# Forming Limit Diagram for Sheet Metal Forming: Review

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**Abstract**

*The accurate description of forming behavior and simulation modeling in deep drawing is indeed essential and it is only possible through having a thorough knowledge of anisotropic behavior of sheet material. In essence, it is possible only by an accurate prediction of sheet metal formability characteristics and the use of correct constitutive model that describes the material behavior under complex loading conditions. An attempt has been made to depict the methodology involved in construction of forming limit diagram of a light weight magnesium alloy AZ31. The construction of FLD by evaluation of limit strains is based on the in-homogeneity factor induced in the sheet metal according to the most widely used M-K model.*

**Keywords***: Simulation Modeling, Deep Drawing, FLD, magnesium alloy AZ31, M-K model*

## 1. Introduction

Most of sheet metal parts are manufactured via sheet metal forming. One of the most widely used sheet metal forming processes is the deep drawing involves a hydraulic or mechanical press in situ having a specially-shaped punch into a matching die with a piece of blank sheet metal in between. Exemplary products made from this process include, but are not limited to, car hood, fender, door, automotive fuel tank, kitchen sink, aluminum can, etc. In deep drawing, the depth of a part being made is generally more than half its diameter. As a result, the blank stretches, thinning in various locations due to the geometry of the part. The part is only good when there are no structural defects such as cracking, tearing, wrinkling, necking, etc. The ability of sheet metal to deform plastically during the forming process can be thoroughly quantified by means of formability. The formability of blank is mainly limited by the occurrence of flow localization and / or plastic instability. In essence, every sheet metal can be deformed without failing only up to a certain limit which is known as forming limit curve. The accuracy of the simulation results are essentially influences the accuracy of the model. Hence, it is essential to develop new material model that is capable of describing the anisotropic behavior as accurate as possible. The computer simulation of the sheet metal forming process needs a quantitative description of the plastic anisotropy by the yield locus. Forming limit curve (FLC) is generally governed by localized necking that eventually leads to ductile fracture. FLC can be empirically determined in the space of principal in-plane strains that defines the boundary between safe strains, where no necking occurs, and unsafe strains prone to necking and sheet rupture. FLC can be represented as a curve of the major strain ($\varepsilon_1$) at the onset of localized necking for all values of the minor strain ($\varepsilon_2$), and the full graph is called as forming limit diagram (FLD). The FLD predictions are strongly influenced by the shape of the yield locus used in the computational model. In 1948, Hill proposed the first yield criterion for anisotropic materials. The mathematical shape of the criterion is a simple quadratic function and the coefficients can be analytically computed. Due to this characteristic, it is the most widely used plasticity model. Since then, many other yield criterions have been proposed in order to improve the fit with experimental data. The experimental research shows that the mechanical parameters are not rigorously constant and knowing the variability of these material characteristics would allow performing numerical simulations not only for the average values, but also for the extreme ones. After the experimental determination of the mechanical parameters, a study related to the influence of this variability upon FLC can be performed. Having this information acquired, a forming limit band can be calculated [1].The aim of this paper is to describe the forming limit band for light weight magnesium alloy AZ31 based on the dispersion of the uniaxial yield stresses and anisotropic coefficients determined at 0°, 45° and 90° with respect to the rolling direction

## 2. Literature review

Room temperature ductility and formability of rolled Mg-Gd-Zn alloy sheets were conducted by D Wu et al., 2011 and researched that these sheets exhibit a large elongation-to-fracture i.e., upto 50%, uniform elongation larger than 30% and a high Eriction value of 8 at room temperature, due to the excellent strain hardening capability, high n value and low r value [2]. D Banabic et al [3] tested advanced materials models as implemented in the finite-element code. The influence of the numbers of the mechanical parameters on the accuracy of the sheet forming simulation has been studied for two materials DC04 steel grade and Ac121-T4 aluminum alloys. The results demonstrate that for an accurate prediction of the sheet metal forming simulation it is crucial to take not only the uni-axial yield stresses and r-values but also the biaxial yield stress into account [4]. M Tisza et al [5] investigated theoretically and experimentally for the forming limit diagrams as a special field of the formability of sheet metals. Various aspects of damage limitations i.e., fracture, local necking and diffuse necking were studied as the limits of formability

