

# FUZZY DOT SUBALGEBRAS AND FUZZY DOT IDEALS OF B-ALGEBRAS

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## ABSTRACT

In this paper, the notion of fuzzy dot subalgebras, fuzzy normal dot subalgebras and fuzzy dot ideals of  $B$ -algebras are introduced and investigated some of their properties. We also introduced the notion of  $n^{\text{th}}$  term of fuzzy dot subalgebras, fuzzy normal dot subalgebras and fuzzy dot ideals of  $B$ -algebras.

**Keywords :-**  $B$ -algebras, fuzzy subalgebra, fuzzy ideal, fuzzy dot subalgebra, fuzzy normal dot subalgebra, fuzzy dot ideal.

## 1 INTRODUCTION

The study of  $BCK/BCI$ -algebras [3] was initiated by Imai and Iseki in 1966 as a generalization of the concept of set theoretic difference and propositional calculus. Neggers and kim [5,6] introduced a new notion, called  $B$ -algebras which is related to several classes of algebras of interest such as  $BCK/BCI$ -algebras. Cho and kim [2] discussed further relations between  $B$ -algebras and other topics especially quasigroups. Jun et al.[4] fuzzyfied (normal)  $B$ -algebras and gave a characterization of a fuzzy  $B$ -algebras. Saeid [7] introduced the concept of interval-valued fuzzy  $B$ -subalgebras of a  $B$ -algebra, and studied some of their properties. Senapati et al. [8,12,13] introduced fuzzy closed ideals of  $B$ -algebras and fuzzy subalgebras of  $B$ -algebras with respect to  $t$ -norm. Also, the authors [1,9,10,11] done lot of works on  $BG$ -algebras which is a generalization of  $B$ -algebras. For the general development of  $B$ -algebras, the ideal theory and subalgebras play important role. In this paper, fuzzy dot subalgebras of  $B$ -algebras are defined and some properties are investigated for set of all  $n$ -terms. The remainder of this article is structured as follows: Section 2 proceeds with a recapitulation of all required definitions and properties. In section 3, the concepts and operations of fuzzy dot subalgebras are introduced and discussed their properties in details. In section 4, some properties of fuzzy normal dot subalgebras are discussed. In section 5, the notion of fuzzy dot ideals of  $B$ -algebras are considered and investigated their properties in details. Finally, in section 6, conclusion and scope for future research are given.

## 2 PRELIMINARIES

In this section, some elementary aspects that are necessary for this paper are included. A  $B$ -algebras is an important class of logical algebras introduced by Neggers and kim [5] and was extensively investigated by several researchers. This algebra is defined as follows.

**2.1 Definition ([5])** A non-empty set  $X$  with a constant  $0$  and a binary operation  $*$  is said to be  $B$ -algebra if it satisfies the following axioms

$$B_1. x * x = 0;$$

$$B_2. x * 0 = x;$$

$$B_3. (x * y) * z = x * (z * (0 * y)), \text{ for all } x, y, z \in X.$$

**2.2 Definition** A non-empty subset  $S$  of a  $B$ -algebra  $X$  is called a subalgebra [6] of  $X$  if  $x * y \in S$  for any  $x, y \in S$ .

**2.3 Definition** A mapping  $f: X \rightarrow Y$  of  $B$ -algebra is called a homomorphism [6] if  $f(x * y) = f(x) * f(y)$  for all  $x, y \in X$ .

**2.4 Definition** A non-empty subset  $N$  of a  $B$ -algebra  $X$  is said to be normal if  $(x * a) * (y * b) \in N$  whenever  $x * y \in N$  and  $a * b \in N$ . Note that any normal subset  $N$  of a  $B$ -algebra  $X$  is a subalgebra of  $X$ , but the converse need not be true [6].

