Fuzzy Graphs Theory: Properties and Types

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Abstract

Fuzzy graph theory has wider range of applications in logic, algebra, topology, operations research, pattern recognition, artificial intelligence, neural networks and several other fields. In this article, recent developments in fuzzy graph theory have been reviewed. After introducing and developing fuzzy set theory, a lot of studies have been done in this field and then a result appeared as a Fuzzy Graph (Combination of graph theory and fuzzy set theory). This is now known as Fuzzy graph theory. In this article we review essential works on different types of fuzzy graph and fuzzy hyper graph.

Keywords: *Graph, Theory, Types, Properties, Set.*

I. INTRODUCTION

Graph theory has several interesting applications in system analysis, operations research and economics. Since most of the time the aspects of graph problems are uncertain, it is nice to deal with these aspects via the methods of fuzzy logic. The concept of fuzzy relation which has a widespread application in pattern recognition in his landmark paper “Fuzzy sets” in 1965 was introduced by Zadeh. Fuzzy graph and several fuzzy analogs of graph theoretic concepts were first introduced by Rosenfeld in 1975. Sense then, fuzzy graph theory is finding an increasing number of applications in modeling real time systems where the level of information inherent in the system varies with different levels of precision.

Fuzzy models are becoming useful because of their aim in reducing the differences between the traditional numerical models used in engineering and sciences and the symbolic models used in expert systems. Mordeson and Peng defined the concept of complement of fuzzy graph and studied some operations on fuzzy graphs. The definition of complement of a fuzzy graph was modified so that the complement of the complement is the original fuzzy graph, which agrees with the crisp graph case. Moreover some properties of self-complementary fuzzy graphs (fuzzy graphs that are isomorphic to their complements) and the complement of the operations of union, join and composition of fuzzy graphs. For more on the previous notions and the following ones, one can see

In 1973, Kaufmann defined fuzzy graphs for the first time. Then Azriel Rosenfeld developed the theory of fuzzy graphs in 1975. Here we have presented the contributions of several authors towards the field of this fast growing fuzzy graph theory.

Definition

A fuzzy graph $G = (\sigma, \mu)$ is a pair of functions $\sigma : V \to [0, 1]$ and $\mu : V \times V \to [0, 1]$ such that $\mu(u, v) \leq \sigma(u) \cap \sigma(v)$ for all u, v in V where V is the vertex set. The fuzzy graph $H = (\tau, \rho)$ is called a fuzzy subgraph of $G = (\sigma, \mu)$ if $\tau(u) \leq \sigma(u)$ for all $u \in V$ and $\rho(u, v)(u, v)$ for all $u, v \in V$. Also H is called a spanning subgraph if $\tau(u) = \sigma(u)$ for all $u \in V$.

II. PROPERTIES OF FUZZY GRAPHS

Mcallister proved that intersection of two fuzzy graphs is again a fuzzy graph. Radha et al. explained the degree of an edge in union and join of fuzzy graph through some illustrations. Nagoor Gani et al. discussed degree of a vertex in composition and Cartesian product. Nirmala et al. explained degree of a vertex in Tensor product and normal product of fuzzy graph. Prabir Bhattacharya et al. presented an algorithm to find the supremum of max-min powers of a map by characterizing the path in a fuzzy graph. Moderson et al., proved a necessary and sufficient condition for a graph to be a Cartesian product of two fuzzy sub graphs and union of two fuzzy subgraphs of a graph is again a fuzzy subgraph. Nair discussed few properties of complete fuzzy graph and fuzzy trees. He proved triangle laws, parallelogram laws and little equivalence of bridges in fuzzy graphs. Sunitha et al. studied certain properties of fuzzy bridges, fuzzy cut nodes and using them they obtained a characterization of fuzzy trees and fuzzy cut node. Nagoor Gani et al. proved the inequality involving order and size of a fuzzy graph. Mordeson has defined the complement of a fuzzy graph. Arindam Dey et al. presented a program to determine the fuzzy complement of a fuzzy graph. Nagoor Gani and Chandrasekaran defined µ-complement of a fuzzy graph. Nagoor Gani et al. discussed few properties of µ-complement of fuzzy graph. They proved that µ-complement of a fuzzy graph has isolated nodes if and only if the given graph is a strong fuzzy graph.

