FUZZY e-COMPACTNESS AND FUZZY e-CLOSED SPACES

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ABSTRACT

The aim of this paper, we used fuzzy e-compactness and fuzzy e-closed spaces are investigated and also fuzzy filterbases are used to characterize these concepts.

Keywords: - *Fuzzy filterbases, fuzzy e-compactness, fuzzy e-closed spaces.*

1.INTRODUCTION AND PRELIMINARIES

The concept of fuzzy has invaded almost all branches of mathematics with theintroduction of fuzzy sets by Zadeh [14] of 1965. The theory of fuzzy topological spaces was introduced and developed by Chang[2]. In 2008, Erdal Ekici[8], has introduced the concept of *e*-open sets in general topology. V.Seenivasan and K.Kamala [11] introduced the concept of Fuzzy e-continuity and fuzzy e-open set 2013.And also in this e-open sets more results introduction by A.Vadivel and M.Palanisamy.For Eample A.Vadiveland M . Palanisamy [12] introduced the concept of fuzzy totally e-continuous function 2014. Using fuzzy flterbases, we characterize fuzzy e-compactness and fuzzy e-closed spaces. We also explore some expected basic properties of these

Definition 1.1

A fuzzy sets u in a fuzzy topological spaces X is said to be;

a) Fuzzy Semiopen set [1] if $u \leq cl\ (int\ (u))$
b) Fuzzy Preopen set [10] if $u \leq int\ (\ cl\ (u))$
c) Fuzzy e-open sets [11] if $\lambda \leq int\ \left(cl_\delta(\lambda)\right) \vee cl\ \left(int_\delta(\lambda)\right)$

It is obvious that each Semi-open and pre-open fuzzy sets implies e-open.

Definition 1.2 [11]

Let u be a fuzzy subset sets in a fuzzy topological spaces X. The fuzzy e-closure (ecl) and e-interior (eint) of u are defined as follows

$ecl\ (u) = \wedge\{A: u \leq A, A\ is\ e\text{-}\ closed\};$

$eint\ (u) = \vee\{A: u \geq A, A\ is\ e\text{-}open\};$

It is obvious that $ecl\ (u') = (eint\ (u))'$ and $eint\ u' = (ecl\ (u))'$.

Definition 1.3

A function $f: X \rightarrow Y$ is said to be fuzzy e-continuous [11] (resp. e-continuous) if the inverse image of every open (resp. e-open) fuzzy set in Y is e-open (resp. e-open) fuzzy set in X.

Lemma 1.4

Let $f: X \to Y$ be a function, then the followings are equivalent

a) f is fuzzy e-continuous.
b) $f(ecl\ u) \leq ecl\ f(u)$, for every fuzzy set u in X.

Proof

(a)⟹(b)

Let u be a fuzzy set of X, then $eclf(u)$ is e-closed. by (a) $f^{-1}\left(ecl\ f(u)\right)$ is e-closed and so

$$f^{-1}\left(ecl\ f(u)\right) = eclf^{-1}\left(ecl\ f(u)\right).$$

Since $u \leq f^{-1}\left(f(u)\right)$,

we have $ecl\ (u) \leq ecl\ [f^{-1}\left(f(u)\right)]$

$$\leq ecl\ [f^{-1}\left(ecl\ f(u)\right)]$$

$$ecl\ (u) = f^{-1}\left(ecl\ f(u)\right).$$

Hence $f(ecl\ u) \leq ecl\ f(u)$.

(b)⟹(a)

Let v be a fuzzy e-closed set in Y. by (b) if $u = f^{-1}(v)$,

Then $eclf^{-1}(v) \leq f^{-1}\left(ecl\ f\left(f^{-1}(v)\right)\right)$

$$\leq f^{-1}(ecl\ v)$$

$$ecl\ f^{-1}(v) = f^{-1}(v).$$

Since $f^{-1}(v) \leq eclf^{-1}(v)$

Then $f^{-1}(v) = ecl\ f^{-1}(v)$

Hence $f^{-1}(v)$ is fuzzy e-closed set in X.

Hence f is fuzzy e-continuous.

