel cylinder taken := 300 mm
Length can be of any dimension, as it is a case of axi-symmetric plain strain problem. We have chosen 600 mm.

Geometrically the entire cylinder is uniform(across the cross section also), material is isotropic in nature. Entire analysis work has been done assuming /neglecting thermal effects. For the cylinder with holes case, the hole is a radial cross bore of dimension is chosen.

The following material properties are chosen.
YOUNG'S MODULUS : 200 GPA
Poission ratio : 0.3
Yield strength : 684 MPA.
The main criteria for failure chosen is maximum strain energy criterion or **von mises failure criteria.** It says that the material will fail when the equivalent stress exceeds the yield point limit.The main criteria for failure chosen is maximum distortion energy criterion or **von Mises yield criteria.**
It says that the material will fail when the equivalent stress exceeds the yield point limit.
For an axi-symmetric problem there are no shear forces. Hence hoop, longitudinal and radial stresses are the principal stresses.

$$\left(\frac{1}{2}\right)\left((\sigma_\theta - \sigma_r)^2 - (\sigma_r - \sigma_z)^2 - (\sigma_z - \sigma_\theta)^2\right) \leq \sigma_y^2 \qquad (1)$$

The above equation is the failure criteria. The left hand side is the equivalent stress or von Mises stress

3.2 Elastic Analysis Of Thick Walled Cylinders

3.2.1 Analysis Of Uniform Cylinders

Cylinder is then subjected to an internal pressure varying gradually(increased in steps) and corresponding maximum von Mises stress values are noted from the analysis results. The iterative procedure is continued till the von Mises stress reaches near about yield strength values. While modeling and carrying analysis in CATIA the following The cylinder with above specified dimensions are chosen and modeled in the software CATIA.
The assumptions are made :
1. Cylinder without end-caps, subjected to internal pressure.
2. Material is perfectly elastic.
3. Default tetrahedral mesh gives enough accuracy.

Theoretical stresses based on lame's equations for elastic analysis are used to validate CATIA outputs.
The general lame's equations are followed for elastic analysis by theory which are shown in mathematical modeling chapter.

That is $$\sigma_{eq} = \sqrt{[\sigma_\theta^2 + \sigma_r^2 - \sigma_r.\sigma_\theta]} \quad (2)$$

There is an important pressure limit to study the thick walled cylinders. This is internal pressure required at the onset of yielding of inner bore surface. That is the load to initiate the plasticity at the internal cylinder radius , often expressed as Elastic load capacity

$$(\gamma_o = \frac{p_o}{\sigma_y})$$

Load capacity of a cylinder :

$$\gamma_o = \frac{1-\beta^2}{\sqrt{3}} = \frac{p_o}{\sigma_y} \quad (3)$$

Where r_0 is the load capacity: β is the radius ratio (R_i / R_0): Po is tne pressure where the plasticity begins at internal walls of cylinder and σ_y is yield strength of material. For above specified dimension β =0.66, σ_{y} = 684 Mpa

Hence Po = [1-(0.66^2)] / √3]*684 Mpa = 220.8 Mpa.

The internal pressure at the inner surface is applied from a starting value of 70 Mpa & slowly is incremented in steps of 10 Mpa . In each case the corresponding maximum equivalent stress is tabulated as depicted in Table 3.1. A screenshot at one of pressures in CATIA is shown in Fig 3.1.

Table 3.1 Variation Of Maximum Equivalent Stress Developed With Internal Pressure In Uniform Cylinder

Pressure (Mpa)	Maximum von Mises stress(Mpa)
70	321.12
80	338.34
90	360.3
100	394.4
110	419.2
120	436.8
130	458.8
140	478.86
150	502.1
160	524.17
170	538.56
180	560.4
190	582.15
200	600.24
210	643.2
220	680