**2.5 Definition** Let  $X$  be the collection of objects denoted generally by  $x$ . Then a fuzzy set [14]  $A$  in  $X$  is defined as  $A = \{(x, \mu_A(x)): x \in X\}$ , where  $\mu_A(x)$  is called the membership value of  $x$  in  $A$  and  $0 \leq \mu_A(x) \leq 1$ . The complement of  $A$  is denoted by  $A^c$  and is given by  $A^c = \{(x, \mu_A^c(x)): x \in X\}$  where  $\mu_A^c = 1 - \mu_A(x)$ . for any two fuzzy set  $A = \{(x, \mu_A(x)): x \in X\}$  and  $B = \{(x, \mu_B(x)): x \in X\}$  in  $X$ , the following operations are defined  $A \subseteq B \iff \mu_A(x) \leq \mu_B(x)$  for all  $x \in X$ ,  $A \cap B = \min\{\mu_A(x), \mu_B(x)\}$  for all  $x \in X$ .

**2.6 Definition ([4])** A fuzzy set  $A$  in  $X$  is called a fuzzy subalgebra if it satisfies the inequality  $\mu_A(x * y) \geq \min\{\mu_A(x), \mu_A(y)\}$  for all  $x, y \in X$ .

**2.7 Definition ([8])** A fuzzy set  $A$  in  $X$  is called a fuzzy ideal of  $X$  if it satisfies the inequality (i)  $\mu_A(0) \geq \mu_A(x)$  and (ii)  $\mu_A(x) \geq \min\{\mu_A(x * y), \mu_A(y)\}$  for all  $x, y \in X$ .

### 3 FUZZY DOT SUBALGEBRAS OF $B$ -ALGEBRAS

In this section, Fuzzy Dot Subalgebras of  $B$ -algebras are defined and some propositions and theorems are presented. It what follows, let  $(X, *, 0)$  or simply  $X$  denote a  $B$ -algebra unless otherwise specified.

**3.1 Definition** Let  $A$  be a fuzzy set in a  $B$ -algebra  $X$ . Then  $A$  is called a fuzzy dot subalgebra of  $X$  if for all  $x, y \in X$ ,  $\mu_A(x * y) \geq \mu_A(x) \cdot \mu_A(y)$ , where  $\cdot$  denotes ordinary multiplication.

**Example 1** Let  $X = \{0,1,2,3\}$  be a set with the following Cayley table:

*	0	1	2	3
0	0	1	2	3
1	1	0	3	2
2	2	3	0	1
3	3	2	1	0

Then  $(X, *, 0)$  is a  $B$ -algebra. Define a fuzzy set  $A$  in  $X$  by  $\mu_A(1) = \mu_A(2) = 0.7$  and  $\mu_A(x) = 0.5$  for all  $x \in X \setminus \{1,2\}$ . Then  $A$  is a fuzzy dot subalgebra of  $X$ .

Note that every fuzzy subalgebra of  $X$  is a fuzzy dot subalgebra of  $X$ , but the converse is not true. In fact, the fuzzy dot subalgebra in above is not a fuzzy subalgebra, since  $\mu_A(1 * 1) = \mu_A(0) = 0.5 < 0.7 = \mu_A(1) = \min\{\mu_A(1), \mu_A(1)\}$

**3.2 Proposition** Every fuzzy dot subalgebras  $A$  of  $X$  satisfies the inequality  $\mu_A(0) \geq (\mu_A(x))^2$  for all  $x \in X$ .

**Proof**

For all  $x, y \in X$ , we have  $x * x = 0$ . Then  $\mu_A(0) = \mu_A(x * x)$

$$\geq \mu_A(x) \cdot \mu_A(x)$$

$$\mu_A(0) = (\mu_A(x))^2$$

**3.3 Theorem** Let  $A$  be a fuzzy dot subalgebra of  $X$ . If there exists a sequence  $\{x_n\}$  in  $X$  such that  $\lim_{n \rightarrow \infty} (\mu_A(x_n))^2 = 1$ , then  $\mu_A(0) = 1$ .

