Fuzzy Almost Contra α-Continuous Function

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Abstract

In this paper we introduced and investigate Fuzzy Almost contra α-Continuous Function and fuzzy α-normal and related properties. And also we used to defined fuzzy α-connected fuzzy strongly normal and fuzzy α-continuous.

Key words:

Fuzzy -closed space, contra- α-continuous, fuzzy contra α-continuity, α-open set, fuzzy contra continuity.

1.Introduction

Joseph and kwack [7] introduce (θ, s)-continous function in order to investigate S-closed space due to Thompson [13] a function f is called (θ, s) continuous if inverse image of each regular open set is closed.Moreover, chang introduced fuzzy s-closed space in 1968.The purpose of this paper is to introduce forms of fuzzy almost contra continuous function and to investigate properties and relationships of fuzzy almost contra-α-continuous function.Also, by using this paper, properties of fuzzy almost contra-continuous function, fuzzy almost contra-free continuous function and fuzzy almost contra-semi continuous function can be obtained with similar way.The class of fuzzy sets on a universe X will be denoted by I^X and fuzzy sets on X will be denoted by Greek letters as $\mu, \rho, \eta,$ etc.A family τ of fuzzy sets in X is called a fuzzy topology for X if and only if (1) $\theta, X \in \tau$,(2) $\mu \wedge \rho \in \tau$ whenever $\mu, \rho, \in \tau$ (3) ,If $\mu_i \in \tau$ for each i$\in I$ then $\vee \mu_i \in \tau$ .Moreover, the pair (X, τ) is called a fuzzy topology space. Every member of τ is called a fuzzy open set (9) .In this paper,X and Y are fuzzy topological space. Let μ be a fuzzy set in X. We denote the interior and the closure of a fuzzy set μ by $int(\mu)$ and $cl(\mu)$, respectively.A fuzzy set μ in a space X is called pre-open [11] (resp, fuzzy semi-open [1]) if $\mu \leq (int(cl(\mu)))$. (resp. $\mu \leq cl(int(\mu))$ .The complement of a fuzzy pre-open (resp. fuzzy semi-open) set is said to be fuzzy pre-closed (resp. fuzzy semi-closed).

A fuzzy set μ in a space X is called fuzzy α-open [8] or fuzzy semi-open [12], if $\mu \leq cl(int(cl(\mu)))$. The complement of fuzzy α-open set is said to be fuzzy α-closed.Let μ be a fuzzy set in topological space X. The fuzzy α-closure and α-interior of μ are defined by as $\wedge \{\eta : \mu \leq \rho, \rho$ is α-closed$\}$, $\vee \{\eta : \mu \leq \rho, \rho$ is α-open$\}$, and denoted by α-$cl(\mu)$ and α-$int(\mu)$, respectively. A fuzzy set in X is called a fuzzy singleton if and only if it takes the value 0 for all $y \in X$ except one say x$\in X$. If its value at x is $\varepsilon(0 < \varepsilon \leq 1)$ we denote this fuzzy singleton by x_ε where the point x is called its support (9).For any fuzzy singleton x_ε and any fuzzy set μ, we write $x_\varepsilon \in \mu$ if and only if $\varepsilon \leq \mu x$.

1.Fuzzy Almost Contra α-Continuous Function

Definition 1.1

Let X and Y be fuzzy topological spaces. A fuzzy function $f: X \to Y$ is said to be fuzzy almost contra α-continuous if inverse image of each fuzzy regular open set in Y is fuzzy α-closed in X.

Theorem 1.2

Let $f: X \to Y$ be fuzzy function and let $g: X \to X \times Y$ be the fuzzy graph function of f, defined by $g(x_\epsilon) = (x_\epsilon, f(x_\epsilon))$ for every $x_\in \in X$. If g is fuzzy almost contra α-continuous.

proof

Let η be set in a fuzzy regular closed in Y, then $X \times \eta$ is a fuzzy regular closed set in $X \times Y$. Since g is fuzzy almost contra α-continuous Then $f^{-1}(\eta) = g^{-1}(X \times \eta)$ is fuzzy - α open in X. Thus f is fuzzy almost contra α-continuous.

Definition 1.3

A fuzzy filter base ∧ is said to be fuzzy α-convergent to a fuzzy singleton x_ϵ in X.thus if for any fuzzy α-open set η in X containing x_ϵ there exists a fuzzy set $\mu \leq \eta$.