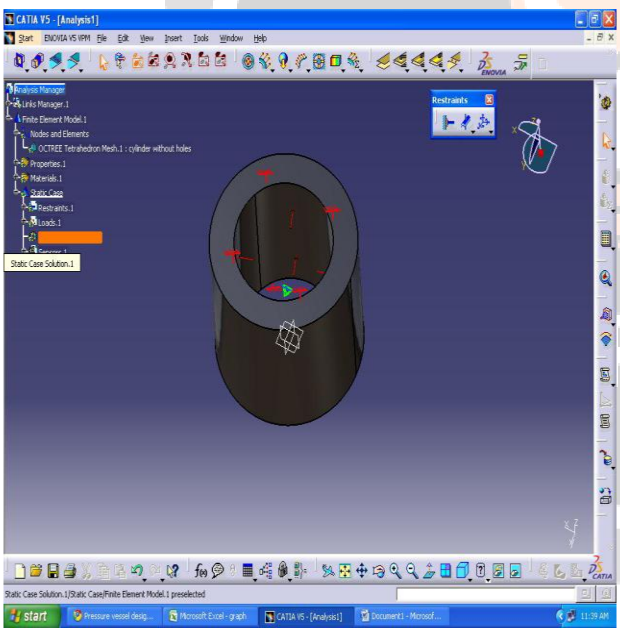

Fig 3.1 A screen shot of cylinder model at one of applied pressures

The above observations shows a linear relationship, confirming elastic behavior as predicted by theory. Corresponding to the value of pressure which initiates the plasticity inside bore , It is observed that the maximum stress induced approaches the yield value. Beyond the value, the analysis is no way correct.

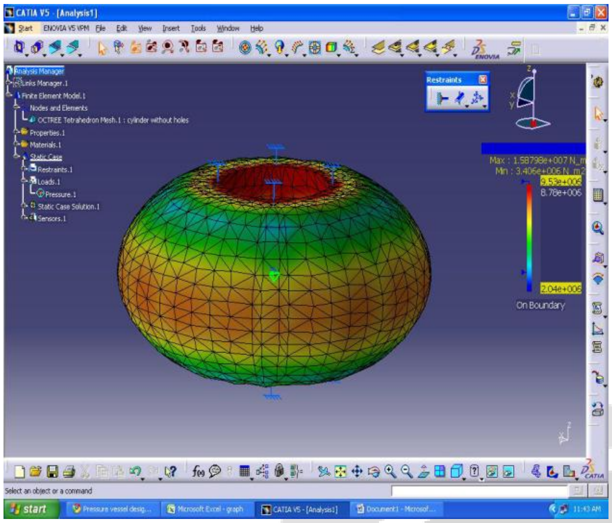

Fig 3.2 Screenshot Of Von Mises Stress Distribution Of Uniform Cylinder In CATIA

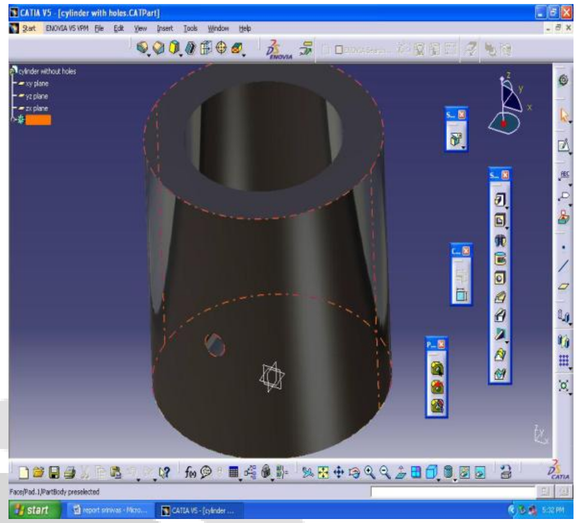

Fig 3.3 A screenshot of cylinder with hole model

3.2.2 Elastic Analysis Of Thick Walled Cylinder With A Radial Hole

As this is again the elastic analysis, expected relationship between pressure and stress should be the same. Now only slope of graph will change as the pressures required to attain maximum stresses are lower. The Fig 3.3 shows the screenshot of CATIA model with radial hole considered. The internal pressure is varied & corresponding equivalent stresses are measured. It is observed that equivalent stress is equal to yield value of material occurs comparably at lower pressures. Fig 3.5 shows the stress variation with pressure for with & without holes within elastic limits

Table 3.2 Variation Of Maximum Equivalent Stress Of Cylinder With Holes, With- -Out Hole, Stress Concentration Factor With Internal Pressure

