# Development of Mathematical Model for Heat Transfer Analysis of Cylindrical Induction Furnace Wall

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# ABSTRACT

Furnace are most almost continuously used for melting of materials. Induction Cylindrical and spherical furnaces are more invaluable as no fuel is required. The purpose of the furnaces is to chemically reduce and physically convert iron oxides into liquid iron it is called as hot metal. Handling of furnace unit is not simple issue. It is a difficulty to search out life cycle of Induction type cylindrical furnace wall beneath load variant. The Induction type cylindrical furnace wall is made-up of silica ramming mass by experimental study which is one variety of refractory material. During the furnace operation various hazards like fire and explosion, furnace gases is more hazardous and explosive in a nature arise. The issue of induction type cylindrical furnace wall loses its properties after some cycles and hours. The problem comes from this Induction type cylindrical furnace is losing its life cycle and disturb production schedule in industries so we needed proper life span of the furnaces. The failure happens because of cyclic thermal stress due to heating and cooling cycle in furnaces. The problem of these furnaces will occur by computational heat transfer analysis by mathematically method. It is explicit finite difference analysis method. We also work on modification and the area of efficiency, cost, material used in furnace and analysis by mathematically solution. We made mathematical model on Turbo  $C^{++}$  software and analysis of cylindrical induction furnace. By which we have find life cycle of furnace wall with various types of refractory materials used making furnace wall. We found forecasting life span of cylindrical and furnace by used of finite difference method and divided in nodes. We are plotted output values on stress v/s life cycle graph.

Key words: Mathematically, Advanced Heat Transfer, Induction Furnace Wall, Cylindrical Furnace, silica ramming mass.

# I. INTRODUCTION

The furnace is an equipment to use for melt the metals for casting or heating of material to changing shape and size like rolling, forging etc. It is also used for change the properties of the metals like heat treatment processes. Generally, furnace is classified into two types according to generating method of heat. It is combustion type and electric type. In combustion type furnace generally used as fuel is oil and coal. Then electric type furnace called as induction furnace. These are generally used in automobile and melting scrap industries.

If we want to solving problem of simple heat transfer involving simple geometries with simple boundary condition. It is solved by analytical method but, when it wills complex boundary condition than we cannot solved analytically. There are several ways of obtains the numerical formulation of heat transfer problem such as finite difference method, finite element method, boundary element method.

## II. DEVELOPMENT OF ADVANCED EXPLICT DIFFERENCE MODEL.

The cylindrical induction furnace wall divided into nodal network is in Fig.1.it is divided into 24 nodes and various radius (r1, r2, r3, r4, r5). we have derived explicit finite difference method for the all nodes with boundary condition. The all furnace wall has thermal conduction heat transfer between difference nodes and it is according to inside melting metal and outside temperature of atmosphere. The outside of furnace is taken convection cause of cooling water is circulate outside of furnace wall.

This is solved by advanced heat transfer problem of cylindrical induction melting furnace wall. Which is made from silica ramming mass (Basic), in that we have consider initial and boundary condition, material property and some basic assumptions.

In refractory materials for cylindrical furnace wall as basic assumption in the mechanics of science.

- Environmental temperature is homogeneous at 27° C.
- > Ignore the influence of heat radiation.
- Ignore the effect of gravity field.
- > The surface of induction melting furnace wall is clean.
- > Scrap material input inside furnace is considered uniform for our analysis.
- The initial temperature of this furnace is 27 ° C.



#### Fig.1 Nodal network for finite difference method

#### NODE:-1

$$\begin{split} & h_{a}\frac{\Delta x}{2}\left(T_{\infty}-T_{1}^{i}\right)+h_{o}\frac{\Delta y}{2}\left(T_{\infty}-T_{1}^{i}\right)+k\frac{\Delta x}{2}\frac{(T_{4}^{i}-T_{1}^{i})}{\Delta y}+\pi k\Delta y\frac{T_{2}^{i}-T_{1}^{i}}{\ln\left(\frac{r_{1}}{r_{2}}\right)}=\rho\frac{\Delta x}{2}\frac{\Delta y}{2}C\frac{(T_{1}^{i+1}-T_{1}^{i})}{\Delta t}\\ & T_{1}^{i+1}=\left(\left(h_{a}\frac{\Delta x}{2}\left(T_{\infty}-T_{1}^{i}\right)+h_{o}\frac{\Delta y}{2}\left(T_{\infty}-T_{1}^{i}\right)+\pi k\Delta y\frac{T_{2}^{i}-T_{1}^{i}}{\ln\left(\frac{r_{1}}{r_{2}}\right)}+k\frac{\Delta x}{2}\frac{T_{4}^{i}-T_{1}^{i}}{\Delta y}\right)\frac{4\Delta t}{\rho(\Delta x\Delta y)}\right)+T_{1}^{i}\\ & T\left[1\right]\left[i+1\right]=\left(\left((0.5^{*}h_{a}^{*}x^{*}(T_{o}-T[1][i])\right)+(h_{o}^{*}y^{*}(T_{a}-T[1][i]/2)+(\pi^{*}k^{*}y^{*}(T[2][i]-T[1][i])/\ln(r_{1}/r_{2}))\right)\\ & +(0.5^{*}k^{*}x^{*}(T[4][i]-T[1][i]/y))^{*}((4^{*}t)/(r^{*}c^{*}x^{*}y)))+T[1][i]; \end{split}$$

$$h_{a}\Delta x \ (T_{\infty}-T_{2}^{i}) + k\Delta x \ \frac{(T_{5}^{i}-T_{2}^{i})}{\Delta y} + \pi k\Delta y \ \frac{T_{1}^{i}-T_{2}^{i}}{\ln\left(\frac{r_{1}}{r_{2}}\right)} + \pi k\Delta y \ \frac{T_{3}^{i}-T_{2}^{i}}{\ln\left(\frac{r_{2}}{r_{3}}\right)} = \rho\Delta x \ \frac{\Delta y}{2} \ C \ \frac{(T_{2}^{i+1}-T_{2}^{i})}{\Delta t}$$

$$\begin{split} \mathbf{T}_{2}^{i+1} &= ((\mathbf{h}_{a}\Delta \mathbf{x} \ (T_{\infty} - \mathbf{T}_{2}^{i}) + \pi k \Delta y \ \frac{T_{1}^{i} - T_{2}^{i}}{\ln(\frac{r_{1}}{r_{2}})} + \pi k \Delta y \ \frac{T_{3}^{i} - T_{2}^{i}}{\ln(\frac{r_{2}}{r_{3}})} + \mathbf{k} \Delta \mathbf{x} \frac{T_{5}^{i} - T_{2}^{i}}{\Delta y} \bigg) \frac{2\Delta t}{\rho (\Delta x \Delta y)} + \mathbf{T}_{2}^{i} \\ \mathbf{T}_{2}^{i} \mathbf{T}_{2}^{i} \mathbf{T}_{1}^{i} \mathbf{T}_{2}^{i} \mathbf{T}_{1}^{i} \mathbf{T}_{1}^{i} \mathbf{T}_{2}^{i} \mathbf{T}_{2}^{i} \mathbf{T}_{1}^{i} \mathbf{T}_{2}^{i} \mathbf{T}_{1}^{i} \mathbf{T}_{2}^{i} \mathbf{T}_{2}^{i} \mathbf{T}_{2}^{i} \mathbf{T}_{1}^{i} \mathbf{T}_{2}^{i} \mathbf{T}_{2}^{i} \mathbf{T}_{1}^{i} \mathbf{T}_{2}^{i} \mathbf{T}_{2}^{i} \mathbf{T}_{1}^{i} \mathbf{T}_{2}^{i} \mathbf{$$