Strong arcs in fuzzy graphs

Bhutani et al. defined strong arcs in fuzzy graphs and proved that a strong arc need not be a bridge whereas a bridge is a strong arc. Sunil Mathew et al. proved that a fuzzy graph is a fuzzy tree iff there exists a unique α-strong path between any two nodes and also proved that an arc in a fuzzy tree is α-strong iff it is an arc of the spanning tree of the fuzzy graph.

Connectivity in a fuzzy graph. Sandeep Narayanan et al. proved that the complement of a fuzzy graph is connected if the given fuzzygraph without m-strong arcs is connected. Also proved that a graph and its complement are connected iff the given graph has atleast one connected spanning fuzzy subgraph without any m-strong arcs.

Blocks and Cycles in fuzzy graphs

Mini Tom et al. proved that the fuzzy graph satisfying the condition that either (u, v) is an δ-arc or µ(u, v) = 0 is a block iff there exists atleast two internally disjoint strongest u v paths. Also they proved that if the underlying graph G is a complete graph then the fuzzy graph G without δ-arc is a block. Mordeson et al. proved that a fuzzy graph which is a cycle is a fuzzy cycle iff it is not a fuzzy tree. They also proved that the fuzzy graph (σ, µ) does not have a fuzzy bridge iff it is a cycle and µ is a constant function assuming that the dimension of the cycle space of the underlying graph (σ , µ) is unity.

Domination in fuzzy graphs

Manjunisha et al. proved that the strong domination number of a non-trivial fuzzy graph is equal to the size of the fuzzy graph iff each node is either an isolated node or has an unique strong neighbour and all arcs are strong.

Automorphism of fuzzy graphs

Bhattacharya obtained a fuzzy analog from graph theory to fuzzy graph theory which states that we can associate a group with fuzzy graph as an automorphism group. Bhutani introduced the concept of isomorphism between fuzzy graphs. He proved that every fuzzy group has an embedding into the fuzzy group of the group of automorphism of a fuzzy graph. Let (σ1, µ1) and (σ2, µ2) be fuzzy subgraphs of graphs (σ1 , µ1) and (σ2 , µ2) respectively. Then Mordeson proved that any weak isomorphism of (σ1, µ1) onto (σ2, µ2) is again an isomorphism of (σ1, µ1) onto (σ2 , µ2). Sunitha et al. proved that the set all automorphisms of a fuzzy graph will be a group when the bi- nary relation is set theoretic composition of maps. Sathyaseelan et al. proved that the order and size of any two isomorphic fuzzy graphs are the same. They also proved that the relation Isomorphism between fuzzy graphs satisfies reflexivity, symmetricity and transitivity. i.e. It is an equivalence relation.

Coloring and Clustering of fuzzy graphs

Eslahchi et al. introduced fuzzy coloring of a fuzzy graph. Arindam Dey et.al introduced an algorithm with an illustration to color the complement of a fuzzy graph through α-cuts by considering three cases.

Nivethana et al. presented executive committee problem as an illustration to find the chromatic number of a fuzzy graph. Anan- thanarayanan et al. explained how to find the chromatic number of a fuzzy graph using α-cuts by considering fuzzy graphs with crisp vertices and fuzzy edges through illustrations. Sameena presented an algorithm for constructing ϵ-clusters using strong arcs and explained the procedure to obtain ϵ-clusters through some illustrations where $0 \leq \epsilon \leq 1$.

Interval valued Fuzzy line graphs

Moderson presented a necessary and sufficient condition for a fuzzy graph to be a fuzzy line graph. Craine analysed various properties of fuzzy interval graphs. Naga Maruthi Kumari et.al proved that the composition of two strong interval valued fuzzy graphs is a strong interval valued fuzzy graph. Hossein Rashmanlou et.al proved that the semi strong product and strong product of two interval valued fuzzy graphs is complete. Akram stated a proposition that Interval valued fuzzy graph is isomorphic to an interval valued fuzzy intersection graph. Sen et al. proved that fuzzy intersection graph is chordal iff for a, b, c, d in the semigroup, some pair from {a, b, c, d} has a right common multiple property.