Pressure(MPA)	Maximum von Mises stress(MPA)	Equivalent stress for without holes	Stress concentration factor( )		Pressure(MPA)	Maximum von Mises stress(MPA)	Equivalent stress for without holes	Stress concentration factor( )
70	330.3	321.12	1.02		140	499.24	478.86	1.048
80	347.6	338.34	1.031		150	536.8	502.1	1.07
90	384.68	360.3	1.06		160	560.56	524.17	1.07
100	412.56	394.4	1.068		170	592.42	538.56	1.1
110	431.16	419.2	1.03		180	630.84	582.15	1.12
120	455.87	436.8	1.04		190	668.34	600.24	1.13
130	479.9	458.8	1.046		194	682.3	613.48	1.14

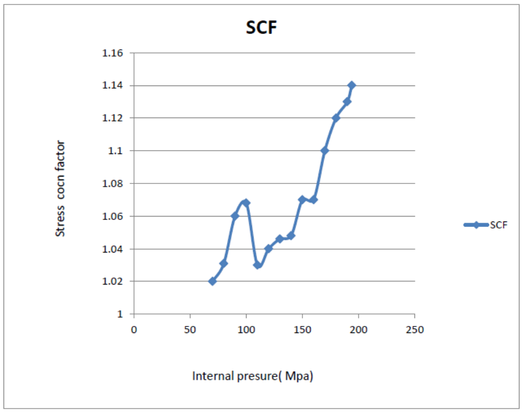

Fig 3.4 Stress concentration factor

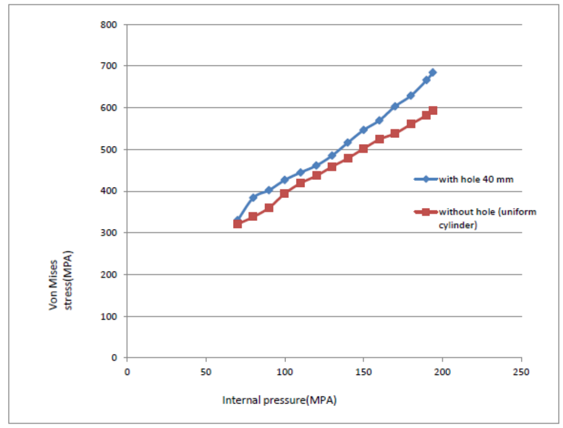

Fig 3.5 Elastic analysis

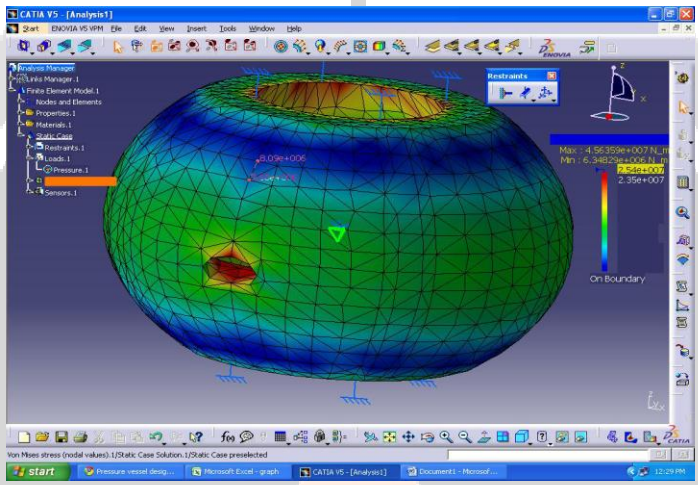

Fig 3.6 Deformed Model with single hole

In pressurized cylinders there may be multiple no. of holes leading to drastic reduction in elastic limit. There are some numerical codes available to estimate the stress concentration factors & corresponding maximum stresses induced at the inner bore surface. Fig 3.6 shows the deformed model at one of the pressures.