Lemma 1.5

Let $f: X \to Y$ be a function, then tha following are equivalent:

(a) f is fuzzy e-continuous.
(b) $f\left(ecl\ (u)\right) \leq cl\ f(u)$, for every fuzzy set u in X.

Proof

Obvious

Definition 1.6 [6].

A collection of fuzzy subsets ε of a X fuzzy topological spaces is said to form a fuzzy filterbases iff for every finite collection $\{A_j : j = 1, \ldots\ldots, n\}$,$\bigwedge_{j=1}^{n} A_j \neq 0_X$

Definition 1.7 [6].

A collection μ of a fuzzy sets in a fuzzy topological spaces X is said to be cover of a fuzzy set u of X iff $\left(\bigvee_{A\in\mu} A\right)(x) = 1_X$, for every $x \in S(u)$. A fuzzy cover μ of a fuzzy set u in a fuzzy topological spaces X is said to have a finite subcover iff there exists a finite subcollection $\eta = \{A_1 A_2 \ldots\ldots A_n\}$ of μ such that $\left(\bigvee_{j=1}^{n} A_j\right)(x) \geq u(x)$, for every $x \in S(u)$.

Definition 1.8

A fuzzy topological spaces X is said to be strongly compact [9] (resp. semicompact [20]) iff every preopen (resp. Semiopen) cover of X has a finite subcover.

Definition 1.9

A fuzzy topological spaces X is said to be almost compact [4] (resp. S-closed [6], s-closed [7], p-closed [13]) iff every open (resp. Semiopen, semi-closures, pre-closures) cover of X has a finite subcollection whose closuers (resp. closures, semi-closures, pre-closures) cover X .

2 FUZZY E-COMPACT SPACE

Definition 2.1 [11]

A fuzzy topological spaces X is said to be fuzzy e-compact iff for every family μ of e-open fuzzy sets such that $\bigvee_{A\in\mu} A = 1_X$ there is a finite subfamily $\eta \subseteq \mu$ such that $\bigvee_{A\in\eta} A = 1_X$.

Definition 2.2

A fuzzy set u in a fuzzy topological spaces X is said to be fuzzy e-compact relative to X iff for every family μ of e-open fuzzy sets such that $\bigvee_{A\in\mu} A \geq u(x)$ there is a finite subfamily $\eta \subseteq \mu$ such that $\bigvee_{A\in\eta} A \geq u(x)$ for every $x \in S(u)$.

Remark 2.3

Since each of Semiopen and preopen fuzzy set implies e-open, it is clear that every fuzzy e-compact space implies each of fuzzy strongly compact space and fuzzy semicompact space. But the converse need not be true.

Theorem 2.4

A fuzzy topological spaces X is e-compact iff for every collection $\{A_j : j \in J\}$ of e-closed fuzzy sets of X having the finite intersection property, $\bigwedge_{j\in J} A_j \neq 0_X$.

Proof

Let $\{A_j : j \in J\}$ be a collection of e-closed fuzzy sets with the finite intersection property.
Suppose that $\wedge_{j\in J} A_j = 0_X$.
Then $\vee_{j\in J} A_j' = 1_X$.
Since $\{A_j' : j \in J\}$ is a collection of e-open fuzzy sets cover of X, then from the e-compactness of X it follows that there exists a finite subset $F \subseteq J$ such that $\vee_{j\in J} A_j' = 1_X$.
Then $\wedge_{j\in F} A_j = 0_X$
which gives a contradiction and
Therefore $\wedge_{j\in J} A_j \neq 0_X$.

Conversely,

Let $\{A_j : j \in J\}$ be a collection of e-open fuzzy sets cover of X. Suppose that for every finite subset $F \subseteq J$, we have $\vee_{j\in F} A_j \neq 1_X$.
Then $\wedge_{j\in F} A_j' \neq 0_X$.

Hence $\{A_j' : j \in J\}$ satisfies the finite intersection property. Then from the hypothesis
we have $\wedge_{j\in J} A_j' \neq 0_X$

Which implies $\wedge_{j\in F} A_j \neq 1_X$ and
This contradicting that $\{A_j : j \in J\}$ is a e-open cover of X. Thus X is fuzzy e-compact.