**NODE:-3**  
$$h_{a}\frac{\Delta x}{2}(T_{\infty}-T_{3}^{i}) + h_{i}\frac{\Delta y}{2}(T_{h}-T_{3}^{i}) + k\frac{\Delta x}{2}\frac{(T_{6}^{i}-T_{3}^{i})}{\Delta y} + \pi k\Delta y\frac{T_{2}^{i}-T_{3}^{i}}{\ln\left(\frac{r_{2}}{r_{3}}\right)} = \rho\frac{\Delta x}{2}\frac{\Delta y}{2}C\frac{(T_{3}^{i+1}-T_{3}^{i})}{\Delta t}$$

$$\begin{split} \mathbf{T}_{3}^{i+1} &= (\mathbf{h}_{a} \frac{\Delta x}{2} (T_{\infty} - \mathbf{T}_{3}^{i}) + \mathbf{h}_{i} \frac{\Delta y}{2} (\mathbf{T}_{h} - \mathbf{T}_{3}^{i}) + \pi k \Delta y \frac{T_{2}^{i} - T_{3}^{i}}{\ln(\frac{r_{2}}{r_{3}})} + \mathbf{k} \frac{\Delta x}{2} \frac{T_{6}^{i} - T_{3}^{i}}{\Delta y} \frac{4\Delta t}{\rho (\Delta x \Delta y)} + \mathbf{T}_{3}^{i} \\ \mathbf{T} [3][i+1] &= (((\mathbf{h}_{a} * \mathbf{x} * (\mathbf{T}_{o} - \mathbf{T}[3][i] * 0.5) + (\mathbf{h}_{i} * \mathbf{y} * (\mathbf{T}_{h} - \mathbf{T}[3][i] * 0.5) + (0.5 * \mathbf{k} * \mathbf{x} * (\mathbf{T}[6][i] - \mathbf{T}[3][i]) / \ln(r_{2}/r_{3}))) * ((4 * t) / (r^{*} \mathbf{c} * \mathbf{x} * y))) + \mathbf{T}[3][i]; \end{split}$$

# NODE:-4

$$h_{o}\Delta y (T_{\infty} - T_{4}^{i}) + k \frac{\Delta x}{2} \frac{T_{1}^{i} - T_{4}^{i}}{\Delta y} + k \frac{\Delta x}{2} \frac{T_{7}^{i} - T_{4}^{i}}{\Delta y} + \pi k \Delta y \frac{T_{5}^{i} - T_{4}^{i}}{\ln(\frac{r_{1}}{r_{2}})} = \rho \frac{\Delta x}{2} \Delta y C \frac{(T_{4}^{i+1} - T_{4}^{i})}{\Delta t}$$

$$\begin{split} \mathbf{T}_{4}^{i+4} &= (\mathbf{h}_{o}\Delta y \ (T_{\infty} - \mathbf{T}_{4}^{i}) + \pi k \Delta y \ \frac{T_{5}^{i} - T_{4}^{i}}{\ln(\frac{r_{1}}{r_{2}})} + \mathbf{k} \frac{\Delta x}{2} \frac{T_{1}^{i} - T_{4}^{i}}{\Delta y} + \mathbf{k} \frac{\Delta x}{2} \frac{T_{7}^{i} - T_{4}^{i}}{\Delta y} \right) \frac{2\Delta t}{\rho C \Delta x \Delta y} + \mathbf{T}_{4}^{i} \\ \mathbf{T} \ [4][i+1] &= (((\mathbf{h}_{o} * \mathbf{y}^{*}(\mathbf{T}_{a} - \mathbf{T}[4][i])) + (\pi^{*}\mathbf{k}^{*}\mathbf{y}^{*}(\mathbf{T}[5][i] - \mathbf{T}[4][i])/\ln(r_{1}/r_{2})) + (0.5^{*}\mathbf{k}^{*}\mathbf{x}^{*}(\mathbf{T}[7][i] - \mathbf{T}[4][i])/y))^{*}((2^{*}t)/(\mathbf{r}^{*}\mathbf{c}^{*}\mathbf{x}^{*}\mathbf{y}))) + \mathbf{T}[4][i]; \end{split}$$

# NODE:-5

$$k\Delta x \frac{T_{2}^{i} - T_{5}^{i}}{\Delta y} + k\Delta x \frac{T_{8}^{i} - T_{5}^{i}}{\Delta y} + \pi k\Delta y \frac{T_{4}^{i} - T_{5}^{i}}{\ln(\frac{r_{1}}{r_{2}})} + \pi k\Delta y \frac{T_{6}^{i} - T_{5}^{i}}{\ln(\frac{r_{2}}{r_{3}})} = \rho \Delta x \Delta y \ C \ \frac{(T_{5}^{i+1} - T_{5}^{i})}{\Delta t}$$

$$T_{5}^{i+1} = ((\pi k\Delta y \frac{T_{4}^{i} - T_{5}^{i}}{\ln(\frac{r_{1}}{r_{2}})} + \pi k\Delta y \frac{T_{6}^{i} - T_{5}^{i}}{\ln(\frac{r_{2}}{r_{3}})} + k\Delta x \frac{T_{2}^{i} - T_{5}^{i}}{\Delta y} + k\Delta x \frac{T_{8}^{i} - T_{5}^{i}}{\Delta y} \right) \frac{\Delta t}{\rho C \Delta x \Delta y} + T_{5}^{i}$$

$$T \ [5][i+1] = (((\pi^{*}k^{*}y^{*}(T[4][i] - T[5][i])/\ln(r_{1}/r_{2})) + (\pi^{*}k^{*}y^{*}(T[6][i] - T[5][i])/\ln(r_{2}/r_{3})) + (k^{*}x^{*}(T[2][i] - T[5][i]/y) + (k^{*}x^{*}(T[8][i] - T[5][i]/y))^{*}((t)/(r^{*}c^{*}x^{*}y))) + T[5][i];$$