The forming limit diagram (FLD) introduced by Keeler and Backofen (1965) and Goodwin (1968) is a constructive concept for characterizing the formability of sheet metal. It has proved to be an essential tool for material selection, design and try out of the tools for deep drawing operations. Sheet metal forming processes often impose forming sequences with severe strain-path changes that drastically influence the forming limits. The deformation mode, loading history and material behavior are essential factors that affect maximum admissible strains. For non proportional strain paths, FLDs are very useful tools to understand the behavior of the material upon complex loading, to estimate the severity of the strain paths imposed to the work-piece and to optimize the shape of the dies to avoid necking. The Marciniak-Kuczinsky (MK) approach (1967) that supposes an infinite sheet containing a region of local imperfection where heterogeneous plastic flow develops and localizes, has become one of the most important tools in predicting the sheet metal formability. The predicted limit strains strongly depend on the constitutive law incorporated in the MK analysis [6, 7,].

## 3. Constitutive Models for Metal Forming

The most popular isotropic yield conditions, verified for many metals, were proposed by Tresca and Von Mises and may be expressed in terms of the principal values of the stress ( $\sigma_i$ ) or the deviatoric stress ( Sj) tensors as

$$\left.\begin{aligned}\emptyset &= |\boldsymbol{\sigma_1}-\boldsymbol{\sigma_2}|^{a}+|\boldsymbol{\sigma_2}-\boldsymbol{\sigma_3}|^{a}+|\boldsymbol{\sigma_3}-\boldsymbol{\sigma_1}|^{a}\\ &= |\boldsymbol{S_1}-\boldsymbol{S_2}|^{a}+|\boldsymbol{S_2}-\boldsymbol{S_3}|^{a}+|\boldsymbol{\sigma_3}-\boldsymbol{\sigma_1}|^{a}\\ &= 2\bar{\boldsymbol{\sigma}}^{a}\end{aligned}\right\} \qquad (1)$$

where $\bar{\sigma}$ defines the effective stress.

For an exponent a = 2 or a = 4 in Eq.1 reduces to Von Mises, whereas for a = 1 it leads to Tresca yield condition.

$$\left.\begin{aligned}\emptyset &= |\boldsymbol{\sigma_1}-\boldsymbol{\sigma_2}|^{a}+|\boldsymbol{\sigma_2}-\boldsymbol{\sigma_3}|^{a}+|\boldsymbol{\sigma_3}-\boldsymbol{\sigma_1}|^{a}\\ &= 2\bar{\boldsymbol{\sigma}}^{a}\end{aligned}\right\} \qquad \textbf{(2)}$$

In terms of deviatoric stress ($S_j$) tensors as

$$|\boldsymbol{S_1}-\boldsymbol{S_2}|^{a}+|\boldsymbol{S_2}-\boldsymbol{S_3}|^{a}+|\boldsymbol{\sigma_3}-\boldsymbol{\sigma_1}|^{a}=2\bar{\boldsymbol{\sigma}}^{a} \qquad (3)$$

defines the effective stress. For an exponent a = 2 or a = 4 in Equation 1 reduces to Von Mises, whereas for a = 1 it leads to Tresca yield condition. For isotropic materials yield criteria have the same firm in any reference frame. For anisotropic materials yielding properties are directional and thus the expression depends on the reference frame. The simplest form of yield criterion is with respect to a coordinate system associated with the axes of symmetry of the material. Hill proposed an extension of the isotropic Mises criterion to orthotropic materials In this equation F, G, H, L, M and N are material constants. The validity of this yield function has been explored in numerous experiments and it is well suited to specific metals and textures like steel. Hill also proposed an extension of the isotropic Mises criterion to materials exhibiting planar isotropy for plane stress states

$$\left.\begin{aligned}\phi &= \left|\sigma_1+\sigma_2\right|^a + \left(1+2\overline{r}\right)\left|\sigma_1-\sigma_2\right|^a \\ &= 2(1+\overline{r})\overline{\sigma}^a\end{aligned}\right\} \quad (4)$$