Theorem 2.5

A fuzzy topological spaces X is fuzzy e-compact iff every filterbases ξ in X, $\wedge_{G\epsilon\xi}\, ecl\ G \neq 0_X$.

Proof

Let μ be a e-open fuzzy set cover of X and μ has no a finite subcover.
Then for every finite subcollection $\{A_1, \dots, A_n\}$ of μ, there exists $x\epsilon X$ such that $A_j(x) < 1$ for every $j = 1, \dots, n$.
Then $A_j'(x) > 0$,
So that $\wedge_{j=1}^{n} A_j'(X) \neq 0_X$.
Thus $\{A_j'(x) : A_j \in \mu\}$ forms a filterbases in X. Since μ is e-open fuzzy set cover of X ,
Then $(\vee_{A_j\epsilon\mu} A_j)(x) = 1_X$ for every $x \in X$ and
Hence $\wedge_{A_j\epsilon\mu}\, ecl\ A_i'(x) = \wedge_{A_j\epsilon\mu} A_j'(x) = 0_X$
which is a contradiction.
Then every e-open fuzzy set cover of X has a finite subcover and hence X is fuzzy e-compact.

Conversely,

suppose there exists a filterbases ξ such that $\wedge_{G\epsilon\xi}\, ecl\ G = 0_X$, so that $(\vee_{G\epsilon\xi}(ecl\ G)')(x) = 1_X$ for every $x\epsilon X$ and hence $\mu = \{(ecl\ G)' : G\epsilon\xi\}$ is a e-open fuzzy set cover of X.

Since X is fuzzy e-compact, then μ has a finite subcover.

Then $(\vee_{j=1}^{n}(ecl\ G_j)')(x) = 1_X$ and

Hence $(\vee_{j=1}^{n} G_j')(x) = 1_X$,

so that $\wedge_{j=1}^{n} G_j = 0_X$

which is a contradictions.
Since the G_j are members of filterbases ξ.
Therefore $\wedge_{G\epsilon\xi} ecl\ G \neq 0_X$ for every fileterbases ξ.

Theorem 2.6

A fuzzy set u in a fuzzy topological spaces X is fuzzy e-compact relative to X iff for every filterbases ξ such that every finite of members of ξ is quasi coincident with u, $(\wedge_{G\epsilon\xi} ecl\ G) \wedge u \neq 0_X$.

Proof

Let u not be fuzzy e-compact relative to X , then there exists a e-open fuzzy set μ cover of u such that μ has no finite subcover η. Then $\left(\vee_{A_j\epsilon\eta} A_j\right)(x) < u(x)$ for some $x\epsilon S(u)$,
So that $(\wedge_{A_j\epsilon\eta} A_j')(x) > u'(x) \geq 0$. Hence $\xi = \{A_j' : A_j\epsilon\mu\}$ forms a filterbases and $\wedge_{A_j\epsilon\eta} A_j' q u$.
By hypothesis $(\wedge_{A_j\epsilon\eta} ecl A_j') \wedge u \neq 0_X$ and hence $(\wedge_{A_j\epsilon\eta} A_j') \wedge u \neq 0_X$.
Then for some $x \epsilon S(u)$, $\left(\wedge_{A_j\epsilon\mu} A_j'\right)(x) > 0_X$,
That is $(\vee_{A_j\epsilon\mu} A_j)(x) < 1_X$,
which is a contradiction.
Hence u is fuzzy e-compact relative to X.
conversely,
Suppose that there exists a filterbases ξ such that every finite of members of ξ is quasi coincident with u and $(\wedge_{G\epsilon\xi} ecl\ G) \wedge u \neq 0_X$.Then for every $x \epsilon S(u)$, $(\wedge_{G\epsilon\xi} ecl\ G)(x) = 0_X$ and hence $(\vee_{G\epsilon\xi} (ecl\ G)')(x) = 1_X$ for every $x \epsilon S(u)$. Thus $\mu = \{(ecl\ G)' : G \epsilon \xi\}$ is e-open fuzzy set cover of u.
Since u is fuzzy e-compact relative to X, then there exists a finite subcover, say $\{(ecl\ G_1)', \dots, (ecl\ G_n)'\}$, such that $\left(\vee_{j=1}^{n} (ecl\ G_j)\right)'(x) \geq u(x)$ for every $x \epsilon S(u)$.
Hence $(\wedge_{j=1}^{n}(ecl\ G_j))(x) \leq u'(x)$ for every $x \epsilon S(u)$,
so that $\wedge_{j=1}^{n}(ecl\ G_j) \tilde{q}\, u$,
which is a contradiction.
Therefore for every filterbases ξ such that every finite of members of ξ is quasi coincident with u, $(\wedge_{G\epsilon\xi} ecl\ G) \wedge u \neq 0_X$.