# NODE:-6

$$h_{i}\Delta y (T_{h}-T_{6}^{i}) + k\frac{\Delta x}{2}\frac{T_{3}^{i}-T_{6}^{i}}{\Delta y} + k\frac{\Delta x}{2}\frac{T_{9}^{i}-T_{6}^{i}}{\Delta y} + \pi k\Delta y \frac{T_{5}^{i}-T_{6}^{i}}{\ln(\frac{r_{2}}{r_{2}})} = \rho \frac{\Delta x}{2}\Delta y C \frac{(T_{6}^{i+1}-T_{6}^{i})}{\Delta t}$$

$$\mathbf{T}_{6}^{i+1} = \left( \left( \mathbf{h}_{i} \Delta y \left( T_{h} - \mathbf{T}_{6}^{i} \right) + \pi k \Delta y \frac{T_{5}^{i} - T_{6}^{i}}{\ln \left(\frac{r_{2}}{r_{3}}\right)} + \mathbf{k} \frac{\Delta x}{2} \frac{T_{3}^{i} - T_{6}^{i}}{\Delta y} + \mathbf{k} \frac{\Delta x}{2} \frac{T_{9}^{i} - T_{6}^{i}}{\Delta y} \right) \frac{2\Delta t}{\rho (\Delta x \Delta y)} + \mathbf{T}_{6}^{i}$$

 $T \ [6][i+1] = (((h_i * y * (T_h - T[6][i])) + (\pi * k * y * (T[5][i] - T[6][i])/ln(r_2/r_3)) + (0.5 * x * k * (T[3][i] - T[6][i]/y) + (0.5 * k * x * (T[9][i] - T[6][i])/y)) * ((2*t)/(r * c * x * y))) + T[6][i];$ 

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# NODE:-7

$$h_{o}\Delta y (T_{\infty} - T_{7}^{i}) + k \frac{\Delta x}{2} \frac{T_{4}^{i} - T_{7}^{i}}{\Delta y} + k \frac{\Delta x}{2} \frac{T_{10}^{i} - T_{7}^{i}}{\Delta y} + \pi k \Delta y \frac{T_{8}^{i} - T_{7}^{i}}{\ln(\frac{r_{1}}{r_{2}})} = \rho \frac{\Delta x}{2} \Delta y \left( \frac{(T_{7}^{i+1} - T_{7}^{i})}{\Delta t} + T_{7}^{i+7} \right) \right)$$

$$T_{7}^{i+7} = \left( (h_{o}\Delta y (T_{\infty} - T_{7}^{i}) + \pi k \Delta y \frac{T_{8}^{i} - T_{7}^{i}}{\ln(\frac{r_{1}}{r_{2}})} + k \frac{\Delta x}{2} \frac{T_{4}^{i} - T_{7}^{i}}{\Delta y} + k \frac{\Delta x}{2} \frac{T_{10}^{i} - T_{7}^{i}}{\Delta y} \right) \frac{2\Delta t}{\rho (\Delta x \Delta y)} + T_{7}^{i}$$

$$\begin{split} T\ [7][i+1] = (((h_o^*y^*(T_a-T[7][i])) + (\ \pi^*k^*y^*(T[8][i]-T[7][i])/ln(r_1/r_2) + (k^*0.5^*x^*(T[4][i]-T[7][i]/y) + (0.5^*k^*x^*(T[10][i]-T[7][i])/y))^*((2^*t)/(r^*c^*x^*y))) + T[7][i]; \end{split}$$

$$\begin{split} & \textbf{NODE:-8} \\ & k\Delta x \, \frac{T_5^i - T_8^i}{\Delta y} + k\Delta x \, \frac{T_{11}^i - T_8^i}{\Delta y} + \pi k\Delta y \, \frac{T_7^i - T_8^i}{\ln\left(\frac{r_1}{r_2}\right)} + \pi k\Delta y \, \frac{T_9^i - T_8^i}{\ln\left(\frac{r_2}{r_3}\right)} = \rho \Delta x \Delta y \, \mathsf{C} \, \frac{(T_8^{i+1} - T_8^i)}{\Delta t} \\ & \mathsf{T}_8^{i+1} = \left( (\pi k\Delta y \, \frac{T_7^i - T_8^i}{\ln\left(\frac{r_1}{r_2}\right)} + \pi k\Delta y \, \frac{T_9^i - T_8^i}{\ln\left(\frac{r_2}{r_3}\right)} + k\Delta x \, \frac{T_5^i - T_8^i}{\Delta y} + k\Delta x \, \frac{T_{11}^i - T_8^i}{\Delta y} \right) \frac{\Delta t}{\rho (\Delta x \Delta y)} + \mathsf{T}_8^{i} \\ & \mathsf{T} \, [8][i+1] = \left( ((\pi^* k^* y^* (\mathsf{T}[7][i] - \mathsf{T}[8][i]) / \ln(r_1 / r_2)) + (\pi^* k^* y^* (\mathsf{T}[9][i] - \mathsf{T}[8][i]) / \ln(r_2 / r_3)) + (k^* x^* (\mathsf{T}[5][i] - \mathsf{T}[8][i]) / y) \right) \\ & \mathsf{T} \, [8][i+1] = ((\pi^* k^* y^* (\mathsf{T}[11][i] - \mathsf{T}[8][i]) / y) + (k^* x^* (\mathsf{T}[5][i] - \mathsf{T}[8][i]) / y) + (k^* x^* (\mathsf{T}[5][i] - \mathsf{T}[8][i]) / y) \right) \\ & \mathsf{T} \, (\mathsf{t}) / (\mathsf{r}^* \mathsf{c}^* \mathsf{x}^* \mathsf{y})) + \mathsf{T} \, \mathsf{T} \, \mathsf{S} \, \mathsf{I}[i]; \end{split}$$

$$\begin{split} & h_{i}\Delta y \ (T_{h}-T_{9}^{i}) + k\frac{\Delta x}{2}\frac{T_{6}^{i}-T_{9}^{i}}{\Delta y} + k\frac{\Delta x}{2}\frac{T_{12}^{i}-T_{9}^{i}}{\Delta y} + \pi k\Delta y \frac{T_{8}^{i}-T_{9}^{i}}{\ln(\frac{r_{2}}{r_{3}})} = \rho \frac{\Delta x}{2}\Delta y \ C \ \frac{(T_{9}^{i+1}-T_{9}^{i})}{\Delta t} \\ & T_{9}^{i+1} = ((h_{i}\Delta y \ (T_{h}-T_{9}^{i}) + \pi k\Delta y \frac{T_{8}^{i}-T_{9}^{i}}{\ln(\frac{r_{2}}{r_{3}})} + k\frac{\Delta x}{2}\frac{T_{6}^{i}-T_{9}^{i}}{\Delta y} + k\frac{\Delta x}{2}\frac{T_{12}^{i}-T_{9}^{i}}{\Delta y} \frac{2\Delta t}{\rho(\Delta x\Delta y)} + T_{9}^{i} \\ & T \ [9][i+1] = ((h_{i}*y*(T_{h}-T[9][i])) + (\pi *k*y*(T[8][i]-T[9][i])/\ln(r_{2}/r_{3})) + (k*0.5*x*(T[6][i]-T[9][i]/y) + (0.5*k*x*(T[12][i]-T[9][i])/y))*((2*t)/(r*c*x*y))) + T[9][i]; \end{split}$$