Definition 1.4

A fuzzy filter base ∧ is said to be fuzzy rc-convergent to a fuzzy singleton x_ϵ in X if for any fuzzy regular closed set η in X containing x_ϵ there exists a fuzzy set $\mu \in A$ such that $\mu \leq . \eta$

Theorem1.5

If a fuzzy function $f: X \to Y$ is fuzzy almost contra e-continuous, then for each fuzzy singleton $x_\epsilon \in X$ and each fuzzy filter base A in X α-converging to x_ϵ, the fuzzy filter base $f(\wedge)$ is fuzzy rc-convergent to $f(x_\epsilon)$.

Proof

For each fuzzy singleton $x_\epsilon \in X$ and each fuzzy filter base A in X α-converging to x_ϵ, the fuzzy filter base $f(A)$ is fuzzy rc-convergent to $f(x_\epsilon)$. Let $x_\epsilon \in X$ and be any fuzzy filter base in X α-converging x_ϵ. Since f is fuzzy almost contra α-continuous, then for any fuzzy regular closed set λ in Ycontaining $f(x_\epsilon)$, There exists fuzzy α-open set μ in X containing x_ϵ such that $f(\mu) \leq \lambda$.

since A is fuzzy α-converging to x_ϵ.There exists a $\xi \epsilon A$ such that a $\xi \leq \mu$ this means $f(\xi) \leq \lambda$ and therefore the filter base $f(A)$ is fuzzy rc-convergent to $f(x_\epsilon)$.

Definition 1.6

A space X is called fuzzy e-connected [3] if X is not the union of two disjoint nonempty fuzzy α-open sets.

Definition 1.7

A space X is called fuzzy connected [10] if X is not the union of two disjoint nonempty fuzzy open sets.

Theorem 1.8

If $f: X \to Y$ is almost contra α-continuous surjection and X is fuzzy α -connected**,** then Y is fuzzy connected**.**

Proof

Suppose that Y is not a fuzzy connected space. There exist nonempty disjoint fuzzy open sets η_1 and η_2 such that $= \eta_1 \vee \eta_2$. Therefore η_1 and η_2 are fuzzy clopen in Y. Since f fuzzy almost contra α-continuous $f^{-1}\eta_2$ and $f^{-1}(\eta_2)$ are fuzzy α-open in X. Moreover $f^{-1}(\eta_1)$ and $f^{-1}(\eta_{2)}$ are nonempty disjoint and $f^{-1}(\eta_1) \vee f^{-1}(\eta_2)$ this shows that X is not fuzzy α-connected.This contradicts that Y is not fuzzy connected assumed. Hence Y is fuzzyconnected.

Definition 1.9

A fuzzy space X is said to be fuzzy α-normal if every pair of nonempty disjoint fuzzy closed sets can be separated by disjoint fuzzy α-open sets.

Definition1.10

A fuzzy space X is said to be fuzzy strongly normal if for every pair of nonempty disjoint fuzzy closed sets μ and ŋ there exist disjoint fuzzy open sets ρ and ξ such that $\mu \leq \rho, \eta \leq \xi$ and cl(ρ)∨cl(ξ)=∅.

Theorem 1.11

If Y is fuzzy strongly normal and $f: X \rightarrow Y$ is fuzzy almost contra α-continuous closed injection, then X is fuzzy α-normal.

Proof

Let η and ρ be disjoint nonempty fuzzy closed sets of X. Since f is injective and closed. $f(\eta)$ and $f(\rho)$ are disjoint fuzzy closed sets. Since Y is fuzzy strongly normal, there exist fuzzy open sets μ and ξ such that $f(\eta) \leq \mu$ and $f(\rho) \leq \xi$ and $cl(\mu)$∧cl(ξ)= ∅. Then, since $cl(\mu)$ and $cl(\xi)$ are fuzzy regular closed and f is fuzzy almost contra- α -continuous, $f^{-1}(cl(\mu))$ and $f^{-1}(cl(\xi)$ are fuzzy α-open set. Since $\eta \leq f^{-1}(cl(\mu))$, $\rho \leq f^{-1}(cl(\xi))$ and $f^{-1}(cl(\mu))$ and $f^{-1}(cl(\xi)$ are disjoint, X is fuzzy α-normal.

Definition1.12

A space X is said to be fuzzy weakly T_2 if each element of X is an intersection of fuzzy regular closed sets**.**

Definition1.13

A space X is said to be fuzzy α-T_2[3] if for each pair of distinct points x_ε and y_v in X, there exits fuzzy α-open set μ containing x_ε and fuzzy α-open set ŋ containing y_v such that $\mu \wedge \eta = \emptyset$.