**Proof**

By the proposition 3.2,

$$\mu_A(0) \geq (\mu_A(x))^2 \text{ for all } x \in X.$$

Therefore,  $\mu_A(0) \geq (\mu_A(x_n))^2$  for every positive integer  $n$ .

Consider,  $1 \geq \mu_A(0) \geq \lim_{n \rightarrow \infty} (\mu_A(x_n))^2$

$$\lim_{n \rightarrow \infty} (\mu_A(x_n))^2 = 1.$$

Hence,  $\mu_A(0) = 1$ .

**3.4 Theorem** Let  $A_1$  and  $A_2$  any two fuzzy dot subalgebras of  $X$ . Then  $A_1 \cap A_2$  is a fuzzy dot subalgebra of  $X$ .

**Proof**

Let  $x, y \in A_1 \cap A_2$ . Then  $x, y \in A_1$  and  $A_2$ .

Now,  $\mu_{A_1 \cap A_2}(x * y) = \min\{\mu_{A_1}(x * y), \mu_{A_2}(x * y)\}$

$$\geq \min\{\mu_{A_1}(x) \cdot \mu_{A_1}(y), \mu_{A_2}(x) \cdot \mu_{A_2}(y)\}$$

$$= (\min\{\mu_{A_1}(x), \mu_{A_2}(x)\}) \cdot (\min\{\mu_{A_1}(y), \mu_{A_2}(y)\})$$

$$\mu_{A_1 \cap A_2}(x * y) = \mu_{A_1 \cap A_2}(x) \cdot \mu_{A_1 \cap A_2}(y).$$

Hence,  $A_1 \cap A_2$  is a fuzzy dot subalgebra of  $X$ .

**3.5 Theorem** Let  $\{A_i \mid i = 1, 2, 3, \dots, n\}$  be a family of fuzzy dot subalgebra of  $X$ . Then  $\cap A_i$  is also a fuzzy dot subalgebra of  $X$ , where  $\cap A_i = \min \mu_{A_i}(x)$ .

**Proof**

Let  $x, y \in A_1 \cap A_2 \cap \dots \cap A_n$ . Then  $x, y \in A_1$  and  $A_2$  and ... and  $A_n$ .

$$\begin{aligned} \text{Now, } \mu_{A_1 \cap A_2 \cap \dots \cap A_n}(x * y) &= \min\{\mu_{A_1}(x * y), \mu_{A_2}(x * y), \dots, \mu_{A_n}(x * y)\} \\ &\geq \min\{\mu_{A_1}(x) \cdot \mu_{A_1}(y), \mu_{A_2}(x) \cdot \mu_{A_2}(y), \dots, \mu_{A_n}(x) \cdot \mu_{A_n}(y)\} \\ &= (\min\{\mu_{A_1}(x), \mu_{A_2}(x), \dots, \mu_{A_n}(x)\}) \\ &\quad \cdot (\min\{\mu_{A_1}(y), \mu_{A_2}(y), \dots, \mu_{A_n}(y)\}) \\ \mu_{A_1 \cap A_2 \cap \dots \cap A_n}(x * y) &= \mu_{A_1 \cap A_2 \cap \dots \cap A_n}(x) \cdot \mu_{A_1 \cap A_2 \cap \dots \cap A_n}(y). \end{aligned}$$

Hence,  $A_1 \cap A_2 \cap \dots \cap A_n$  is a fuzzy dot subalgebra of  $X$ .

Hence,  $\cap A_i$  is also a fuzzy dot subalgebra of  $X$ .

**4 FUZZY NORMAL DOT SUBALGEBRAS OF B-ALGEBRAS**

In this section, fuzzy normal dot subalgebras of  $B$ -algebras are defined and discussed the relationship between fuzzy normal dot subalgebra, fuzzy normal subalgebra and fuzzy dot subalgebra of  $B$ -algebras.