Intuitionistic Fuzzy Graphs

Deng-Feng li proposed two linear dissimilarity measures between intuitionistic fuzzy sets. Akram et.al discussed few metric aspects of intuitionistic fuzzy graphs. Nagoor Gani et.al proved that the sum of the degree of membership value of all vertices in an intuitionistic fuzzy graph is two times the sum of the membership value of all edges and the sum of the degree of non-membership value of all vertices in an intuitionistc fuzzy graph is two times the sum of the non-membership value of all edges. Karunambigai et.al discussed three cases where strong path is a strongest path in intuitionistic fuzzy graphs. Akram et.al proved that the join of two strong intuitionistic fuzzy graphs is again a strong intuitionistic fuzzy graph. Karunambigai et.al proved that the order and size of two isomorphic intuitionistic fuzzy graphs are same. He proved that every complete intuitionistic fuzzy graph is balanced.

Parvathi et al. proved a necessary and sufficient condition for a dominating set to be a minimal dominating set in intuitionistic fuzzy graph.

III. TYPES OF FUZZY GRAPHS

Regular Fuzzy Graph A

NagoorGani and K.Radha (2008) introduced regular fuzzy graphs on paper "On Regular Fuzzy Graphs". In their paper they have introduced regular fuzzy graphs, total degree and totally regular fuzzy graphs. Regular fuzzy graphs and totally regular fuzzy graphs are compared through various examples. A necessary and sufficient condition for equivalence and characterization of regular fuzzy graph was provided.

Complementary Fuzzy Graph

Moderson (1994) introduced the concept of complement of fuzzy graphs and M.S. Sunitha and A. Vijayakumar(2001) gave a modified definition of complement of fuzzy graph. R. Sattanathan and S. Lavanya(2009) studied about complementary fuzzy graphs and fuzzy chromatic number. In their paper they find the fuzzy chromatic number of complement of fuzzy graphs and gave the bounds for sum and product of fuzzy chromatic number of fuzzy graph and its complement.

Antipodal Fuzzy Graph A

NagoorGani and J.Malarvizhi (2010) defined concept of Antipodal Graph. In crisp graph theory the concept of antipodal graph of a given graph G was introduced by Smith. The condition on the graph G, for A(G) = G and A(G) = G are discussed by Aravamudhan and Rajendran. As a fuzzy analog to this, in their paper antipodal fuzzy graph was defined and its nature was discussed.

Constant intuitionistic fuzzy graphs

M. G. Karunambigai, R. Parvathi and R. Buvaneshwari (2011) introduced Constant intuitionistic fuzzy graphs. In their work, Constant Intuitionistic Fuzzy Graphs (IFGs), and totally constant IFGs were introduced. A necessary and sufficient condition for equivalence was provided. A characterization of constant IFGs on acycle was also given. Some properties of constant IFGs with suitable illustrations were also discussed.

Fuzzy Graph Structures

T. Dinesh and T. V. Ramakrishnan (2011) introduced the concept of a fuzzy graph structure based on the concept of graph structure. A new concept, namely, graph structure has been introduced by E. Sampathkumar which, in particular, is a generalization of the notions like graphs, signed graphs and edge-coloured graphs with the colourings. According to him, G = (V, R1, R2,..., Rk) is a graph structure if V is a nonempty set and R1, R2,..., Rk are relations on V which are mutually disjoint such that each Ri, i = 1, 2, 3,..., k, is symmetric and irreflexive. This is the motivation for the study of fuzzy graph structures. New concepts like pi-edge, pi-cycle, pi-tree, pi-forest, fuzzy pi-cycle, fuzzy pi-tree, fuzzy pi-forest, pi-connectedness etc. are introduced and studied.

Product Intuitionistic Fuzzy Graph

The first definition of intuitionistic fuzzy graphs was proposed by Atanassov. Dr. V. Ramaswamy and Poomima. B introduce the concept of product fuzzy graph. N. Vinoth Kumar and G.GeethaRamani(2011) develop the concept of Product Intuitionistic fuzzy graphs of intuitionistic fuzzy graphs. Further investigate properties Product Intuitionistic fuzzy graphs.