## NODE:-10

$$\begin{split} & h_{o}\Delta y \ (T_{\infty} - T_{10}{}^{i}) + k \frac{\Delta x}{2} \frac{T_{7}^{i} - T_{10}^{i}}{\Delta y} + k \frac{\Delta x}{2} \frac{T_{15}^{i} - T_{10}^{i}}{\Delta y} + \pi k \Delta y \frac{T_{11}^{i} - T_{10}^{i}}{\ln(\frac{T_{1}}{T_{2}})} = \rho \frac{\Delta x}{2} \Delta y \ C \ \frac{(T_{10}^{i+1} - T_{10}^{i})}{\Delta t} \\ & T_{10}{}^{i+10} = ((h_{o}\Delta y \ (T_{\infty} - T_{10}{}^{i}) + \pi k \Delta y \frac{T_{11}^{i} - T_{10}^{i}}{\ln(\frac{T_{1}}{T_{2}})} + k \frac{\Delta x}{2} \frac{T_{7}^{i} - T_{10}^{i}}{\Delta y} + k \frac{\Delta x}{2} \frac{T_{15}^{i} - T_{10}^{i}}{\Delta y} \Big) \frac{2\Delta t}{\rho (\Delta x \Delta y)} + T_{10}{}^{i} \\ & T \ [10][i+1] = (((h_{o}^{*}y^{*}(T_{a} - T[10][i])) + (\pi^{*}k^{*}y^{*}(T[11][i] - T[10][i])/\ln(r_{1}/r_{2})) + (0.5^{*}k^{*}x^{*}(T[15][i] - T[10][i])/y))^{*}((2^{*}t)/(r^{*}c^{*}x^{*}y))) + T[10][i]; \end{split}$$

## NODE:-11

$$\begin{aligned} k\Delta x \, \frac{T_8^i - T_{11}^i}{\Delta y} + k\Delta x \, \frac{T_{16}^i - T_{11}^i}{\Delta y} + \pi k \, \Delta y \, \frac{T_{10}^i - T_{11}^i}{\ln\left(\frac{r_1}{r_2}\right)} + \pi k\Delta y \, \frac{T_{12}^i - T_{11}^i}{\ln\left(\frac{r_2}{r_3}\right)} &= \rho \Delta x \Delta y \, \mathbb{C} \, \frac{(T_{11}^{i+1} - T_{11}^i)}{\Delta t} \\ T_{11}^{i+1} &= \left((\pi k \, \Delta y \, \frac{T_{10}^i - T_{11}^i}{\ln\left(\frac{r_1}{r_2}\right)} + \pi k\Delta y \, \frac{T_{12}^i - T_{11}^i}{\ln\left(\frac{r_2}{r_3}\right)} + k\Delta x \, \frac{T_8^i - T_{11}^i}{\Delta y} + k\Delta x \, \frac{T_{16}^i - T_{11}^i}{\Delta y}\right) \frac{\Delta t}{\rho C \Delta x \Delta y}\right) + T_{11}^{i} \\ T \, [11][i+1] &= \left((\pi^* k^* y^* (T[10][i] - T[11][i]) / \ln(r_1/r_2)) + (\pi^* k^* y^* (T[12][i] - T[11][i]) / \ln(r_2/r_3)) + (k^* x^* (T[8][i] - T[8][i] - T[8][i] - T[8][i]) / \ln(r_2/r_3)) + (k^* x^* (T[8][i] - T[8][i] - T[8][i] / \ln(r_2/r_3)) + (k^* x^* (T[8][i] - T[8][i] - T[8][i] / \ln(r_2/r_3)) + (k^* x^* (T[8][i] - T[8][i] - T[8][i] / \ln(r_2/r_3)) + (k^* x^* (T[8][i] - T[8][i] / \ln(r_2/r_3)) +$$

#### NODE:-12

$$\begin{split} & k\frac{\Delta x}{2}\frac{T_{9}^{i}-T_{12}^{i}}{\Delta y} + k\Delta x\frac{T_{17}^{i}-T_{12}^{i}}{\Delta y} + h_{i}\frac{\Delta x}{2}(T_{h}-T_{12}^{i}) + h_{i}\frac{\Delta y}{2}(T_{h}-T_{12}^{i}) + \pi k\Delta y\frac{T_{11}^{i}-T_{12}^{i}}{\ln(\frac{r_{2}}{r_{3}})} + \pi k\Delta y\frac{T_{13}^{i}-T_{12}^{i}}{\ln(\frac{r_{3}}{r_{4}})} = \rho\frac{3\Delta x\Delta y}{4}C\frac{(T_{12}^{i+1}-T_{12}^{i})}{\Delta t} \\ & T_{12}^{i+1} = ((h_{i}\frac{\Delta x}{2}(T_{h}-T_{12}^{i}) + h_{i}\frac{\Delta y}{2}(T_{h}-T_{12}^{i}) + \pi k\Delta y\frac{T_{11}^{i}-T_{12}^{i}}{\ln(\frac{r_{3}}{r_{3}})} + \pi k\Delta y\frac{T_{13}^{i}-T_{12}^{i}}{\ln(\frac{r_{3}}{r_{4}})} + k\frac{\Delta x}{2}\frac{T_{9}^{i}-T_{12}^{i}}{\Delta y} + k\Delta x\frac{T_{17}^{i}-T_{12}^{i}}{\Delta y}\right)\frac{4\Delta t}{\rho(\Delta x\Delta y)}\right) + T_{12}^{i} \\ & T [12][i+1] = (((0.5*h_{i}*x*(T_{h}-T[12][i])) + (0.5*h_{i}*y*(T_{h}-T[12][i])) + (\pi*k*y*(T[11][i]-T[12][i])/\ln(r_{2}/r_{3})) \\ & + (\pi*k*y*(T[13][i]-T[12][i])/\ln(r_{3}/r_{4})) + (0.5*k*x*(T[9][i]-T[12][i])/y) + (k*x*(T[17][i]-T[12][i])/\ln(r_{2}/r_{3})) \\ & T [12][i])/y) * ((4*t)/(3*r*c*x*y))) + T [12][i]; \end{split}$$