3.3 Elastic-Plastic Analysis

3.3.1 Elastic-Plastic Analysis Of Thick Walled Cylinder

When cylinder is loaded to such pressures, yielding begins at inner wall. So here the relative pressures load that initiates the plastic state from inner wall is obtained from earlier elastic analysis. Using theoretical relations, the hoop & radial stress distributions during loading & unloading are generated according to a simple matlab program(Table 3.3). The outputs of the programare shown in Fig 3.7.Elastic-plastic analysis requires finite element modeling in order to comprehend with theoretical results.
Hence a bilinear kinematic hardening model is chosen on ANSYS and corresponding program is generated to do the necessary analysis. Table 3.4 shows the ANSYS command line code to obtain the solution for axisymmetric stress analysis.

Table 3.3 MATLAB program for residual stress

```
Ri = 300e-3;
Ro= 450e-3;
sy= 684e6;
Rp = sqrt(Ri*Ro);
% LOADING DISTRIBUTION
```

```
% HOOP STRESS
i=1;
for r=Ri:10e-5:Ro
  if r<=Rp
        stheta(i) = sy*[1+ log(r/Rp)-0.5*(1-(Rp/Ro)^2) ];
        sradial(i) = sy*[ log(r/Rp)-0.5 * (1-(Rp/Ro)^2) ];
        i = i+1;
else
        p3 = sy*Rp^2/(2*Ro^2/Rp^2);
        stheta(i)= p3*(1-Ro^2/r^2);
        sradial(i)= p3*(1+ Ro^2/r^2);
        i=i+1;
 end
end
```

```
% UNLOADING DISTRIBUTION
i=1;
for r=Ri: 10e-5 :Ro
m=Rp/Ri; k=Ro/Ri;
pa = 0.5*sy*(1-m^2/k^2) + sy*log(m);
stheta2(i) = pa*(1+(Ro/r)^2)/(k^2-1);
sradial2(i) = pa*(1-(Ro/r)^2)/(k^2-1);
i=i+1;
end

% Residual stresses S2-S1
i=1;
for r=Ri: 10e-5:Ro
sthere(i) = stheta2(i)-stheta(i);
sradre(i) = sradial2(i)-sradial(i);
i=i+1;
end
Radius = Ri : 10e-5 : Ro;
figure(1);

plot (Radius, stheta,Radius,stheta2,'--',Radius,sthere,'.-');
xlabel('radius along wall thickness');
ylabel('Hoop residual stress');
legend('loading','unloading','residual',2);
grid on;
figure(2);

plot(Radius,sradial,Radius,sradial2,'--',Radius,sradre,'.-');
xlabel( ' wall thickness');
ylabel ( ' Radial residual stress');
legend('loading','unloading','residual',2);
grid on;
```