**4.1 Definition** Let  $A$  be a fuzzy set in a  $B$ -algebra  $X$ . Then  $A$  is called a fuzzy normal dot subalgebra of  $X$  if  $\mu_A((x * a) * (y * b)) \geq \mu_A(x * y) \cdot \mu_A(a * b)$  for all  $x, y \in X$ .

**4.2 Definition** A fuzzy set  $A$  in  $X$  is called a fuzzy normal dot  $B$ -algebra if it is a fuzzy dot  $B$ -algebra which is fuzzy normal.

**4.3 Theorem** Every fuzzy normal dot subalgebra is a fuzzy dot subalgebra.

**Proof**

Let  $A$  be a fuzzy normal dot subalgebra and  $x, y \in X$ .

$$\begin{aligned} \text{Then } \mu_A(x * y) &= \mu_A((x * y) * (0 * 0)) \\ &\geq \mu_A(x * 0) \cdot \mu_A(y * 0) \\ \mu_A(x * y) &= \mu_A(x) \cdot \mu_A(y). \text{ Consequently, } A \text{ be a fuzzy dot subalgebra.} \end{aligned}$$

**4.4 Proposition** If a fuzzy set  $A$  in  $X$  is a fuzzy normal dot  $B$ -algebra with  $\mu_A(0) = 1$ , then  $\mu_A(x * y) = \mu_A(y * x)$  for all  $x, y \in X$ .

**Proof**

Let  $x, y \in X$ . Then by the definition of  $B$ -algebra,

$$\begin{aligned} \mu_A(x * y) &= \mu_A((x * y) * (x * x)) \\ &\geq \mu_A(x * x) \cdot \mu_A(y * x) \end{aligned}$$

$$= \mu_A(0) \cdot \mu_A(y * x)$$

$$\mu_A(x * y) = \mu_A(y * x)$$

**4.5 Theorem** Let  $A_1$  and  $A_2$  be two fuzzy normal dot subalgebras of  $X$ . Then  $A_1 \cap A_2$  is a fuzzy normal dot subalgebra of  $X$ .

**Proof**

Let  $x, y \in A_1 \cap A_2$ . Then  $x, y \in A_1$  and  $A_2$ .

Now,

$$\begin{aligned} \mu_{A_1 \cap A_2}((x * a) * (y * b)) &= \min\{\mu_{A_1}((x * a) * (y * b)), \mu_{A_2}((x * a) * (y * b))\} \\ &\geq \min\{\mu_{A_1}(x * y) \cdot \mu_{A_1}(a * b), \mu_{A_2}(x * y) \cdot \mu_{A_2}(a * b)\} \\ &= (\min\{\mu_{A_1}(x * y), \mu_{A_2}(x * y)\}) \\ &\quad \cdot (\min\{\mu_{A_1}(a * b), \mu_{A_2}(a * b)\}) \end{aligned}$$

$$\mu_{A_1 \cap A_2}((x * a) * (y * b)) = \mu_{A_1 \cap A_2}(x * y) \cdot \mu_{A_1 \cap A_2}(a * b)$$

Hence,  $A_1 \cap A_2$  is a fuzzy normal dot subalgebra of  $X$ .

**4.6 Theorem** Let  $\{A_i \mid i = 1, 2, 3 \dots n\}$  be a family of fuzzy normal dot subalgebra of  $X$ . Then  $\cap A_i$  is also a fuzzy normal dot subalgebra of  $X$ , where  $\cap A_i = \min \mu_{A_i}(x)$ .