Theorem 2.7

Every e-closed fuzzy subset of a fuzzy e-compact space is fuzzy e-compact relative to X.

Proof

Let ξ be a fuzzy filterbases in X such that $uq \wedge \{G : G\epsilon\lambda\}$ holds for every finite subcollection λ of ξ and a e-closed fuzzy set u. Consider $\xi^* = \{u\} \cup \xi$.
For any finite subcollection λ^* of ξ^*, if $u \notin \lambda^*$, then $\wedge \lambda^* \neq 0_X$.
If $u\epsilon\lambda^*$ and since $uq \wedge \{G : G \in \lambda^* - u\}$, then $\wedge \lambda^* \neq 0_X$.
Hence λ^* is a fuzzy filterbases in X.
Since X is fuzzy e-compact, then $\wedge_{G\epsilon\xi^*} ecl \neq 0_X$,

So that $(\wedge_{G\epsilon\xi} ecl\ G) \wedge u = (\wedge_{G\epsilon\xi} ecl\ G) \wedge ecl\ u$

$= 0_X$

Hence by theorem 2.6, we have u is fuzzy e-compact relative to X.

Theorem 2.8

If a function $f : X \longrightarrow Y$ is fuzzy e-continuous and u is fuzzy e-compact relative to X, then so is $f(u)$.

Proof

Let $\{A_j : j \in J\}$ be a e-open fuzzy set cover of $S(f(u))$.

For $x \epsilon S(u), f(x) \epsilon f(S(u) = S(f(u))$. Since f is fuzzy e-continuous, then $\{f^{-1}(A_j) : j \in J\}$ is e-open fuzzy set cover of $S(u)$. Since u is fuzzy e-compact relative to X, there is a finite subfamily $\{f^{-1}(A_j) : j = 1, \dots, n\}$ such that $S(u) \leq \vee_{j=1}^{n} f^{-1}(A_j)$.

which implies $S(u) \leq f^{-1}(\vee_{j=1}^{n} A_j)$ and

That $S(f(u)) = f(S(u))$

$\leq f f^{-1}(\vee_{j=1}^{n} A_j)$

$\leq \vee_{j=1}^{n} A_j$

Therefore $f(u)$ is fuzzy e-compact relative to Y.

Lemma 2.9

If $f : X \to Y$ is fuzzy open and fuzzy continuous function , then f is fuzzy e-continuous.

Proof

Let v be an e-open fuzzy set in Y,

That $v \leq cl\ int\ cl\ v$.

So $f^{-1}(v) \leq f^{-1}(cl\ int\ cl\ v)$

$\leq cl(f^{-1}(int\ cl\ v))$.

Since f is fuzzy continuous,

Then $f^{-1}(int\ cl\ v) = int(f^{-1}(cl\ v))$.

$f^{-1}(int\ cl\ v) = int(f^{-1}(int\ cl\ v))$

$\leq int(f^{-1}(cl\ v))$

$f^{-1}(int\ cl\ v) \leq int\ cl\ (f^{-1}(v))$.

Thus $f^{-1}(v) \leq cl(f^{-1}(int\ cl\ v))$

$f^{-1}(v) \leq cl\ int\ cl(f^{-1}(v))$.

Definition 2.10

A function $f : X \to Y$ is said to be fuzzy e-open iff the image of every e-open fuzzy set in X is e-open in Y.

Theorem 2.11

Let $f: X \rightarrow Y$ be a fuzzy e-open bijective function and Y is fuzzy e-compact, then X is fuzzy e-compact.