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#### NODE:-13

$$h_{i} \Delta x (T_{h} - T_{13}^{i}) + k\Delta x \frac{T_{18}^{i} - T_{13}^{i}}{\Delta y} + \pi k\Delta y \frac{T_{12}^{i} - T_{13}^{i}}{\ln(\frac{r_{3}}{r_{4}})} + \pi k\Delta y \frac{T_{14}^{i} - T_{13}^{i}}{\ln(\frac{r_{4}}{r_{5}})} = \rho \Delta x \frac{\Delta y}{2} C \frac{(T_{13}^{i+1} - T_{13}^{i})}{\Delta t}$$

$$T_{13}^{i+1} = ((h_{i}\Delta x (T_{h} - T_{13}^{i}) + \pi k\Delta y \frac{T_{12}^{i} - T_{13}^{i}}{\ln(\frac{r_{3}}{r_{4}})} + \pi k\Delta y \frac{T_{14}^{i} - T_{13}^{i}}{\ln(\frac{r_{4}}{r_{5}})} + k\Delta x \frac{T_{18}^{i} - T_{13}^{i}}{\Delta y}) \frac{2\Delta t}{\rho (\Delta x \Delta y)} + T_{13}^{i}$$

$$\begin{split} T\ [13][i+1] &= ((h_i^*x^*(T_h - T[13][i])) + (\pi^*k^*y^*(T[12][i] - T[13][i])/ln(r_3/r_4)) + (\pi^*k^*y^*(T[14][i] - T[13][i])/ln(r_4/r_5)) + (k^*x^*(T[18][i] - T[13][i]/y)^*((2^*t)/(r^*c^*x^*y))) + T[13][i]; \end{split}$$

$$\begin{split} & h_{i}\frac{\Delta x}{2}(T_{h}-T_{14}^{\ i}) + h_{i}\frac{\Delta y}{2}(T_{h}-T_{14}^{\ i}) + k\frac{\Delta x}{2}\frac{T_{19}^{i}-T_{14}^{i}}{\Delta y} + \pi k\Delta y\frac{T_{13}^{i}-T_{14}^{i}}{\ln\left(\frac{r_{4}}{r_{5}}\right)} = \rho\frac{\Delta x}{2}\frac{\Delta y}{2}C\frac{(T_{14}^{i+1}-T_{14}^{i})}{\Delta t} \\ & T_{14}^{\ i+1} = ((h_{i}\frac{\Delta x}{2}(T_{h}-T_{14}^{\ i}) + h_{i}\frac{\Delta y}{2}(T_{h}-T_{14}^{\ i}) + \pi k\Delta y\frac{T_{13}^{i}-T_{14}^{i}}{\ln\left(\frac{r_{4}}{r_{5}}\right)} + k\frac{\Delta x}{2}\frac{T_{19}^{i}-T_{14}^{i}}{\Delta y} + k\frac{\Delta x}{2}\frac{T_{19}^{i}-T_{14}^{i}}{\Delta y} + L_{14}^{i} \\ & T\left[14\right][i+1] = ((0.5^{*}h_{i}^{*}x^{*}(T_{h}-T[14][i])) + (0.5^{*}h_{i}^{*}y^{*}(T_{h}-T[14][i])) + (\pi^{*}k^{*}y^{*}(T[13][i]-T[14][i])/\ln(r_{4}/r_{5})) + (k^{*}0.5^{*}x^{*}(T[19][i]-T[14][i]/y))^{*}((4^{*}t)/(r^{*}c^{*}x^{*}y))) + T[14][i]; \end{split}$$

## **NODE: -15**

$$\begin{split} & h_{o}\Delta y \ (T_{\infty} - T_{15}{}^{i}) + k \frac{\Delta x}{2} \frac{T_{10}^{i} - T_{15}^{i}}{\Delta y} + k \frac{\Delta x}{2} \frac{T_{20}^{i} - T_{15}^{i}}{\Delta y} + \pi k \Delta y \ \frac{T_{16}^{i} - T_{15}^{i}}{\ln(\frac{r_{1}}{r_{2}})} + \rho \frac{\Delta x}{2} \ \Delta y \ C \ \frac{(T_{15}^{i+1} - T_{15}^{i})}{\Delta t} \\ & T_{15}{}^{i+15} = ((h_{o}\Delta y \ (T_{\infty} - T_{15}{}^{i}) + \pi k \Delta y \ \frac{T_{16}^{i} - T_{15}^{i}}{\ln(\frac{r_{1}}{r_{2}})} + k \frac{\Delta x}{2} \frac{T_{10}^{i} - T_{15}^{i}}{\Delta y} + k \frac{\Delta x}{2} \frac{T_{20}^{i} - T_{15}^{i}}{\Delta y} \right) \frac{2\Delta t}{\rho (\Delta x \Delta y)} + T_{15}{}^{i} \\ & T \ [15][i+1] = (((h_{o}^{*}y^{*}(T_{a} - T[15][i])) + (\pi^{*}k^{*}y^{*}(T[16][i] - T[15][i])/\ln(r_{1}/r_{2})) + (0.5^{*}k^{*}x^{*}(T[10][i] - T[15][i])/y) \\ & + (0.5^{*}k^{*}x^{*}(T[20][i] - T[15][i])/y))^{*}((2^{*}t)/(r^{*}c^{*}x^{*}y))) + T[15][i]; \end{split}$$

## NODE:-16

$$k\Delta x \frac{T_{11}^{i} - T_{16}^{i}}{\Delta y} + k\Delta x \frac{T_{21}^{i} - T_{16}^{i}}{\Delta y} + \pi k \Delta y \frac{T_{15}^{i} - T_{16}^{i}}{\ln(\frac{r_{1}}{r_{2}})} + \pi k\Delta y \frac{T_{17}^{i} - T_{16}^{i}}{\ln(\frac{r_{2}}{r_{3}})} = \rho \Delta x \Delta y C \frac{(T_{16}^{i+1} - T_{16}^{i})}{\Delta t}$$

$$T_{16}^{i+1} = ((\pi k \Delta y \frac{T_{15}^{i} - T_{16}^{i}}{\ln(\frac{r_{1}}{r_{2}})} + \pi k\Delta y \frac{T_{17}^{i} - T_{16}^{i}}{\ln(\frac{r_{2}}{r_{3}})} + k\Delta x \frac{T_{11}^{i} - T_{16}^{i}}{\Delta y} + k\Delta x \frac{T_{21}^{i} - T_{16}^{i}}{\Delta y} + K\Delta x \frac{T_{11}^{i} - T_{16}^{i}}{\Delta y} + K\Delta x \frac{T_{11}^{i} - T_{16}^{i}}{\Delta y} + T_{16}^{i}$$

$$T [16][i+1] = ((\pi^{*}k^{*}y^{*}(T[15][i] - T[16][i])/\ln(r_{1}/r_{2})) + (\pi^{*}k^{*}y^{*}(T[17][i] - T[16][i])/\ln(r_{2}/r_{3})) + (k^{*}x^{*}(T[11][i] - T[16][i])/y))^{*}((t)/(r^{*}c^{*}x^{*}y))) + T_{16}^{i}$$

