ELZAKI TRANSFORMATION OF SOME SIGNIFICANT INFINITE POWER SERIES

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ABSTRACT

In Mathematics, a power series in one variable is an infinite series. In this paper, we will find the Elzaki Transformation of some power series. The purpose of paper is to prove the applicability of Elzaki transform to some significant infinite power series.

Keywords: *Elzaki transformation, power series.*

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1. INTRODUCTION

Elzaki transformation is a mathematical tool used to obtain the solutions of differential equations without finding their general solutions. It has applications in nearly all engineering disciplines [1, 2, 3,]. It also comes out to be very effective tool to find the Elzaki Transformation of some power series. In this paper, we present a new approach called Elzaki transform approach to find the Elzaki Transformation of some power series.

2. BASIC DEFINITIONS

2.1 Elzaki Transform

If the function $\hbar(y)$, $y \geq 0$ is having an exponential order and is a piecewise continuous function on any interval, then the Elzaki transform of $\hbar(y)$ is given by

$$E\{\hbar(z)\} = \bar{\hbar}(p) = p\int_0^\infty e^{-\frac{z}{p}}\hbar(y)dy.$$

The Elzaki Transform [1, 2, 3,] of some of the functions are given by

- $E\{z^n\} = n!\,p^{n+2}$, where $n = 0,1,2,..$
- $E\{e^{az}\} = \frac{p^2}{1-ap}$,
- $E\{\sin az\} = \frac{ap^3}{1+a^2p^2}$,
- $E\{\cos az\} = \frac{ap^2}{1+a^2p^2}$,
- $E\{\sinh az\} = \frac{ap^3}{1-a^2p^2}$,
- $E\{\cosh az\} = \frac{ap^2}{1-a^2p^2}$.

2.2 Inverse Elzaki Transform

The Inverse Elzaki Transform of some of the functions are given by

- $E^{-1}\{p^n\} = \frac{z^{n-2}}{(n-2)!}$, $n = 2, 3, 4 \ldots$
- $E^{-1}\{\frac{p^2}{1-ap}\} = e^{az}$

- $E^{-1}\{\frac{p^3}{1+a^2p^2}\} = \frac{1}{a}\sin az$
- $E^{-1}\{\frac{p^2}{1+a^2p^2}\} = \frac{1}{a}\cos az$
- $E^{-1}\{\frac{p^3}{1-a^2p^2}\} = \frac{1}{a}\sin haz$
- $E^{-1}\{\frac{p^2}{1-a^2p^2}\} = \frac{1}{a}\cos haz$

2.3 Power series [4, 5, 6,]:

$$\sum_{n=0}^{\infty} b_n z^n = b_0 + b_1 z + b_2 z^2 + \cdots b_n z^n$$

2.4 Maclaurin series [4, 5, 6,]:

$$y = \sum_{n=0}^{\infty} \frac{y^{(n)}}{n!} z^n = y_0 + \frac{y_0{}'}{1!} z + \frac{y_0{}''}{2!} z^2 + \frac{y_0{}'''}{2!} z^3 \ldots \ldots \ldots$$

3. METHODOLOGY

3.1 Elzaki Transformation of Geometric Series later than the expanding to power series appearance [4, 5, 6,]:

$$\frac{1}{1-z} = \sum_{n=0}^{\infty} z^n = f(z)$$

$$E\{f(z)\} = E\left\{\sum_{n=0}^{\infty} z^n\right\}$$

$$= p\int_0^{\infty} e^{-\frac{z}{p}} \sum_{n=0}^{\infty} z^n \, dz$$

$$= \sum_{n=0}^{\infty} p\int_0^{\infty} e^{-\frac{z}{p}} z^n dz$$

$$= \sum_{n=0}^{\infty} E\{z^n\}$$

$$= \sum_{n=0}^{\infty} n!\, p^{n+2}$$

Hence,

$$E\{f(z)\} = \sum_{n=0}^{\infty} n!\, p^{n+2}$$

3.2 Elzaki Transformation of the Power series expansion of e^z later than the expanding to power series appearance[4, 5, 6,]:

$$e^z = \sum_{n=0}^{\infty} \frac{z^n}{n!} = f(z)$$

$$E\,\{f(z)\} = E\left\{\sum_{n=0}^{\infty} \frac{z^n}{n!}\right\}$$

$$= p\int_0^{\infty} e^{-\frac{z}{p}} \left\{\sum_{n=0}^{\infty} \frac{z^n}{n!}\right\} dz$$

$$= \sum_{n=0}^{\infty} p \int_0^{\infty} e^{-\frac{z}{p}} \frac{z^n}{n!} dz$$
$$= \sum_{n=0}^{\infty} \frac{1}{n!} \left[p \int_0^{\infty} e^{-\frac{z}{p}} z^n dz \right]$$
$$= \sum_{n=0}^{\infty} \frac{1}{n!} E\{z^n\} = \sum_{n=0}^{\infty} \frac{1}{n!} . n!\, p^{n+2}$$

Hence, $$E\{f(z)\} = \sum_{n=0}^{\infty} p^{n+2}$$

3.3 Elzaki Transformation of the Power series expansion of $\log(1+z)$later than the expanding to power series appearance [4, 5, 6,]:

$$\log(1+z) = \sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{n} z^n = f(z)$$
$$E\{f(z)\} = E\left\{ \sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{n} z^n \right\}$$
$$= p \int_0^{\infty} e^{-\frac{z}{p}} \left\{ \sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{n} z^n \right\} dz$$
$$= \sum_{n=1}^{\infty} p \int_0^{\infty} e^{-\frac{z}{p}} \frac{(-1)^{n+1}}{n} z^n dz$$
$$= \sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{n} \left[p \int_0^{\infty} e^{-\frac{z}{p}} z^n dz \right]$$
$$= \sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{n} E\{z^n\}$$
$$= \sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{n} n!\, p^{n+2}$$
$$= \sum_{n=1}^{\infty} (-1)^{n+1} (n-1)!\; p^{n+2}$$

Hence ,

$$E\{f(z)\} = \sum_{n=1}^{\infty} (-1)^{n+1} (n-1)!\; p^{n+2}$$

3.4 Elzaki Transformation of the Power series expansion of $\log(1+z)$ later than the expanding to power series appearance[4, 5, 6,]:

$$\log(1+z) = \sum_{n=1}^{\infty} \frac{(-1)^{2n-1}}{n} z^n = f(z)$$
$$E\{f(z)\} = E\left\{ \sum_{n=1}^{\infty} \frac{(-1)^{2n-1}}{n} z^n \right\}$$
$$= p \int_0^{\infty} e^{-\frac{z}{p}} \left\{ \sum_{n=1}^{\infty} \frac{(-1)^{2n-1}}{n} z^n \right\} dz$$
$$= \sum_{n=1}^{\infty} p \int_0^{\infty} e^{-\frac{z}{p}} \frac{(-1)^{2n-1}}{n} z^n dz$$

$$= \sum_{n=1}^{\infty} \frac{(-1)^{2n-1}}{n} p \int_0^{\infty} e^{-\frac{z}{p}} z^n dz$$
$$= \sum_{n=1}^{\infty} \frac{(-1)^{2n-1}}{n} E\{z^n\}$$
$$= \sum_{n=1}^{\infty} \frac{(-1)^{2n-1}}{n} n!\, p^{n+2}$$
$$= \sum_{n=1}^{\infty} (-1)^{2n-1} (n-1)!\ p^{n+2}$$

Hence ,

$$E\{f(z)\} = \sum_{n=1}^{\infty} (-1)^{2n-1} (n-1)!\ p^{n+2}$$

3.5 Elzaki Transformation of the Power series expansion of $\log\frac{(1+z)}{(1-z)}$ later than the expanding to power series appearance[4, 5, 6,]:

$$\log\frac{(1+z)}{(1-z)} = \sum_{n=1}^{\infty} \frac{2}{2n-1} z^{2n-1} = f(z)$$
$$E\,\{f(z)\} = E\left\{\sum_{n=1}^{\infty} \frac{2}{2n-1} z^{2n-1}\right\}$$