Bipolar Fuzzy Hypergraphs

In 1994, Zhang initiated the concept of bipolar fuzzy sets as a generalization of fuzzy sets. Bipolar fuzzy sets are an extension of fuzzy sets whose range of membership degrees [-1,1] . In bipolar fuzzy set, membership degree 0 of an element means that the element is irrelevant to the corresponding property, the membership degree (0,1] of an element indicates that the element somewhat satisfies the property, and the membership degree [-1,0) of an element indicates the element somewhat satisfies the implicit counter property.

Irregular Fuzzy Graphs

A. NagoorGani and S. R. Latha (2012) defined Neighbourly total irregular fuzzy graphs, highly irregular fuzzy graphs and highly total irregular fuzzy graphs were introduced. A comparative study between neighbourly irregular and highly irregular fuzzy graphs was made. Also some results on neighbourly irregular fuzzy graphs were studied.

Irregular Bipolar Fuzzy Graphs

S. Samanta and M .Pal (2012) define irregular bipolar fuzzy graphs and its various classifications. Size of regular bipolar fuzzy graphs was derived. The relation between highly and neighbourly irregular bipolar fuzzy graphs were established. Some basic theorems related to the stated graphs have also been presented.

Fuzzy Labeling Graph

A NagoorGani and D. Rajalaxmi (a) Subahashini (2012) introduced a new concept of fuzzy labeling. A graph is said to be a fuzzy labeling graph if it has fuzzy labeling. Fuzzy sub graph, union, fuzzy bridges, fuzzy end nodes, fuzzy cut nodes and weakest arc of fuzzy labeling graphs have been discussed. And number of weakest arc, fuzzy bridge, cut node and end node of a fuzzy labeling cycle has been found. It is proved that $\Delta(G\omega)$ is a fuzzy cut node and $\delta(G\omega)$ is a fuzzy end node of fuzzy labeling graph. Also it was proved that If Go is a connected fuzzy labeling graph then there exists a strong path between any pair of nodes.

Strong Intuitonistic Fuzzy Graphs

Muhammad Akram and BijanDavvaz (2012) introduce the notion of strong intuitionistic fuzzy graphs and investigate some of their properties. They discuss some propositions of self-complementary and self-weak complementary strong intuitionistic fuzzy graphs. They introduced the concept of intuitionistic fuzzy line graphs.

Interval-valued Fuzzy Graph

Muhammad Akram and Wieslaw A. Dudek (2012) define the operations of Cartesian product, composition, union and join on interval-valued fuzzy graphs and investigate some properties. They gave concept of isomorphism (resp. weak isomorphism) between interval- valued fuzzy graphs. They also introduced the notion of interval-valued fuzzy complete graphs and present some properties of self-complementary and self-weak complementary interval-valued fuzzy complete graphs.

Complete interval-valued Fuzzy Graph

HosseinRashmanlou and Young Bae Jun (2013) In their paper, they provided three new operations on interval-valued fuzzy graphs; namely direct product, semi strong product and strong product. They gave sufficient conditions for each one of them to be complete.

Balanced Intuitionistic Fuzzy Graph

Al-Hawary introduced the concept of balanced fuzzy graphs and studied some operations of fuzzy graphs. Atanassov introduced the concept of intuitionistic fuzzy relations and intuitionistic fuzzy graphs (IFGs). Parvathy and Karunambigai introduced the concept of IFG elaborately and analysed its components. Articles motivated us to analyze balanced IFGs and their properties. Their paper deals with the significant properties of balanced IFG. The necessary condition for an IFG to be a Balanced IFG if the graph G is complete, strong, regular and self-complementary IFG were discussed. They also discussed some properties of complementary and self-complementary balanced IFGs. Also deals with direct product, semi strong product and strong product of intuitionistic fuzzy graphs and their properties with suitable illustrations were given.

Fuzzy Dual Graph

Researchers introduced Fuzzy dual graph. In their paper the definition of fuzzy dual graphs was considered with the following properties were obtained, which was the dual of the dual of fuzzy graph is the fuzzy graph itself, and the dual of fuzzy bipartite graph is Eulerian fuzzy graph.

IV. CONCLUSION

Several results on fuzzy graphs have been referred and this will be a compendium for the researchers to work in the field of fuzzy graph theory. In this paper we are willing to summarize types of fuzzy graph, fuzzy graph structure and also type of intuitionistic fuzzy graph along with persons related to respective topic. We hope this paper will help the researchers to look the field of Fuzzy Graph at a glance.

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