For the materials exhibiting orthotropic symmetry Hosford proposed the yield criteria as

$$\left.\begin{aligned}\phi = F\left|\sigma_{yy}-\sigma_{zz}\right|^a + G\left|\sigma_{zz}-\sigma_{xx}\right|^a \\ +\left|\sigma_{xx}-\sigma_{yy}\right|^2 \\ = \overline{\sigma}^a\end{aligned}\right\} \quad (5)$$

The important drawback of Hill's criteria is that they do not take into account the shear stresses and hence it cannot useful where the plastic properties the material continuously varies. Metals with hexagonal close packed (HCP) crystal deform plasticity by slip and twinning. As opposed to slip, twinning is a directional shear mechanism. Shear in one direction can produce twinning while shear in the opposite direction cannot. Thus yield surface are not symmetric w.r.t. the stress free condition. Since HCP metal sheets exhibit strong basal textures a pronounced anisotropy in yielding is observed. To account for both strength differential (SD) effect and the anisotropy displaced by HCP metals. Hosford proposed the following modification of Hill's orthotropic yield criterion.

$$\left.\begin{aligned}\boldsymbol{A\sigma_{xx} + B\sigma_{yy} - (A+B)\sigma_{zz} + F\left(\sigma_{yy}-\sigma_{zz}\right)^2} \\ \boldsymbol{+G(\sigma_{zz}-\sigma_{xx})^2 + H(\sigma_{xx}-\sigma_{yy})^2 = 1}\end{aligned}\right\} \quad (6)$$

where A, B, F, G, H are material coefficients and x, y, z, are normal to the mutually orthogonal planes of symmetry of the material. Since the criterion does not involve shear stresses, it cannot account for the continuous variation of the plastic properties between the material axes of symmetry.

## 4. Various Experimental Tests

In most of the sheet metal forming processes, the sheet metal is subjected to multi-axial loads. Therefore multi-axial loading experiments are highly desirable for validating the plasticity models to be used for simulations. Servo-controlled testing machines are essential for such experiments. Some of the important experimental methods used for the measurement of multi axial stresses that are helpful in modeling of anisotropic plastic behavior of the sheet are described here.

### a. Uniaxial Tensile Test

Tensile test specimens were cut as per ASTM E8 standard. At least two samples at each direction ($0^0$, $45^0$, $90^0$) with respect to rolling directions were tested according to ASTM E517-00 standard [8]. Tensile test was carried out under constant strain rate of $1\times10^{-3}$ s at room temperature. Although r-value is introduced as the ratio of width strain $\varepsilon_w$ to thickness strain $\varepsilon_t$, the thickness strain in thin sheets can't be accurately measured. Hence, by measuring longitudinal strain $\varepsilon_l$ and width strain $\varepsilon_w$ and by implementing the principle of volume constancy i.e,

$$\varepsilon_l + \varepsilon_w + \varepsilon_t, = 0 \quad (7)$$

from the above equation

$$\boldsymbol{\varepsilon_t, = -(\varepsilon_l + \varepsilon_w)} \quad (8)$$

The strain ratio, r-value can be calculated for different directions as r0, r45 and r90 as

$$r_{x} = \varepsilon_{w,x} / \varepsilon_{t,x} \quad (9)$$

where, x is angle relative to the rolling direction. Subsequently, the normal anisotropy

$$r = \frac{\varepsilon_0 + 2\varepsilon_{45\_} + \varepsilon_{90}}{4} \qquad (10)$$

and planar anisotropy

$$\Delta r = \frac{\varepsilon_0 - 2\varepsilon_{45\_} + \varepsilon_{90}}{2} \qquad (11)$$

Can be calculated can be calculated according to ASTM E517-00.

### 4.2. Biaxial compression test

Biaxial compression tests are effective in observing yielding behavior in the π plane. One of the disadvantages of the biaxial compression test is that the difficulty in obtaining accurate stress-strain relations due to friction between the specimen and tool. In addition, whenever the plastic deformation mechanism of the material is influenced by the hydraulic component of stress, yield locus shapes can be obtained from the results of biaxial tension test.