let, $2n-1 = m$

$2n = m+1$

$n = \frac{m+1}{2}$

$$= p\int_0^{\infty} e^{-\frac{z}{p}} \left\{\sum_{n=1}^{\infty} \frac{2}{2n-1} z^{2n-1}\right\} dz$$
$$= \sum_{n=1}^{\infty} p\int_0^{\infty} e^{-\frac{z}{p}} \frac{2}{2n-1} z^{2n-1} dz$$
$$= \sum_{n=1}^{\infty} \frac{2}{2n-1}\left[p\int_0^{\infty} e^{-\frac{z}{p}} z^{2n-1} dz\right]$$
$$= \sum_{n=1}^{\infty} \frac{2}{2n-1} E\{z^{2n-1}\}$$
$$= \sum_{n=1}^{\infty} \frac{2}{2n-1} (2n-1)!\, p^{2n-1+2}$$
$$= \sum_{n=1}^{\infty} 2(2n-2)!\, p^{2n+1}$$

Hence,

$$E\,\{f(z)\} = \sum_{n=1}^{\infty} 4(n-1)!\, p^{2n+1}$$

3.6 Elzaki Transformation of the Power series expansion of Cosx later than the expanding to power series appearance [4, 5, 6,]:

$$\cos x = \sum_{n=0}^{\infty} \frac{(-1)^n}{2n!} z^{2n} = f(z)$$

$$E\{F(z)\} = E\left\{\sum_{n=0}^{\infty}\frac{(-1)^n}{2n!}z^{2n}\right\}$$
$$= p\int_0^{\infty} e^{-\frac{z}{p}}\left\{\sum_{n=0}^{\infty}\frac{(-1)^n}{2n!}z^{2n}\right\}dz$$
$$= \sum_{n=0}^{\infty} p\int_0^{\infty} e^{-\frac{z}{p}}\frac{(-1)^n}{2n!}z^{2n}dz$$
$$= \sum_{n=0}^{\infty}\frac{(-1)^n}{2n!}\left[p\int_0^{\infty} e^{-\frac{z}{p}}z^{2n}dz\right]$$
let $2n = u$
$$= \sum_{n=0}^{\infty}\frac{(-1)^n}{2n!}E\,\{z^u\}$$
$$= \sum_{n=0}^{\infty}\frac{(-1)^n}{2n!}u!\,p^{u+2}$$
$$= \sum_{n=0}^{\infty}\frac{(-1)^n}{2n!}2n!\,p^{2n+2}$$
$$\text{Hence}, E\{f(t)\} = \sum_{n=0}^{\infty}(-1)^n\,p^{2n+2}$$

3.7 Elzaki Transformation of the Power series expansion of Sinx later than the expanding to power series appearance $[\mathbf{4},\mathbf{5},\mathbf{6},]$:

$$\text{Sinx} = \sum_{n=0}^{\infty}\frac{(-1)^n}{(2n+1)!}z^{2n+1} = f(z)$$
$$E\{f(z)\} = E\left\{\sum_{n=0}^{\infty}\frac{(-1)^n}{(2n+1)!}z^{2n+1}\right\}$$
$$= p\int_0^{\infty} e^{-\frac{z}{p}}\left\{\sum_{n=0}^{\infty}\frac{(-1)^n}{(2n+1)!}z^{2n+1}\right\}dz$$
$$= \sum_{n=0}^{\infty} p\int_0^{\infty} e^{-\frac{z}{p}}\frac{(-1)^n}{(2n+1)!}z^{2n+1}dz$$
$$= \sum_{n=0}^{\infty}\frac{(-1)^n}{(2n+1)!}\left[p\int_0^{\infty} e^{-\frac{z}{p}}z^{2n+1}dz\right]$$
$$= \sum_{n=0}^{\infty}\frac{(-1)^n}{(2n+1)!}E\,\{z^{2n+1}\}$$
$$= \sum_{n=0}^{\infty}\frac{(-1)^n}{(2n+1)!}(2n+1)!\,p^{2n+1+2}$$
$$\text{Hence}, E\{f(t)\} = \sum_{n=0}^{\infty}(-1)^n\,p^{2n+3}$$