Definition 1.14

A space X is said to be fuzzy α-T_1[3] if for each pair of distinct fuzzy singletons x_ε and y_v in X, there exits fuzzy α-open set μ and ŋ containing x_ε and y_v, respectively, such that $y_v \notin \mu$ and $x_\varepsilon \notin \eta$.

Theorem 1.15

If $f: X \rightarrow Y$ is almost contra α-continuous injection and Y is fuzzy Urysohn, then X fuzzy α-T_2.

Proof

suppose that Y fuzzy Urysohn. By the injective of f it follows that $f(x_\varepsilon) \neq f(y_\varepsilon)$ for any distinct fuzzy singletons x_ε and y_v in X. since Y is fuzzy Urysohn there exist fuzzy open sets η and ρ such that $f(x_\varepsilon) \in \eta$, $f(x_\varepsilon) \in \rho$ and $cl(\eta) \wedge cl(\rho)=\emptyset$. Since f is almost contra α -continuous there fuzzy α-open sets μ and ξ in X containig x_ε and y_v respectively, such that $f(\mu) \leq cl(\eta)$ and $f(\xi) \leq cl(\rho)$. Hence $\mu \wedge \xi = \emptyset$. Show that X is fuzzy α-T_2.

Theorem 1.16

If $f: X \to Y$ is almost contra e-continuous injection and Y is fuzzy weakly T_2, than X is fuzzy α-T_2.

Proof

Suppose that Y fuzzy weakly T_2. For any distinct points x_ϵ and y_v in X there exits fuzzy regular closed sets η, ρ in Y such that $f(x_\epsilon) \in \eta$ $f(y_v) \notin \eta$, $f(x_\epsilon) \notin \rho$ and $f(y_v) \in \rho$. since f is fuzzy almost contra α -continuous by theorem[2] $f^{-1}(\eta)$ and $f^{-1}(\rho)$ are fuzzy α-open subsets of X such that $x_\varepsilon \in f^{-1}(\eta)$, $y_v \notin f^{-1}(\eta)$, $x_\varepsilon \notin f^{-1}(\rho)$ and $y_v \in f^{-1}(\rho)$.This shows X is α-T_2.

Theorem1.17

let (X_{i,τ_i}) be a fuzzy topological space for all $i \in I$ and I finite suppose that $(\prod_{i\in I} X_{i,\sigma})$ is a product space and $f: (X,\tau) \longrightarrow (\prod_{i\in I} X_{i,\sigma})$ is any fuzzy function. If f is fuzzy almost contra α -continuous, then $pr_i \circ f$ is fuzzy almost contra α-continuous where pr_i is projection function for each $i \in I$.

Proof

Let $x_\varepsilon \in X$ and $(pr_i \circ f)(x_\varepsilon) \in \rho_i$ and ρ_i be fuzzy regular closed set in (X_{i,τ_i}). Then $f(x_\varepsilon) \in pr_i^{-1}(\rho_i) = \rho_i \times \prod_{j\neq i} X_j$ a fuzzy regular closed sets in $(\prod_{i\in I} X_{i,\sigma})$.Since f is fuzzy almost contra α-continuous, there exists a fuzzy e-open set μ containing x_ε such that $f(\mu) \leq \rho_i \times \prod_{j\neq i} X_j = pr_i^{-1}(\rho_i)$ and hence $\mu \leq (pr_i \circ f)^{-1}(\rho_i)$ and obtain that $pr_i \circ f$ is fuzzy almost contra e-continuous for each $i \in I$.

Definition1.18

The fuzzy graph $G(f)$ of a fuzzy function $f: X \to Y$ is said to be fuzzy strongly contra -α-closed if for each $x_\varepsilon, y_v \in (X,Y) \setminus G(f)$, there exist a fuzzy α-open set μ in X containing y_v such that $\mu \times \eta \wedge G(f) = \emptyset$.

Theorem 1.19

If $f: X \to Y$ is fuzzy almost contra α-continuous and Y is fuzzy Uryshon ,$G(f)$ is fuzzy contra-α- closed in $X \times Y$.