**Proof**

Let  $x, y \in A_1 \cap A_2 \cap \dots \cap A_n$ . Then  $x, y \in A_1$  and  $A_2$  and ... and  $A_n$ .

$$\begin{aligned} \text{Now, } \mu_{A_1 \cap A_2 \cap \dots \cap A_n}((x * a) * (y * b)) &= \min \left\{ \begin{array}{l} \mu_{A_1}((x * a) * (y * b)), \dots, \dots, \\ \mu_{A_n}((x * a) * (y * b)) \end{array} \right\} \\ &\geq \min\{\mu_{A_1}(x * y) \cdot \mu_{A_1}(a * b), \dots, \mu_{A_n}(x * y) \cdot \mu_{A_n}(a * b)\} \\ &= (\min\{\mu_{A_1}(x * y), \mu_{A_2}(x * y), \dots, \mu_{A_n}(x * y)\}) \\ &\quad \cdot (\min\{\mu_{A_1}(a * b), \mu_{A_2}(a * b), \dots, \mu_{A_n}(a * b)\}) \end{aligned}$$

$$\mu_{A_1 \cap A_2 \cap \dots \cap A_n}((x * a) * (y * b)) = \mu_{A_1 \cap A_2 \cap \dots \cap A_n}(x * y) \cdot \mu_{A_1 \cap A_2 \cap \dots \cap A_n}(a * b)$$

Hence,  $A_1 \cap A_2 \cap \dots \cap A_n$  is a fuzzy normal dot subalgebra of  $X$ .

Hence,  $\cap A_i$  is also a fuzzy normal dot subalgebra of  $X$ .

### 5 FUZZY DOT IDEALS OF B-ALGEBRAS

In this section , fuzzy dot ideals of  $B$ -algebras are defined and studied some of its results.

**5.1 Definition** Let  $A$  be a fuzzy set in a  $B$ -algebra  $X$ . Then  $A$  is called a fuzzy dot ideal of  $X$  if it satisfies

$$B_4 \quad \mu_A(0) \geq \mu_A(x),$$

$$B_5 \quad \mu_A(x) \geq \mu_A(x * y) \cdot \mu_A(y) \quad \text{for all } x, y \in X.$$

**Example 2** Let  $X = \{0,1,2,3\}$  be a set with the following Cayley table:

*	0	1	2	3
0	0	1	2	3
1	1	0	3	2
2	2	3	0	1
3	3	2	1	0

Then  $(X, *, 0)$  is a  $B$ -algebra. Define a fuzzy set  $A$  in  $X$  by  $\mu_A(0) = \mu_A(1) = 0.8$ ,  $\mu_A(2) = 0.6$  and  $\mu_A(3) = 0.5$  for all  $x \in X$ . Then  $A$  is a fuzzy dot ideal of  $X$ .

Note that every fuzzy ideal of  $X$  is a fuzzy dot ideal of  $X$ , but the converse is not true. In fact, the fuzzy dot ideal in above example is not a fuzzy ideal, since  $0.5 = \mu_A(3) \not\geq \min\{0.6, 0.8\} = \min\{\mu_A(3 * 1), \mu_A(1)\}$ .

**5.2 Theorem** If  $A$  is a fuzzy ideal of  $X$  such that  $\mu_A(0 * x) \geq \mu_A(x)$  for all  $x \in X$ . Then  $A$  is a fuzzy dot ideal of  $X$ .

**Proof**

$$\text{Let } x, y \in X. \text{ Then } (x * y) * (0 * y) = x.$$

By the lemma, If  $X$  is a  $B$ -algebra, then  $(x * y) * (0 * y) = x$  for all  $x, y \in X$ .

We have

$$\begin{aligned} \mu_A(x) &= \mu_A((x * y) * (0 * y)) \\ &\geq \mu_A(x * y) \cdot \mu_A(0 * y) \\ \mu_A(x) &\geq \mu_A(x * y) \cdot \mu_A(y) \end{aligned}$$

Hence,  $A$  is a fuzzy dot ideal of  $X$ .

**5.3 Proposition** Let  $A$  be a fuzzy dot ideal of  $X$ . If  $x \leq y$  in  $X$ , then  $\mu_A(x) \geq \mu_A(0) \cdot \mu_A(y)$  for all  $x, y \in X$ .