3.8 Elzaki Transformation of the Power series expansion of Coshx later than the expanding to power series appearance $[\mathbf{4},\mathbf{5},\mathbf{6},]$:

$$\text{Coshx} = \sum_{n=0}^{\infty}\frac{1}{2n!}z^{2n} = f(z)$$

$$E\{f(z)\} = E\left\{\sum_{n=0}^{\infty} \frac{1}{2n!} z^{2n}\right\}$$
$$= p\int_0^{\infty} e^{-\frac{z}{p}} \left\{\sum_{n=0}^{\infty} \frac{1}{2n!} z^{2n}\right\} dz$$
$$= \sum_{n=0}^{\infty} p\int_0^{\infty} e^{-\frac{z}{p}} \frac{1}{2n!} z^{2n} dz$$
$$= \sum_{n=0}^{\infty} \frac{1}{2n!}\left[p\int_0^{\infty} e^{-\frac{z}{p}} z^{2n} dz\right]$$
let $2n = u$
$$= \sum_{n=0}^{\infty} \frac{1}{2n!} E\{z^u\}$$
$$= \sum_{n=0}^{\infty} \frac{1}{2n!} u!\, p^{u+2}$$
$$= \sum_{n=0}^{\infty} \frac{1}{2n!}\ 2n!\, p^{2n+2}$$
$$\text{Hence, } E\{f(t)\} = \sum_{n=0}^{\infty} p^{2n+2}$$

3.9 Elzaki Transformation of the Power series expansion of Sinx later than the expanding to power series appearance [4, 5, 6,]:

$$\text{Sinhx} = \sum_{n=0}^{\infty} \frac{1}{(2n+1)!} z^{2n+1} = f(z)$$
$$E\{f(z)\} = E\left\{\sum_{n=0}^{\infty} \frac{1}{(2n+1)!} z^{2n+1}\right\}$$
$$= p\int_0^{\infty} e^{-\frac{z}{p}} \left\{\sum_{n=0}^{\infty} \frac{1}{(2n+1)!} z^{2n+1}\right\} dz$$
$$= \sum_{n=0}^{\infty} p\int_0^{\infty} e^{-\frac{z}{p}} \frac{1}{(2n+1)!} z^{2n+1} dz$$
$$= \sum_{n=0}^{\infty} \frac{1}{(2n+1)!} p\int_0^{\infty} e^{-\frac{z}{p}} z^{2n+1} dz$$
$$= \sum_{n=0}^{\infty} \frac{1}{(2n+1)!} E\{z^{2n+1}\}$$
$$= \sum_{n=0}^{\infty} \frac{1}{(2n+1)!} (\,2n+1)!\, p^{2n+3}$$
$$\text{Hence, } E\{f(t)\} = \sum_{n=0}^{\infty} p^{2n+3}$$

3.10 If $f(z)$ is a power series expansion at the point b, where b is any constant, $b \in R$, Its Taylor's series expansion [5, 6] is

$$\mathbf{f(z) = \sum_{n=0}^{\infty} b_n\,(z-b)^n}$$

Then, The Elzaki transformation of f(z) is given in the form of power series as

$$E\{f(z)\} = E\left[\sum_{n=0}^{\infty} b_n\,(z-b)^n\right]$$
$$= p\int_0^{\infty} e^{-\frac{z}{p}}\left\{\sum_{n=0}^{\infty} b_n\,(z-b)^n\right\}dz$$
$$= p\sum_{n=0}^{\infty} b_n \int_0^{\infty} e^{-\frac{z}{p}}\{(z-b)^n\}dz$$
$$= p\sum_{n=0}^{\infty} b_n \int_0^{\infty} e^{-\frac{u+b}{p}}\{(u)^n\}dz$$
$$= p\sum_{n=0}^{\infty} b_n\,e^{-\frac{b}{p}} \int_0^{\infty} e^{-\frac{u}{p}}\{(u)^n\}dz$$
$$= \sum_{n=0}^{\infty} b_n\,e^{-\frac{b}{p}}\left[p\int_0^{\infty} e^{-\frac{u}{p}}\{(u)^n\}dz\right]$$
$$= \sum_{n=0}^{\infty} b_n\,e^{-\frac{b}{p}}\,E(u)^n$$
$$E\sum_{n=0}^{\infty} b_n\,(z-b)^n = \sum_{n=0}^{\infty} b_n\,e^{-\frac{b}{p}}\,n!\,p^{n+2}$$