$$k\Delta x \frac{T_{12}^{i} - T_{17}^{i}}{\Delta y} + k\Delta x \frac{T_{22}^{i} - T_{17}^{i}}{\Delta y} + \pi k\Delta y \frac{T_{16}^{i} - T_{17}^{i}}{\ln(\frac{r_{2}}{r_{3}})} + \pi k\Delta y \frac{T_{18}^{i} - T_{17}^{i}}{\ln(\frac{r_{3}}{r_{4}})} = \rho\Delta x \Delta y C \frac{(T_{17}^{i+1} - T_{17}^{i})}{\Delta t}$$
$$T_{17}^{i+1} = ((\pi k\Delta y \frac{T_{16}^{i} - T_{17}^{i}}{\ln(\frac{r_{2}}{r_{3}})} + \pi k\Delta y \frac{T_{18}^{i} - T_{17}^{i}}{\ln(\frac{r_{3}}{r_{4}})} + k\Delta x \frac{T_{12}^{i} - T_{17}^{i}}{\Delta y} + k\Delta x \frac{T_{22}^{i} - T_{17}^{i}}{\Delta y} \frac{\Delta t}{\rho(\Delta x\Delta y)} + T_{17}^{i}$$
$$T [17][i+1] = ((\pi k k^{2} k^{2} k^{2} k^{2} (T_{11}^{i} - T_{17}^{i}) + T_{17}^{i}) + T_{17}^{i} (T_{11}^{i} - T_{17}^{i}) + T$$

 $T [17][i+1] = ((\pi^*k^*y^*(T[16][i]-T[17][i])/\ln(r_2/r_3)) + (\pi^*k^*y^*(T[18][i]-T[17][i])/\ln(r_3/r_4)) + (k^*x^*(T[12][i]-T[17][i])/y) + (k^*x^*(T[22][i]-T[17][i])/y)) + ((t)/(r^*c^*x^*y))) + T[17][i];$ 

**NODE:** -18  

$$k\Delta x \frac{T_{13}^{i} - T_{18}^{i}}{\Delta y} + k\Delta x \frac{T_{23}^{i} - T_{18}^{i}}{\Delta y} + \pi k\Delta y \frac{T_{17}^{i} - T_{18}^{i}}{\ln(\frac{r_{3}}{r_{4}})} + \pi k\Delta y \frac{T_{19}^{i} - T_{18}^{i}}{\ln(\frac{r_{4}}{r_{5}})} = \rho\Delta x \Delta y C \frac{(T_{18}^{i+1} - T_{18}^{i})}{\Delta t}$$

$$T_{18}^{i+1} = ((\pi k\Delta y \frac{T_{17}^{i} - T_{18}^{i}}{\ln(\frac{r_{3}}{r_{4}})} + \pi k\Delta y \frac{T_{19}^{i} - T_{18}^{i}}{\ln(\frac{r_{5}}{r_{5}})} + k\Delta x \frac{T_{13}^{i} - T_{18}^{i}}{\Delta y} + k\Delta x \frac{T_{23}^{i} - T_{18}^{i}}{\Delta y} - \frac{\lambda t}{\rho C\Delta x \Delta y} + T_{18}^{i}$$

$$T_{18}^{i+1} = ((\pi^{*}k^{*}y^{*}(T_{117})) + T_{18}^{i}) + T_{18}^{i})$$

 $T [18][i+1] = ((\pi^*k^*y^*(T[17][i]-T[18][i])/\ln(r_3/r_4)) + (\pi^*k^*y^*(T[19][i]-T[18][i])/\ln(r_4/r_5)) + (k^*x^*(T[13][i]-T[18][i]/y) + (k^*x^*(T[23][i]-T[18][i]/y) + ((t)/(r^*c^*x^*y))) + T[18][i];$ 

# NODE:-19

$$h_{i}\Delta y (T_{h}-T_{19}^{i}) + k\frac{\Delta x}{2}\frac{T_{14}^{i}-T_{19}^{i}}{\Delta y} + k\frac{\Delta x}{2}\frac{T_{24}^{i}-T_{19}^{i}}{\Delta y} + \pi k\Delta y \frac{T_{18}^{i}-T_{19}^{i}}{\ln\left(\frac{r_{4}}{r_{5}}\right)} = \rho \frac{\Delta x}{2} \Delta y C \frac{(T_{19}^{i+1}-T_{19}^{i})}{\Delta t}$$

$$T_{19}^{i+1} = ((h_{i}\Delta y (T_{h}-T_{19}^{i}) + \pi k\Delta y \frac{T_{18}^{i}-T_{19}^{i}}{\ln\left(\frac{r_{4}}{r_{5}}\right)} + k\frac{\Delta x}{2}\frac{T_{14}^{i}-T_{19}^{i}}{\Delta y} + k\frac{\Delta x}{2}\frac{T_{24}^{i}-T_{19}^{i}}{\Delta y} - k\frac{\Delta x}{2}\frac{T_{24}^{i}-T_{19}^{i}}{\Delta y} + T_{19}^{i} + T_$$

$$\begin{split} T\ [19][i+1] = ((h_i * y * (T_h - T[19][i])) + (\pi * k * y * (T[18][i] - T[19][i]) / ln(r_4/r_5)) + (k * 0.5 * x * (T[14][i] - T[19][i]/y) + (0.5 * k * x * (T[24][i] - T[19][i]/y)) * ((2 * t) / (r * c * x * y))) + T[19][i]; \end{split}$$