## 4.3. Biaxial tension test using cruciform specimen

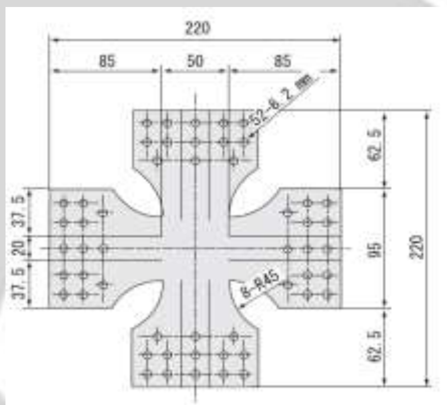


Figure 2: Biaxial tension test specimen (mm)

A servo controlled hydraulic testing machine is used for biaxial testing machine in which cruciform specimen is bi-axially stretched in X direction (rolling direction) and Y direction (transverse direction T D). In order to obtain uniform deformation at the centre of the cruciform specimen in the arbitrary multi-axial stress conditions, a feedback load control is applied in the hydraulic servo system. Tensile loads are measured by load cell and displacements are measured by displacement transducers. Biaxial strains were measured with strain gauges bonded on the surface of the specimen. A variety of cruciform specimens for biaxial tension tests had been proposed in the literature.

Cruciform specimens are suitable for studying the forming behavior of sheet metals in small plastic strain range of less than seven percent. There are different types of biaxial tension specimens classified as Type A, Type B and Type C. Type A specimens can be made of as-received sheet materials. The arms of this type have no slits. Type B specimens have a gauge section thinner than the periphery and this thicker periphery may prohibit uniform deformation of the gauge section. The type C specimens can be made of as received sheet materials but the merit of type C specimen is that it is simple to determine biaxial stress components in the gauge section by virtue of slits in the arms or welded thin strips.

### 5. Experimental Determination of Mechanical Properties

The dispersion of the mechanical parameters can be analyzed for the case of a magnesium alloy AZ31 sheet, with a nominal thickness of 0.7 mm. In order to establish the mechanical parameters of the AZ31, uni-axial tensile can be performed along three directions corresponding to 0°, 45° and 90° angles measured from the rolling direction as shown in the Figure 2 in order to investigate the material characteristics in all directions. Table 1 shows the average values of the mechanical parameters. The Swift's hardening law used can be as follows.

$$Y = K(\varepsilon_0 + \bar{e})^n \tag{12}$$

Due to the dispersion of the mechanical parameters, the strains of the sheet metals spread between an upper and a lower boundary defining a forming limit band. In order to calculate the band, the influence of each mechanical parameter should be studied. In the case of magnesium alloy AZ31 sheet the computational tests have shown that increased values of the parameters n, $r_0$, $Y_0$ and $Y_{90}$ cause the raising of the limit strains. On the other hand, an increased value of the parameter $r_{90}$ will reduce the formability.

Table 1. Tensile properties of AZ31 in different loading directions at room temperature. [9]

| Tensile direction | Tensile Yield stress (MPa) | Ultimate tensile stress (MPa) | Elon gation (%) | r-value |
|---|---|---|---|---|
| RD | 181 | 304 | 22.73 | 1.09 |
| $45^0$ | 166 | 302 | 25.15 | 1.02 |
| TD | 192 | 310 | 20.24 | 1.17 |

The computation can be performed using the alternate formulation of Hora's model coupled with Hill 1948 formulation of the equivalent stress and Swift's hardening law. In order to evaluate the performance of the necking criterion, some experimental points to be plotted on the diagram shown in Fig 5. These points represent limit strains determined through bulging and punch stretching tests.