**Proof**

$$\text{Let } x, y \in X \text{ such that } x \leq y. \text{ Then } x * y = 0, \text{ and thus}$$

$$\begin{aligned} \mu_A(x) &\geq \mu_A(x * y) \cdot \mu_A(y) \\ \mu_A(x) &= \mu_A(0) \cdot \mu_A(y). \end{aligned}$$

**5.4 Proposition** Let  $A$  be a fuzzy dot ideal of  $X$ . If the inequality  $x * y \leq z$  holds in  $X$ , then  $\mu_A(x) \geq \mu_A(0) \cdot \mu_A(y) \cdot \mu_A(z)$  for all  $x, y, z \in X$ .

**Proof**

Let  $x, y, z \in X$  such that  $x * y \leq z$ . Then  $(x * y) * z = 0$  and thus

$$\begin{aligned} \mu_A(x) &\geq \mu_A(x * y) \cdot \mu_A(y) \\ &\geq \mu_A((x * y) * z) \cdot \mu_A(z) \cdot \mu_A(y) \\ &= \mu_A(0) \cdot \mu_A(z) \cdot \mu_A(y) \\ \mu_A(x) &= \mu_A(0) \cdot \mu_A(y) \cdot \mu_A(z). \end{aligned}$$

**5.5 Proposition** If  $A$  is a fuzzy dot ideal of  $X$ , then  $A^m$  ( $m$  is any positive integer) is a fuzzy dot ideal of  $X$ .

**Proof**

For any  $x \in X$ ,  $A^m$  is a fuzzy set in  $X$  defined by  $A^m(x) = \mu_A^m(x)$ , where  $m$  is any positive integer. Let  $A$  be a fuzzy dot ideal of  $X$ .

Then  $\mu_A(0) \geq \mu_A(x)$  and  $\mu_A(x) \geq \mu_A(x * y) \cdot \mu_A(y)$  for all  $x, y \in X$ .

We have

$$\begin{aligned} \mu_A^m(0) &= [\mu_A(0)]^m \geq [\mu_A(x)]^m \\ \mu_A^m(0) &= \mu_A^m(x) \text{ and } \mu_A^m(x) = [\mu_A(x)]^m \\ &\geq [\mu_A(x * y) \cdot \mu_A(y)]^m \\ &= [\mu_A(x * y)]^m \cdot [\mu_A(y)]^m \\ \mu_A^m(x) &= \mu_A^m(x * y) \cdot \mu_A^m(y). \end{aligned}$$

Hence,  $A^m$  is a fuzzy dot ideal of  $X$ .

**5.6 Proposition** If  $A$  and  $A^c$  are both fuzzy dot ideal of  $X$ , then  $A$  is a constant function.

**Proof**

Let  $A$  and  $A^c$  be both fuzzy dot ideal of  $X$ .

Then  $\mu_A(0) \geq \mu_A(x)$  and  $\mu_A^c(0) \geq \mu_A^c(x)$

$$\begin{aligned} \Rightarrow 1 - \mu_A(0) &\geq 1 - \mu_A(x) \\ \Rightarrow \mu_A(0) &\leq \mu_A(x) \text{ for all } x \in X. \end{aligned}$$

Therefore,  $\mu_A(0) = \mu_A(x)$ . Hence,  $A$  is a constant function.

**5.7 Theorem** If  $A_1$  and  $A_2$  be two fuzzy dot ideals of  $X$ , then  $A_1 \cap A_2$  is also a fuzzy dot ideal of  $X$ .

**Proof**

Let  $A_1$  and  $A_2$  be two fuzzy dot ideals of  $X$ .

Then for any  $x \in X$ ,  $\mu_{A_1}(0) \geq \mu_{A_1}(x)$  and  $\mu_{A_2}(0) \geq \mu_{A_2}(x)$ .