$$\begin{aligned} \mathbf{NODE:-20} \\ \mathbf{h}_{o}\frac{\Delta x}{2} \left(T_{\infty} - \mathbf{T}_{20}^{i}\right) + \left(\left(\mathbf{h}_{o}\frac{\Delta y}{2} \left(T_{\infty} - \mathbf{T}_{20}^{i}\right) + \pi k \Delta y \frac{T_{21}^{i} - T_{20}^{i}}{\ln\left(\frac{\tau_{1}}{r_{2}}\right)} + \mathbf{k}\frac{\Delta x}{2} \frac{T_{15}^{i} - T_{20}^{i}}{\Delta y} = \rho \frac{\Delta x}{2} \frac{\Delta y}{2} \mathbf{C} \frac{\left(T_{20}^{i+1} - T_{20}^{i}\right)}{\Delta t} \\ \mathbf{T}_{20}^{i+20} &= \left(\left(\mathbf{h}_{o}\frac{\Delta x}{2} \left(T_{\infty} - \mathbf{T}_{20}^{i}\right) + \left(\left(\mathbf{h}_{o}\frac{\Delta y}{2} \left(T_{\infty} - \mathbf{T}_{20}^{i}\right) + \pi k \Delta y \frac{T_{21}^{i} - T_{20}^{i}}{\ln\left(\frac{\tau_{1}}{r_{2}}\right)} + \mathbf{k}\frac{\Delta x}{2} \frac{T_{15}^{i} - T_{20}^{i}}{\Delta y} + \mathbf{k}\frac{\Delta x}{2} \frac{T_{15}^{i} - T_{20}^{i}}{\Delta y}\right) + \mathbf{T}_{20}^{i} \mathbf{T}_{20}^{i+20} \\ &= \left(\mathbf{M}_{o}\frac{\Delta x}{2} \left(T_{\infty} - \mathbf{T}_{20}^{i}\right) + \left(\mathbf{M}_{o}\frac{\Delta y}{2} \left(T_{\infty} - \mathbf{T}_{20}^{i}\right) + \pi k \Delta y \frac{T_{21}^{i} - T_{20}^{i}}{\ln\left(\frac{\tau_{1}}{r_{2}}\right)} + \mathbf{K}\frac{\Delta x}{2} \frac{T_{15}^{i} - T_{20}^{i}}{\Delta y}\right) + \left(\mathbf{M}_{o}\frac{\Delta x}{2} \left(T_{\infty} - \mathbf{T}_{20}^{i}\right) + \mathbf{K}\frac{\Delta x}{2} \frac{T_{10}^{i} - T_{20}^{i}}{\ln\left(\frac{\tau_{1}}{r_{2}}\right)} + \mathbf{K}\frac{\Delta x}{2} \frac{T_{10}^{i} - T_{20}^{i}}{\Delta y}\right) + \left(\mathbf{M}_{o}\frac{\Delta x}{2} \left(T_{\infty} - \mathbf{T}_{20}^{i}\right) + \left(\mathbf{M}_{o}\frac{\Delta x}{2} \left(T_{\infty} - \mathbf{T}_{20}^{i}\right) + \left(\mathbf{M}_{o}\frac{\Delta x}{2} \left(T_{\infty} - \mathbf{T}_{20}^{i}\right) + \mathbf{M}_{o}\frac{\Delta x}{2} \left(T_{\infty} - \mathbf{T}_{20}^{i}\right)\right) + \left(\mathbf{M}_{o}\frac{\Delta x}{2} \left(T_{\infty} - \mathbf{T}_{20}^{i}\right) + \mathbf{M}_{o}\frac{\Delta x}{2} \left(T_{\infty} - \mathbf{T}_{20}^{i}\right)\right) + \left(\mathbf{M}_{o}\frac{\Delta x}{2} \left(T_{\infty} - \mathbf{T}_{20}^{i}\right) + \mathbf{M}_{o}\frac{\Delta x}{2} \left(T_{\infty} - \mathbf{T}_{20}^{i}\right)\right) + \left(\mathbf{M}_{o}\frac{\Delta x}{2} \left(T_{\infty} - \mathbf{T}_{20}^{i}\right) + \mathbf{M}_{o}\frac{\Delta x}{2} \left(T_{\infty} - \mathbf{T}_{20}^{i}\right)\right) + \left(\mathbf{M}_{o}\frac{\Delta x}{2} \left(T_{\infty} - \mathbf{T}_{20}^{i}\right) + \mathbf{M}_{o}\frac{\Delta x}{2} \left(T_{\infty} - \mathbf{T}_{20}^{i}\right)\right) + \left(\mathbf{M}_{o}\frac{\Delta x}{2} \left(T_{\infty} - \mathbf{T}_{20}^{i}\right) + \mathbf{M}_{o}\frac{\Delta x}{2} \left(T_{\infty} - \mathbf{T}_{20}^{i}\right)\right) + \left(\mathbf{M}_{o}\frac{\Delta x}{2} \left(T_{\infty} - \mathbf{T}_{20}^{i}\right) + \left(\mathbf{M}_{o}\frac{\Delta x}{2} \left(T_{\infty} - \mathbf{T}_{20}^{i}\right)\right) + \left(\mathbf{M}_$$

$$\begin{split} T & [20][i+1] = (((0.5*h_o*x*(T_a-T[20][i])) + (0.5*h_o*y*(T_a-T[20][i])) + (\pi*k*y*(T[21][i]-T[20][i])/ln(r_1/r_2)) \\ & + (0.5*x*k*(T[15][i]-T[20][i]/y))*((4*t)/(r*c*x*y))) + T[20][i]; \end{split}$$

$$\begin{split} & h_{o}\Delta x \; (T_{\infty} - T_{21}{}^{i}) + k \frac{\Delta x}{2} \frac{T_{16}^{i} - T_{21}^{i}}{\Delta y} + \pi k \Delta y \; \frac{T_{20}^{i} - T_{21}^{i}}{\ln(\frac{r_{1}}{r_{2}})} + \pi k \Delta y \; \frac{T_{22}^{i} - T_{21}^{i}}{\ln(\frac{r_{2}}{r_{3}})} = \rho \; \Delta x \; \frac{\Delta y}{2} \; \mathsf{C} \; \frac{(T_{21}^{i+1} - T_{21}^{i})}{\Delta t} \\ & T_{21}^{i+1} = ((h_{o}\Delta x \; (T_{\infty} - T_{21}^{i}) + \pi k \Delta y \; \frac{T_{20}^{i} - T_{21}^{i}}{\ln(\frac{r_{1}}{r_{2}})} + \pi k \Delta y \; \frac{T_{22}^{i} - T_{21}^{i}}{\ln(\frac{r_{2}}{r_{3}})} + k \Delta x \; \frac{T_{16}^{i} - T_{21}^{i}}{\Delta y} \Big) \frac{2\Delta t}{\rho \mathsf{C} \Delta x \Delta y} \Big) + T_{21}^{i} \\ & T \; [21][i+1] = ((h_{o}^{*} x^{*}(T_{a} - T[21][i])) + (\pi^{*} k^{*} y^{*}(T[20][i] - T[21][i])/\ln(r_{1}/r_{2})) + (\pi^{*} k^{*} y^{*}(T[22][i] - T[21][i])/\ln(r_{1}/r_{2})) + (\pi^{*} k^{*} y^{*}(T[22][i] - T[21][i])/\ln(r_{1}/r_{2})) + T_{21}^{i} \\ & T \; [21][i]/\ln(r_{2}/r_{3})) + (k^{*} x^{*}(T[16][i] - T[21][i]/y))^{*}((2^{*} t)/(r^{*} c^{*} x^{*} y))) + T_{21}^{i} [21][i]; \end{split}$$