Proof

Let $\{A_j : j \in J\}$ be a collection of e-open fuzzy set cover of X, then $\{f(A_j): j \in J\}$ is e-open fuzzy set covering of Y. Since Y is fuzzy e-compact, there is a finite subset $F \subseteq J$ such that $\{f(A_j): j \in F\}$ is an cover of Y.

But $1_X = f^{-1}(1_Y)$

$= f^{-1}f(\vee_{j \in F} A_j)$

$= \vee_{j \in F} A_j$

Therefore X is fuzzy e-compact.

3.FUZZY E-CLOSED SPACES

Definition 3.1 [17].

A fuzzy set u in a fuzzy topological spaces X is said to be a eq-nbd of a fuzzy point x_t in X if there exists a e-open fuzzy set $A \leq u$ such that $x_t q A$.

Theorem 3.2

Let x_t be a fuzzy point in a fuzzy yopological spaces X and u be any fuzzy set of X, then $x_t \in ecl\ u$ iff for every eq-nbd H of x_t, Hqu.

Proof

Let $x_t \in ecl\ u$ and there exists a e-q-nbd H of $x_t, H\tilde{q}u$. Then there exists a e-open fuzzy set $A \leq H$ in X such that $x_t q A$, which implies $A\tilde{q}u$ and hence $u \leq \acute{A}$. Since $\acute{A}$ is e-closed fuzzy set, then $ecl\ u \leq \acute{A}$.
Since $x_t \notin \acute{A}$, then $x_t \notin ecl\ u$, which is a contradiction.
Conversely ,
Let $x_t \notin ecl\ u = \wedge\{A: A\ is\ e - closed\ in\ X, A \geq u\}$. Then there exists a e-closed fuzzy set $A \geq u$ such that $x_t \notin A$. Hence $x_t q \acute{A} = H$, where H is a e-open fuzzy set in X and $H\tilde{q}u$. Then there exists a $eq - nbd\ H\ of\ x_t$ with $H\tilde{q}u$. Hence the result.

Definition 3.3

A fuzzy topological spaces X is said to be e-closed iff for every family μ of e-open fuzzy set such that $\vee_{A \epsilon \mu} A = 1_X$ there is a finite subfamily $\eta \subseteq \mu$ such that $(\vee_{A \epsilon \eta} ecl\ A)(x) = 1_x$, for every $x \epsilon X$.

Remark 3.4

From the above definition and other types of fuzzy compactness, one can draw the following diagram:

F-semicompact → F s-closed → F s-closed

↑ ↑

Fe-compact → F e-closed

↓ ↓

F-strongly compact → FP-closed

Where F=fuzzy.

Example 3.5

Let $X \neq 0_X$ be a set and $u_n(x) = 1 - \frac{1}{n}$ for every $x \in X$ and $n \in N^+$.

The collection $\{u_n : n \in N^+\}$ is a base for a fuzzy topology on X. The collection $\{u_n : n \in N^+\}$ is obviously a e-open fuzzy set cover of X. On the other hand we have $ecl\ u = 1_X$ for every $n \geq 3$. Hence X is e-closed but not fuzzy e-compact,(see [5]).

4.CONCLUSION

We have discussed about the fuzzy e-compactness and fuzzy e-closed spaces are investigated and also fuzzy filterbases are used to characterize these concepts .A comparison between these types and some different forms of compactness in fuzzy topology is established. In my future research work a new fuzzy filter is presented for the noise reduction of image corrupted with additive noise. The filter consists of two stages. The first stage computes a fuzzy derivative for eight different directions. The second stage uses these fuzzy derivatives to perform fuzzy smoothing by weighting the contributions of neighboring pixel values. Both stages are based on fuzzy rules which make use of membership function. The filter can be applied iteratively to effectively reduce heavy noise. In particular, the shape of the membership functions is adapted according to the remaining noise level after each iteration, making use of the distribution of the homogeneity in the image. A statistical model for the noise distribution can be incorporated to relate the homogeneity to the adaptation scheme of the membership functions. Experimental results are obtained to show the feasibility of the proposed approach. These results are also compared to other filters by numerical measures and visual inspection.

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