Proof

Suppose that Y is fuzzy Uryshon. let $x_\varepsilon, y_v \in (X,Y) \setminus G(f)$, it follows that $f(x_\varepsilon) \neq y_v$. since Y is fuzzy Uryshon, there exists fuzzy open sets η and ρ such that $f(x_\varepsilon) \in \eta$, $y_v \in \rho$ and $cl(\eta) \wedge (cl(\rho) = \emptyset$.Since$f$ fuzzy almost contra- α -continuous, there exists a fuzzy e-open set μ in X containing x_ε such that $f(\mu) \leq cl(\eta)$ .therefore, $f(\mu) \leq cl(\rho) = \emptyset$ and $G(f)$ is fuzzy strongly contra-α- closed in $X \times. Y$

2 **The Relationships**

In this section, the relationships between fuzzy almost contra α-continuous functions and the other forms are investigated**.**

Definition 2.1

A function $f: X \to Y$ is called fuzzy weakly almost contra e-continuous if for each $x \in X$ and each fuzzy regular closed set η of Ycontaining$f(x_\varepsilon)$, there exists a fuzzy α-open set μ in X x_ε such that $int(f(\mu)) \leq \eta$.

Definition 2.2

A function $f: X \to Y$ is called fuzzy (α, s) -open if the image of each fuzzy α-open set is fuzzy semi-open.

Theorem 2.3

If a function $f: X \to Y$ is fuzzy weakly almost contra α-continuous and fuzzy (α, s) -open, then f almost contra α-continuous.

Proof

Let $x_\varepsilon \in X$ and η be a fuzzy regular closed set containing $f(x_\varepsilon)$.Since f is fuzzy weakly almost contra α-continuous, there exists a fuzzy α-open set μ in X containing x_ε such that $int(f(\mu) \leq \eta)$. Since f is a (α, s) -open, $f(\mu)$ is a semi-open set in Y and $f(\mu) \leq cl(int(f(\mu))) \leq \eta$.This shows f almost contra α-continuous.

Definition 2.4

let X and Y be fuzzy topological spaces. A fuzzy function $f: X \to Y$ is said to be

(1) Fuzzy almost contra-pre-continuous if inverse image of each fuzzy regular open set in Y is fuzzy pre-closed in X.

(2) Fuzzy almost contra-semi-continuous if inverse image of each fuzzy regular open set in Y is fuzzy semi-closed in X.

(3) Fuzzy almost contra-continuous if inverse image of each fuzzy regular open set in Y is fuzzy closed in X.

Definition 2.5

A fuzzy space is said to be fuzzy P_Σ if for any fuzzy open set μ of X and each $x_\varepsilon \in \mu$, there exists fuzzy regular closed set ρ containing x_ε such that $x_\varepsilon \in \rho \leq \mu$.

Definition 2.6

A fuzzy function $f: X \to Y$ is said to be fuzzy e-continuous, if $f^{-1}(\mu)$ is fuzzy α-open in X for every fuzzy open set μ in Y.

Theorem 2.7

Let $f: X \to Y$ is said to be fuzzy function. Then if f is a fuzzy almost contra α-continuous and Y is fuzzy P_Σ , then f is fuzzy α-continuous.

Proof

Let μ be any fuzzy open set in Y. Since is fuzzy P_{Σ}, there exists a family ψ whose members are fuzzy regular closed set of f such that $\mu = \vee\{\rho : \rho \in \Psi\}$. Since f is fuzzy almost contra α-continuous, $f^{-1}(\rho)$ is fuzzy α-open in X for each $\rho \in \Psi$ and $f^{-1}(\mu)$ is fuzzy α-open in X Therefore, f is almost contra α-continuous.

Definition 2.8

A space is said to be fuzzy weakly P_{Σ} if for any fuzzy regular open set μ and each $x_{\varepsilon} \in \mu$, there exists a fuzzy regular closed set ρ containing x_{ε} such that $x_{\varepsilon} \in \rho \leq \mu$.

Definition 2.9

A fuzzy function $f: X \to Y$ is said to be fuzzy almost contra α- continuous at $x_{\varepsilon} \in X$ if for each fuzzy open set η containing $f(x_{\varepsilon})$, there exists a fuzzy α-open set μ containing x_{ε} such that $f(\mu) \leq int(cl(\eta))$.

Theorem 2.10

Let $f: X \to Y$ be a fuzzy almost contra α-continuous function. If Y fuzzy weakly P_{Σ} , then f fuzzy almost contra- α-continuous.

Proof

Let μ be any fuzzy regular open set of Y. Since Y is fuzzy weakly P_{Σ} there exists a family Ψ whose members are fuzzy regular closed set of Y such that $\mu = \vee\{\rho : \rho \in \Psi\}$. Since f is a almost contra α-continuous, $f^{-1}(\rho)$ is fuzzy α-open in X for each $\rho \in \Psi$ and f^{-1} is α-open in X. Hence , f is fuzzy almost α-continuous.