Now  $\mu_{A_1 \cap A_2}(0) = \min\{\mu_{A_1}(0), \mu_{A_2}(0)\}$

$$\geq \min\{\mu_{A_1}(x), \mu_{A_2}(x)\}$$

$$\mu_{A_1 \cap A_2}(0) = \mu_{A_1 \cap A_2}(x).$$

Again for any  $x, y \in X$ ,

we have  $\mu_{A_1 \cap A_2}(x) = \min\{\mu_{A_1}(x), \mu_{A_2}(x)\}$

$$\geq \min\{\mu_{A_1}(x * y) \cdot \mu_{A_1}(y), \mu_{A_2}(x * y) \cdot \mu_{A_2}(y)\}$$

$$= (\min\{\mu_{A_1}(x * y), \mu_{A_2}(x * y)\}) \cdot (\min\{\mu_{A_1}(y), \mu_{A_2}(y)\})$$

$$\mu_{A_1 \cap A_2}(x) = \mu_{A_1 \cap A_2}(x * y) \cdot \mu_{A_1 \cap A_2}(y).$$

Hence,  $A_1 \cap A_2$  is a fuzzy dot ideal of  $X$ .

**5.8 Theorem** Let  $\{A_i \mid i = 1, 2, 3, \dots, n\}$  be a family of fuzzy dot ideal of  $X$ . Then  $\cap A_i$  is also a fuzzy dot ideal of  $X$ , where  $\cap A_i = \min \mu_{A_i}(x)$ .

**Proof**

Let  $A_1$  and  $A_2$  and ..... and  $A_n$  be fuzzy dot ideal of  $X$ .

Then for any  $x \in X$ ,  $\mu_{A_1}(0) \geq \mu_{A_1}(x)$  and  $\mu_{A_2}(0) \geq \mu_{A_2}(x)$  and ..... and  $\mu_{A_n}(0) \geq \mu_{A_n}(x)$ .

Now  $\mu_{A_1 \cap A_2 \cap \dots \cap A_n}(0) = \min\{\mu_{A_1}(0), \mu_{A_2}(0), \dots, \mu_{A_n}(0)\}$

$$\geq \min\{\mu_{A_1}(x), \mu_{A_2}(x), \dots, \mu_{A_n}(x)\}$$

$$\mu_{A_1 \cap A_2 \cap \dots \cap A_n}(0) = \mu_{A_1 \cap A_2 \cap \dots \cap A_n}(x).$$

Again for any  $x, y \in X$ , we have  $\mu_{A_1 \cap A_2 \cap \dots \cap A_n}(x) = \min\{\mu_{A_1}(x), \mu_{A_2}(x), \dots, \mu_{A_n}(x)\}$

$$\geq \min\{\mu_{A_1}(x * y) \cdot \mu_{A_1}(y), \mu_{A_2}(x * y) \cdot \mu_{A_2}(y), \dots, \mu_{A_n}(x * y) \cdot \mu_{A_n}(y)\}$$

$$= (\min\{\mu_{A_1}(x * y), \mu_{A_2}(x * y), \dots, \mu_{A_n}(x * y)\})$$

$$\cdot (\min\{\mu_{A_1}(y), \mu_{A_2}(y), \dots, \mu_{A_n}(y)\})$$

$$\mu_{A_1 \cap A_2 \cap \dots \cap A_n}(x) = \mu_{A_1 \cap A_2 \cap \dots \cap A_n}(x * y) \cdot \mu_{A_1 \cap A_2 \cap \dots \cap A_n}(y).$$

Hence,  $A_1 \cap A_2 \cap \dots \cap A_n$  is a fuzzy dot ideal of  $X$ .

Hence,  $\cap A_i$  is also a fuzzy dot ideal of  $X$ .

**6 CONCLUSION**

In the present paper, the notions of fuzzy dot subalgebras, fuzzy normal dot subalgebras and fuzzy dot ideals of  $B$ -algebras has been introduced and some important properties of it are also studied. We also proved that the  $n^{th}$  term of fuzzy dot subalgebras, fuzzy normal dot subalgebras and fuzzy dot ideals of  $B$ -algebras. In our opinion, the future study of fuzzy structure of  $B$ -algebras should be related to cubic subalgebras.



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