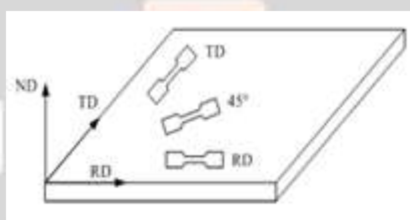


Figure 3: Geometry of uni-axial tension tests

## 6. Marcniak and Kuczyanki ( MK Model)

The first theoretical method was proposed by Swift and Hill assuming that the homogeneous sheet metals fails due to either by diffuse necking or by localized necking. Shortly after the publication of FLD concept on the basis of strain localization concept, Marcinik (1965) and Marciniak and Kuczynski (1967) were proposed a model taking into account the sheet metals are non-homogeneous nature on both geometrical and microstructure point of view. This method introduced in 1967 is known as M-K method. It assumes an inclined band in the investigated plane sheet with smaller thickness which denotes an imperfection. In this model limit strains can be calculated for non-proportional forming. But the disadvantage of this model is that the calculated limit strains are sensitive to the magnitude of imperfection. Later, different forming concepts were came into existence. Storen and Rice were developed a model based on the bifurcation theory. In addition to that Dudzinski and Molinari used the method of linear perturbations for analyzing the strain localization and computing their limit strains. At present the most widely used model for the computation of the limit strains remains is the same as proposed by Marciniak and Kuczynski.

According to Marciniak's hypothesis, sheet metal may have manufacturing and geometrical imperfections such as either thickness or structural imperfections like inclusions, gaps etc. In the forming process these imperfections progressively evolves and the plastic forming of the sheet metal is almost completely localized in them leading to the necking of the sheet metal. Marciniak also made a deep analysis of the strain localization phenomenon from the right side of the FLD and extended his initial model to cover this area. In M-K method, it is to be assumed that the necking can usually be initiated by a geometrical non-homogeneity of the material and it is associated with the variation of the sheet thickness. This variation is usually due to some defects in the theoretical procedure used to obtain the sheet metal. The theoretical model that was proposed by Marcinak and Kuczynski is as shown in Figure 4.

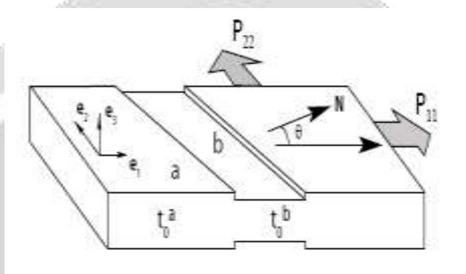


Figure 4: Geometric shape of M-K imperfection model

It is assumed in this approach that the specimen has two regions. The region 'a' is having a uniform thickness $t_0^a$, and region 'b' is having the thickness $t_0^b$. The initial geometrical non-homogeneity of the specimen is described by the so-called 'coefficient of geometrical non-homogeneity' f, expressed as the ratio of the thickness in the two regions as f = $t_0^b/ t_0^a$. In M-K model, the strain and stress states in the two regions are analyzed and the principle strain $\boldsymbol{\varepsilon_1}^{\mathbf{b}}$ in region "b" in relation with the principal strain $\boldsymbol{\varepsilon_1}^{\mathbf{a}}$ in region 'a' is monitored. When the two ratio of these strains $t_0^b/ t_0^a$ becomes too large, one may consider that the entire straining of the specimen is localized in region 'b'. The shape and position of the curve $\varepsilon_1^a - \varepsilon_1^b$ depends on the value of the coefficient f if f = 1, the curve becomes coincident with the first bisector. The general state of the material is described by the power law

$$\bar{\sigma} = K\bar{\varepsilon}^n \, \dot{\varepsilon}^m \tag{13}$$

Where n is the strain hardening coefficient, m is the strain sensitivity coefficient. The ratio of principal stresses and strains are defined as

$$\frac{\sigma_y}{\sigma_x}, \rho = \frac{\varepsilon_y}{\varepsilon_x} = \frac{d\varepsilon_y}{d\varepsilon_x} \tag{14}$$

the effective stress and strain are defined as

$$\bar{\sigma}\bar{\varepsilon} = \varepsilon_x\varepsilon_x + \varepsilon_y\varepsilon_y = \sigma_x\varepsilon_x(1 + \alpha\rho) \tag{15}$$

The associative flow rule is given by:

$$d\varepsilon_{ij} = d\lambda \frac{\partial\bar{\sigma}}{\partial\sigma_{ij}} \tag{16}$$

From the associative flow rule and the constant volume condition

$$d\varepsilon_x + d\varepsilon_y + d\varepsilon_z = 0 \tag{17}$$

expressions for $d\varepsilon_x , d\varepsilon_y , d\varepsilon_z$ are obtained.