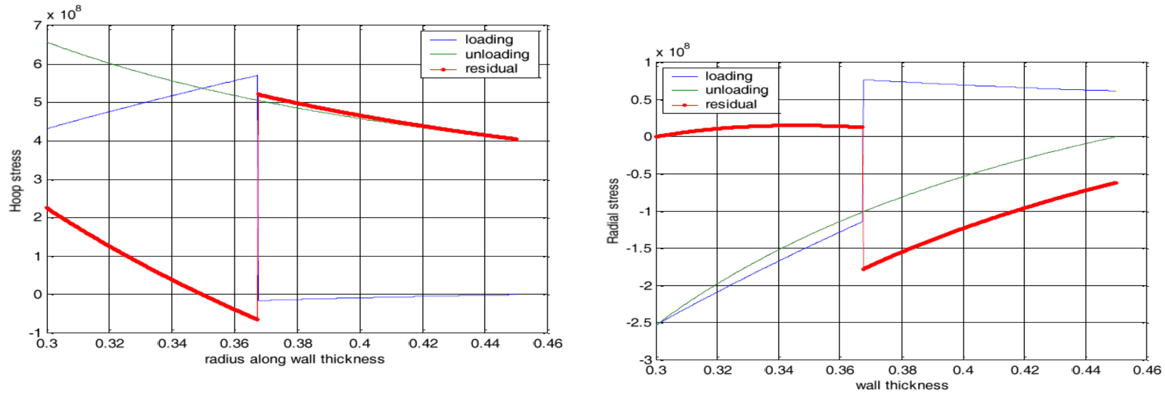

Fig 3.7 Residual stress distribution on MATLAB

Table 3.4 Ansys command line code for elastic-plastic analysis

```
/PREP7
/TITLE, PLASTIC LOADING OF A THICK-WALLED CYLINDER UNDER PRESSURE
ET,1,PLANE42,,1,1                     !AXISYMMETRIC SOLID, SUPPRESS EXTRA
                                      SHAPES
ET,2,SURF153,,,1,1                    !AXISYMMETRIC 2-D SURFACE EFFECT
                                      ELEMENT
MP,EX,1,2e11                          ! BILINEAR KINEMATIC HARDENING
MP,NUXY,1,.3
TB,BKIN,1,1
TBTEMP,70
TBDATA,1,205e6,0                      ! YIELD STRESS AND ZERO TANGENT
                                      MODULUS
N,1,300e-3                            ! DEFINE NODES
N,6,450e-3
FILL
NGEN,2,10,1,6,1,,1
E,11,1,2,12                           ! DEFINE ELEMENTS
EGEN,5,1,1
CPNGN,1,UY,11,16                      ! COUPLE NODES
TYPE,2                                ! CREATE SURF153 TO APPLY SURFACE
PRESSURE                              ! SELECT SURF153 ELEMENTS TO APPLY SURFACE
SFE,ALL,1,PRES,,100e-6                ! LOADING FOR ELASTIC ANALYSIS
ESEL,ALL
OUTPR,BASIC,1
SOLVE
FINISH
```

Fig 3.8 shows equivalent stress distribution obtained at one of the pressures leading to stress above the yield value (684 Mpa). The theoretical equations say that the load to initiate plasticity is Po=220.8 Mpa.

Table 3.5 Shows Variation Of Stresses With Increase

Pressure(MPA)	Von mises stress(MPA)		Pressure(MPA)	Von mises stress(MPA)
220	682		270	286
230	455		280	277
240	369		290	272
250	326		300	270
260	301		320	269

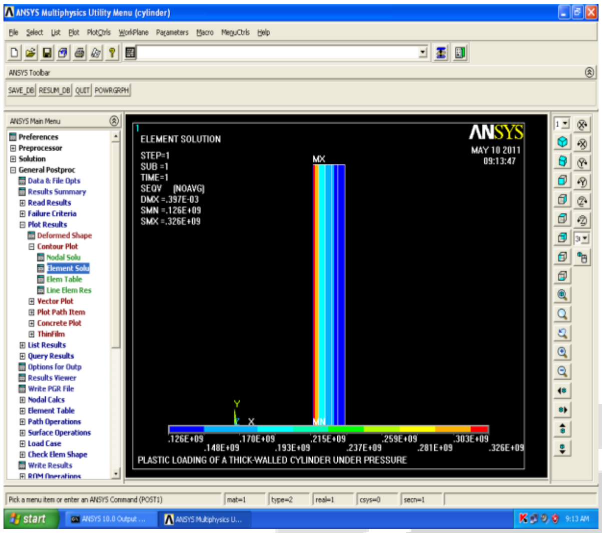

Fig 3.8 Screenshot Of Equivalent Stress Of Distribution In ANSYS

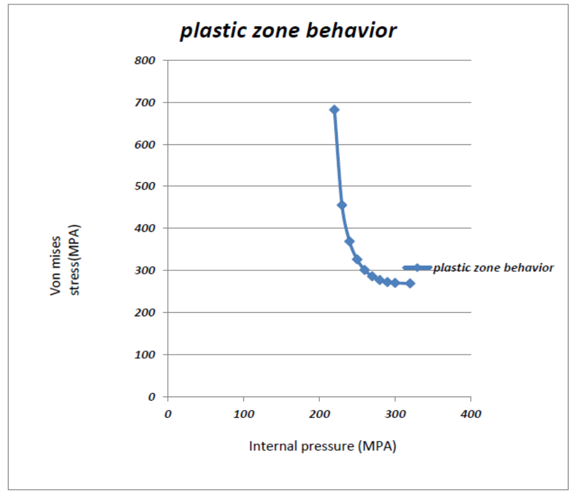

Fig 3.9

Fig. 3.9 Behavior Complies That Of Ductile Behavior Of Steel

It is observed that in plastic zone (220.9-320 MPA) the equivalent(von Mises) stress decreases and becomes constant. This behavior complies that of ductile behavior of steel(see Fig 3.9)