# NODE:-22

$$\begin{aligned} h_{o}\Delta x \ (T_{\infty} - T_{22}^{i}) + k\Delta x \ \frac{T_{17}^{i} - T_{22}^{i}}{\Delta y} + \pi k\Delta y \ \frac{T_{21}^{i} - T_{22}^{i}}{\ln(\frac{r_{2}}{r_{3}})} + \pi k\Delta y \ \frac{T_{23}^{i} - T_{22}^{i}}{\ln(\frac{r_{3}}{r_{4}})} &= \rho \ \Delta x \ \frac{\Delta y}{2} \ C \ \frac{(T_{22}^{i+1} - T_{22}^{i})}{\Delta t} \\ T_{22}^{i+1} &= ((h_{o}\Delta x \ (T_{\infty} - T_{22}^{i}) + \pi k\Delta y \ \frac{T_{21}^{i} - T_{22}^{i}}{\ln(\frac{r_{3}}{r_{3}})} + \pi k\Delta y \ \frac{T_{23}^{i} - T_{22}^{i}}{\ln(\frac{r_{3}}{r_{4}})} + k\Delta x \ \frac{T_{17}^{i} - T_{23}^{i}}{\Delta y} \right) \ \frac{2\Delta t}{\rho(\Delta x\Delta y)} + T_{22}^{i} \\ T \ [22][i+1] &= ((h_{o}^{*}x^{*}(T_{a} - T[22][i])) + (\pi^{*}k^{*}y^{*}(T[21][i] - T[22][i])/\ln(r_{3}/r_{4})) + (\pi^{*}k^{*}y^{*}(T[23][i] - T[22][i])/\ln(r_{4}/r_{5})) + (k^{*}x^{*}(T[17][i] - T[22][i]/y))^{*}((2^{*}t)/(r^{*}c^{*}x^{*}y))) + T[22][i]; \\ \textbf{NODE: -23 +} \\ h_{o}\Delta x \ (T_{\infty} - T_{23}^{i}) + \pi k\Delta y \ \frac{T_{22}^{i} - T_{23}^{i}}{\ln(\frac{r_{3}}{r_{4}})} + \pi k\Delta y \ \frac{T_{22}^{i} - T_{23}^{i}}{\ln(\frac{r_{4}}{r_{5}})} + k\Delta x \ \frac{T_{18}^{i} - T_{23}^{i}}{\Delta y} = \rho \ \Delta x \ \frac{\Delta y}{2} \ C \ \frac{(T_{23}^{i+1} - T_{23}^{i})}{\Delta t} \\ T_{23}^{i+1} &= ((h_{o}\Delta x \ (T_{\infty} - T_{23}^{i}) + \pi k\Delta y \ \frac{T_{22}^{i} - T_{23}^{i}}{\ln(\frac{r_{3}}{r_{4}})} + \pi k\Delta y \ \frac{T_{22}^{i} - T_{23}^{i}}{\ln(\frac{r_{3}}{r_{5}})} + k\Delta x \ \frac{T_{18}^{i} - T_{23}^{i}}{\Delta y} \ \frac{\Delta t}{\rho(\Delta \Delta \Delta y)} + T_{23}^{i} \\ T \ [23][i+1] &= ((h_{o}^{*}x^{*}(T_{a} - T[23][i])) + (\pi^{*}k^{*}y^{*}(T[22][i] - T[23][i])/\ln(r_{3}/r_{4})) + (\pi^{*}k^{*}y^{*}(T[24][i] - T[22][i])/\ln(r_{3}/r_{4})) + (\pi^{*}k^{*}y^{*}(T[24][i] - T[22][i])/\ln(r_{3}/r$$

 $T[23][i])/ln(r_4/r_5)) + (k*x*(T[18][i]-T[23][i]/y))*((2*t)/(r*c*x*y))) + T[23][i];$ 

$$\begin{aligned} &\text{NODE: -24} \\ &\text{h}_{o}\frac{\Delta x}{2} \left(T_{\infty} - T_{24}^{i}\right) + \text{h}_{i}\frac{\Delta y}{2} \left(T_{h} - T_{24}^{i}\right) + \pi k \Delta y \frac{T_{23}^{i} - T_{24}^{i}}{\ln\left(\frac{r_{4}}{r_{5}}\right)} + k \frac{\Delta x}{2} \frac{T_{19}^{i} - T_{24}^{i}}{\Delta y} = \rho \frac{\Delta x}{2} \frac{\Delta y}{2} C \frac{(T_{24}^{i+1} - T_{24}^{i})}{\Delta t} \\ &T_{24}^{i+1} = \left(\left(\text{h}_{o}\frac{\Delta x}{2} \left(T_{\infty} - T_{24}^{i}\right) + \text{h}_{i}\frac{\Delta y}{2} \left(T_{h} - T_{24}^{i}\right) + \pi k \Delta y \frac{T_{23}^{i} - T_{24}^{i}}{\ln\left(\frac{r_{4}}{r_{5}}\right)} + k \frac{\Delta x}{2} \frac{T_{19}^{i} - T_{24}^{i}}{\Delta y} \right) \frac{4\Delta t}{\rho(\Delta x \Delta y)} + T_{24}^{i} \\ &T_{24}^{i+1} = \left(\left((0.5 + \text{h}_{o} * x^{*}(T_{a} - T[24][i])\right) + (0.5 + \text{h}_{i} * y^{*}(T_{h} - T[24][i])\right) + (\pi + k + y^{*}(T[23][i] - T[24][i])/\ln(r_{4}/r_{5})) \end{aligned}$$

 $+(0.5^{*}k^{*}x^{*}(T[19][i]-T[24][i]/y))^{*}((4^{*}t)/(r^{*}c^{*}x^{*}y)))+T[24][i];$ 

# **IV. EXPERIMENTAL VALIDATION**

We have seen that during experimental of cylindrical type induction furnace for melting basic ferrous alloy with wall of furnace made by silica ramming mass (Basic).it is initially temperature starting from 15°C-20°C and increase within 1-minute 30°C - 35°C. This temperature is change up to 25 minute of material melting. Then after induction furnace is increase temperature with 40 °C - 45°C up to 46 minute of melting process. Here is increase at that time stress also increases.



Fig.2 Cylindrical Induction Furnace during melting process

In 46 minutes, metal is fully melted in furnace then there is pouring process is start it depends on shape, size and length of mould. The minimum time is taken 5 seconds and maximum is 5 minutes.



Fig.3 After pouring process

After pouring process the furnace is decrease temperature 30 °C - 35°C with in every minute. Here, temperature is decreases then stress also decreases. In that next melting cycle is start.

## **V. RESULT OUTPUT**





## **Fig. 4** Temperature v/s Time for Silica Ramming Mass



# VI. CONCLUSION

Induction type cylindrical and spherical furnace are highly use for melting of different kinds of material in that problem comes from the zirconia of losing its properties and failure occurs with life cycle <u>200</u> hours of life time. It will disturb production schedule in industries. So, it is required finding of life cycle of zirconia material wall furnace by explicit finite different analysis method. The condition is plotted on S-log N-curves are plotted for life span prediction. We found life span of different materials like, zirconia, silica, fire-bricks, high alumina by

this mathematical program and exactly predicts and failure of induction furnace refractory wall. The accuracy of the fatigue life prediction for the induction type cylindrical and spherical melting furnaces refractory wall depend upon temperature and thermal stress calculated at critical point by this method.

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