The MK model incorporates a compatibility condition

$$d\varepsilon_y{}^A = d\varepsilon_y{}^B \tag{18}$$

Furthermore, the sheet metal being deformed will always be in equilibrium. This is represented by the force balance equation

$$\varphi_A(\bar{\varepsilon}^A + d\bar{\varepsilon}^A)\varepsilon^{\dot{m}}{}_A = f\varphi_B(\bar{\varepsilon}^B + d\bar{\varepsilon}^B\varepsilon^{\dot{m}}{}_B) \tag{19}$$

where $\varphi = \frac{\sigma_x}{\bar{\sigma}}$ and $f = \frac{t_A}{t_B}$, $\mathbf{t_A}$, $\mathbf{t_B}$ denote the instantaneous thickness of regions A and B. this ratio can be found by using the equation:

$$f = f_0 \exp(\varepsilon_Z{}^A - \varepsilon_Z{}^B) \tag{20}$$

Initially values of $f_0$ and ρ are assumed. Small strain increments of $\varepsilon_x{}^B$ are imposed in the groove region. The values of the $\boldsymbol{d\varepsilon_y{}^A}, \boldsymbol{d\varepsilon_y{}^B}$ are found using the corresponding equations described above. Assuming a value for $\boldsymbol{d\varepsilon_y{}^A}$ , the values of $\boldsymbol{d\varepsilon_y{}^A}, \boldsymbol{d\varepsilon_x{}^A}$ are computed. The equality of the force balance equation is checked. If the equality is satisfied, then the necking criterion is checked. If the necking criterion is also satisfied, then that particular strain state of region A corresponds to a point on the FLC. If the assumed value of $\boldsymbol{d\varepsilon_x{}^A}$ does not correspond to equal values of left and right hand sides of the force balance equation, the assumed value is changed and the process is repeated. This procedure is done for different values of $f_0$ and ρ to plot the full FLC.

Thus this theory cannot model the strain localization for geometrically homogeneous sheets. The value of the principal strain $\varepsilon_1{}^a$ in region "a" corresponding to non-significant straining of this region as compared to region "b" represents the limit strain $\varepsilon_1{}^a$. This strain together with the second principal strain $\varepsilon_2{}^a$ in region "a" defines a point belonging to the FLC. Assuming different strain ratios $\varepsilon = d\varepsilon_2/d\varepsilon_1$, one obtains different points on the FLC. Spanning the range $0 < \varepsilon < 1$, one gets the FLC for biaxial tension ($\varepsilon_1 > 0$, $\varepsilon_2 > 0$). In this domain, the orientation of the geometrical non-homogeneity with respect to the principal directions is assumed to be the same during the entire forming process.

## 7. Evaluation of Forming Limit Diagram

A superior way to assess the formability of sheets based on experiments prescribing different strain paths and limiting major strains determined independent of the minor strains. A hemispherical punch can be applied on a work-piece until the strain localization or fracture initiation takes place in the sheet. The work-piece has to securely clamped at the outer periphery between holding and drawing dies and the test results were represented by the forming limit curve (FLC) in the forming limit diagram (FLD).

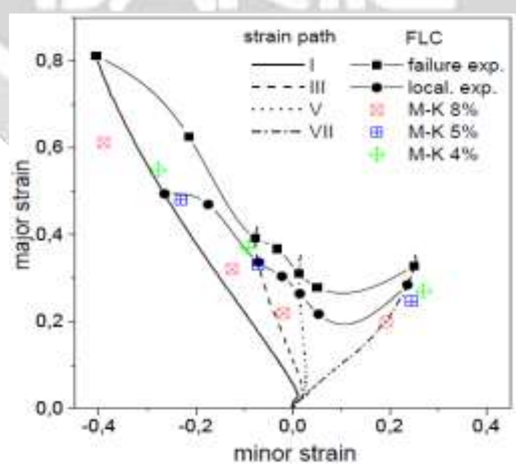


Figure 5: FLD representing experimental and numerically predicted limit strains at different geometrical imperfections [10]

## 8. Conclusions

Different models can be used for evaluation of FLC in order to predict limit strains accurately. The results of M-K model under different geometrical imperfections predicted are to be in good agreement with the standard results found in the literature.

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