3.3.2 Elastoplastic Analysis Of Cylinder With Radial Hole

The analysis is carried out in Finite Element Method using ANSYS. A cylindrical segment is loaded by internal pressure on the internal surface and along the radial hole. A 8 noded solid -45 three dimensional element is employed to mesh the segment. The three surfaces were applied with symmetry boundary conditions An axial thrust

$$Q = p * \frac{\beta^2}{1-\beta^2}$$

is applied at the 4th surface, simulates reactions of cylinder heads. Fig 3.10 shows the meshed model of the segment in Ansys. Pressure is varied slightly & corresponding stress distribution along the hole surface is shown in Fig 3.11 It is observed that unlike uniform cylinder the higher stresses are noticed at the same pressure values.

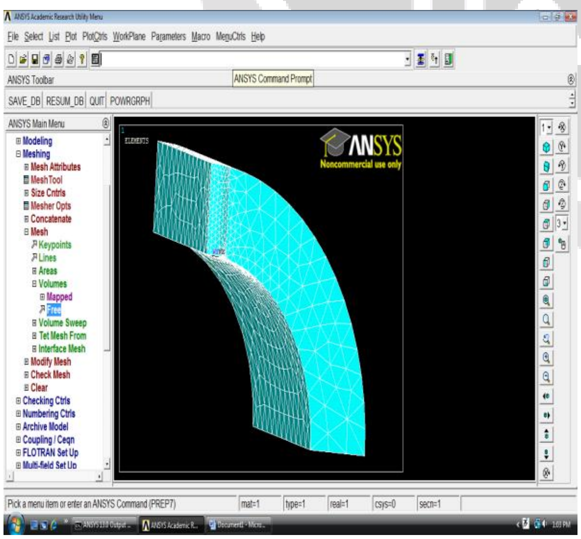

Fig.3.10 The Meshed Model Of The Segment In Ansys

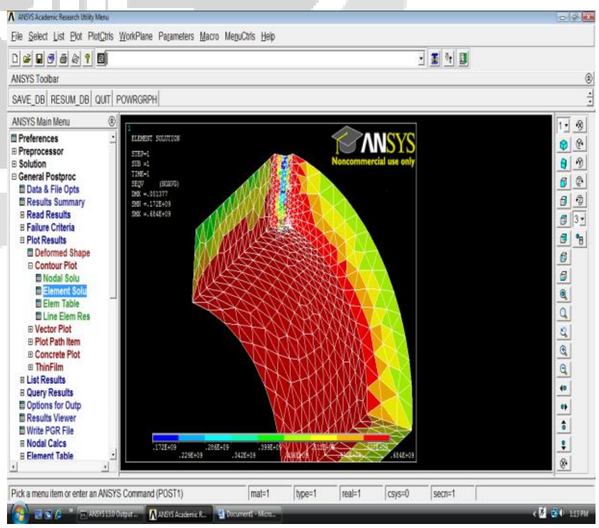

Fig. 3.11 Pressure Is Varied Slightly & Corresponding Stress Distribution Along The Hole Surface

4.CONCLUSIONS

4.1 Summery

An attempt has been made to know the load capacity of a cylinder with radial holes. The work is organized under elastic & elastic-plastic analysis. Classical book work formulas have been employed to obtain the stress distribution in cylinder without holes subjected to internal pressure. Being a new problem the elastoplastic analysis of cylinders with radial hole, there were no theoretical relations. Based on available finite element models, three dimensional analysis has been carried out to predict the actual stress behavior along the cylinder wall especially at the cylinder bore. MATLAB, CATIA & ANSYS software have been used as per requirements.

4.2 Future Scope Of Work

Even though the work is attempted with a single hole vessel, the methodology can be extended to multiple hole case. The results can be compared with standard codes available. The unloading behavior of thick walled cylindrical pressure vessel with holes is another extension for this work.

The load cycles can be increased to know local plastic shakedown limit. Ansys command level code may be developed to carry out this shakedown analysis also. Finally the effect of hole dimensions as well as cylinder wall thickness on the maximum stresses induced may be modeled using nural network.

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