Definition 2.11

Let $f: X \to Y$ be a fuzzy is called fuzzy α-irresolute [4] if inverse image of each fuzzy α-open set is fuzzy α-open.

Theorem 2.12

Let X, Y, Z be a fuzzy topological space and let $f: X \to Y$ and $g: X \to Y$ be a fuzzy function. If f is fuzzy α-irresolute and g is fuzzy almost contra α- continuous then $g \circ f: X \to Z$ is a fuzzy almost contra-α continuous functions.

Proof

Let $\mu \leq Z$ be any fuzzy regular closed set and let $(g \circ f)\,(x_{\varepsilon}) \in \mu$. Then $g(f\,(x_{\varepsilon})) \in \mu$. Since g is fuzzy almost contra α- continuous function, it follows that there exists a fuzzy α-open ρ containing $f(x_{\varepsilon})$ such that $g(\rho) \leq \mu$. Since f is fuzzy α-irresolute function. It follows that there exists a fuzzy α-open set η containing x_{ε} such that $f(\eta) \leq \rho$ from here we obtain that $(g \circ f)(\eta) = g(f(\eta) \leq g(\rho) \leq \mu$.

This, we show that $g \circ f$ is a fuzzy almost contra-α-continuous.

Definition 2.13

A fuzzy function $f: X \to Y$ is called a fuzzy α-open set [3] if image of each fuzzy e-open set is fuzzy α-open.

Theorem 2.14

If $f: X \to Y$ is surjective fuzzy α-open function $g \circ f: X \to Y$ a is fuzzy almost contra-α-continuous, then g is fuzzy almost contra-α-continuous.

Proof

Suppose that x_ε is a fuzzy singleton in X. Let η be a regular closed set in Z containing $(g \circ f)(x_\varepsilon)$.Then there exists a fuzzy α-open set μ in X containing x_ε such that $g(f(\mu)) \leq \eta$. Since f is fuzzy α-open ,$f(\mu)$ is a fuzzy α-open set in Y containing $f(x_\varepsilon)$ such that $g\big(f(\mu)\big) \leq \eta$. Thus implies that g is fuzzy almost contra-α-continuous.

Definition 2.15

A space X is said to be fuzzy α-compact [6] (fuzzy S-closed [2] if every fuzzy α-open (respectively fuzzy regular closed) cover of X has a finite subcover.

Theorem 2.16

The fuzzy almost contra-α-continuous images of fuzzy α-compact spaces are S-closed.

Proof

Suppose that $f: X \to Y$ is a fuzzy almost contra-e-continuous surjection. Let $\{\eta_i : i \in I\}$ be any fuzzy regular closed cover of Y.since f is fuzzy almost contra-α-continuous , then $\{f^{-1}\,(\eta_{i)}: i \in I\}$ is a fuzzy α-open cover of X and hence there exists a finite subset I_o of I such that $X = \vee \{f^{-1}(\eta_i): i \in I_o\}$.Therefore, we have $Y = \vee \{f^{-1}(\eta_i): i \in I_o\}$ and Y is fuzzy S-closed.

Definition 2.17

A space X is said to be

(1) Fuzzy α-closed- compact [3] if every fuzzy α-closed cover of X has a finite subcover,

(2) Fuzzy nearly compact (5) if every fuzzy regular open cover of X has a finite subcover.

Theorem 2.18

The fuzzy almost contra-α-continuous images of fuzzy α-closed-compact space are fuzzy nearly compact.

Proof

Suppose that $f: X \to Y$ is a fuzzy almost contra-e-continuous surjection. Let $\{\eta_i : i \in I\}$ be any fuzzy regular open cover of Y. Since f is fuzzy almost contra-α-continuous, then $\{f^{-1}\,(\eta_{i)}: i \in I\}$ is a fuzzy α-closed cover of X. Since X is fuzzy α -closed-compact .There exists a finite subset I_o of I such that $X = \vee \{f^{-1}(\eta_i): i \in I_o\}$.Thus, we have $Y = \vee \{\eta_i : i \in I_o\}$ and Y is fuzzy nearly compact.

Conclusion:

In this paper discussed Fuzzy Almost contra α-Continuous Function and fuzzy α-normal and related properties. And also we used to defined fuzzy α-connected fuzzy strongly normal and fuzzy α-continuous.

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