3.11 If $f(z)$ is a power series expansion at thepoint 0, where 0, Its Power series expansion is[5, 6, 7,]:

$$\mathbf{f(z) = \sum_{n=0}^{\infty} b_n\,(z)^n}$$

Then, The Elzaki transformation of f(z) is given in the form of power series as

$$E\{f(z)\} = E\left[\sum_{n=0}^{\infty} b_n\,(z)^n\right]$$
$$= p\int_0^{\infty} e^{-\frac{z}{p}}\left\{\sum_{n=0}^{\infty} b_n\,(z)^n\right\}dz$$
$$= p\sum_{n=0}^{\infty} b_n \int_0^{\infty} e^{-\frac{z}{p}}\{(z)^n\}dz$$
$$= \sum_{n=0}^{\infty} b_n\,p \int_0^{\infty}\{(z)^n\}dz$$
$$= \sum_{n=0}^{\infty} b_n\;E(u)^n$$
$$= \sum_{n=0}^{\infty} b_n\;n!\,p^{n+2}$$

3.12 Elzaki Transformation of the Power series expansion of e^{t^2} later than the expanding to power series appearance[5, 6, 7,]:

$$f(z) = \mathbf{e^{t^2}} = \sum_{\mathbf{n=0}}^{\infty} \frac{\mathbf{z^{2n}}}{\mathbf{n!}}$$

$$E[f(z)] = p\int_0^{\infty} e^{-\frac{z}{p}} \left\{\sum_{\mathbf{n=0}}^{\infty} \frac{\mathbf{z^{2n}}}{\mathbf{n!}}\right\} dz$$

$$= p\sum_{n=0}^{\infty} \frac{1}{n!}\int_0^{\infty} e^{-\frac{z}{p}} \{(z)^{2n}\} dz$$

$$= \sum_{n=0}^{\infty} \frac{1}{n!}\left[p\int_0^{\infty} e^{-\frac{z}{p}} \{(z)^{2n}\} dz\right]$$

$$= \sum_{n=0}^{\infty} \frac{1}{n!} E(z)^{2n}$$

$$= \sum_{n=0}^{\infty} \frac{1}{n!} 2n!\, p^{2n+2}$$

3.13 Elzaki transformation of Convergence Series [4, 5, 6,]:

$$1 + \frac{c+z}{1!} + \frac{(c+2z)^2}{2!} + \frac{(c+3z)^3}{3!} + \cdots$$

$$= \sum_{n=0}^{\infty} \frac{(c+nz)^n}{n!} = f(z)$$

$$\text{So, } E\{f(z)\} = E\left\{\sum_{n=0}^{\infty} \frac{(c+nz)^n}{n!}\right\}$$

$$p\int_0^{\infty} e^{-\frac{z}{p}} \left\{\sum_{n=0}^{\infty} \frac{(c+nz)^n}{n!}\right\} dz \quad ,$$

$$\text{let } c + nz = t$$

$$= \sum_{n=0}^{\infty} p\int_0^{\infty} e^{-\frac{z}{p}} \frac{(c+nz)^n}{n!} dz$$

$$= \sum_{n=0}^{\infty} p\int_0^{\infty} e^{-\frac{1}{p}(\frac{t-c}{n})} \frac{t^n}{n!}\frac{dt}{n}$$

$$= \sum_{n=0}^{\infty} p\, e^{\frac{c}{np}} \int_0^{\infty} e^{-\frac{1}{p}(\frac{t}{n})} \frac{t^n}{n!}\frac{dt}{n} \quad , \quad \text{let } \frac{t}{n} = u$$

$$= \sum_{n=0}^{\infty} p e^{\frac{c}{np}} \int_0^{\infty} e^{-\frac{u}{p}} \frac{n^n u^n}{n!}\frac{ndu}{n}$$

$$= \sum_{n=0}^{\infty} e^{\frac{c}{np}} \frac{n^n}{n!} p\int_0^{\infty} e^{-\frac{u}{p}} u^n du$$

$$= \sum_{n=0}^{\infty} e^{\frac{c}{np}} \frac{n^n}{n!} E(u^n)$$

Hence,

$$E\left\{\sum_{n=0}^{\infty} \frac{(c+nz)^n}{n!}\right\} = \sum_{n=0}^{\infty} e^{\frac{c}{np}} n^n p^{n+2}$$

Conclusion:

In this paper, we have found the Elzaki Transformation of some power series and it comes out to be very foremost and effective tool to find the Elzaki Transformation of some power series.

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