GENERALIZED REGULAR CONNECTEDNESS IN INTUITIONISTIC FUZZY TOPOLOGICAL SPACE

B.Boomathi[1] , M.Palanisamy[2]

[1] *Researchscholar, Department Of Mathematics, Vivekanandha College of Arts and Sciences For Women (Autonomous), Nammakal, Tamilnadu, India-637205.*

[2] *Assistant Professor, Department Of Mathematics, Vivekanandha College of Arts and Sciences For Women (Autonomous), Nammakal, Tamilnadu, India-637205.*

ABSTRACT

The aim of this paper, we have introduced the intuitionistic fuzzy generalized regular connected space, intuitionistic fuzzy generalized regular super connected space and intuitionistic fuzzy generalized regular extremally disconnected space. We investigated some of their properties. Also we characterized the intuitionistic fuzzy generalized regular super connected space.

Keywords: - *Intuitionistic fuzzy topology, intuitionistic fuzzy generalized regular connected space, intuitionistic fuzzy generalized regular super connected space.*

1.INTRODUCTION

Zadeh[11] introduced the notion of fuzzy sets. Fuzzy topological space was introduced by chang[2]. After that there have been a number of generalizations of this fundamental concept. Atanassov[1] introduced the notion of intuitionistic fuzzy sets. Using the notion of intuitionistic fuzzy sets, Coker[3] introduced the notion of intuitionistic fuzzy topological space. Connectedness in intuitionistic fuzzy special topological spaces was introduced by Ozcag and coker[5]. Thakur and chanturvedi[8] discussed intuitionistic fuzzy regular openness and intuitionistic fuzzy regular continuity.

In this paper we have introduced intuitionistic fuzzy generalized regular connected space, intuitionistic fuzzy generalized regular super connected space and intuitionistic fuzzy generalized regular extremally disconnected space. We investigated some of their properties. Also we characterized the intuitionistic fuzzy generalized regular super connected space.

2.PRELIMINARIES

Definition 2.1

An intuitionistic fuzzy set (IFS for short) A in X is an object having the form

$$A = \{ \langle x, \mu_A(x), v_A(x) \rangle / x \in X \}$$

where the function $\mu_A : X \to [0,1]$ and $v_A : X \to [0,1]$ denote the degree of membership (namely $\mu_A(x)$) and the degree of non-membership (namely $v_A(x)$) of each element $x \in X$ to the set A , respectively , and $0 \le \mu_A(x) + v_A(x) \le 1$ for each $x \in X$. Denote by IFS (X), the set of all intuitionistic fuzzy sets in X.

Definition 2.2

Let A and B be IFSs of the form $A = \{\langle x, \mu_A(x), \nu_A(x)\rangle / x \in X\}$ and

$B = \{\langle x, \mu_B(x), \nu_B(x)\rangle / x \in X\}$. Then

(a) $A \subseteq B$ if and only if $\mu_A(x) \leq \mu_B(x)$ and $\nu_A(x) \geq \nu_B(x)$ for all $x \in X$;
(b) $A = B$ if and only if $A \subseteq B$ and $B \subseteq A$;
(c) $A^c = \{\langle x, \nu_A(x), \mu_A(x)\rangle / x \in X\}$;
(d) $A \cap B = \{\langle x, \mu_A(x) \wedge \mu_B(x), \nu_A(x) \vee \nu_B(x)\rangle / x \in X\}$;
(e) $A \cup B = \{\langle x, \mu_A(x) \vee \mu_B(x), \nu_A(x) \wedge \nu_B(x)\rangle / x \in X\}$.

The intuitionistic fuzzy sets $0_\sim = \{\langle x, 0, 1\rangle / x \in X\}$ and $1_\sim = \{\langle x, 1, 0\rangle / x \in X\}$ are respectively the empty set and the whole set of X.

For the sake of simplicity, we shall use the notation $A = \langle x, \mu_A, \nu_A\rangle$ instead of $A = \{\langle x, \mu_A(x), \nu_A(x)\rangle / x \in X\}$.

Definition 2.3

An intuitionistic fuzzy topology (IFT for short) on X is a family τ of IFSs in X satisfying the following axiom:

i. $0_\sim, 1_\sim \in \tau$;
ii. $G_1 \cap G_2 \in \tau$ for any $G_1, G_2 \in \tau$;
iii. $\cup G_i \in \tau$ for any family $\{G_i / i \in J\} \subset \tau$.

In this case the pair (X, τ) is called an intuitionistic fuzzy topological space (IFTS for short) and any IFS in τ is known as an intuitionistic fuzzy open set (IFOS for short) in X. The complement A^c of an IFOS A in IFTS (X, τ) is called an intuitionistic fuzzy closed set (IFCS for short) in X.

Definition 2.4

An IFS $A = \langle x, \mu_A, \nu_A\rangle$ in an IFTS (X, τ) is said to be

(i) Intuitionistic fuzzy regular open set $\lambda = int(cl(\lambda))$
(ii) Intuitionistic fuzzy regular closed set $\lambda = cl(int(\lambda))$

Note that an IFS A is an IFRCS if and only if $int(cl(int(A))) \subseteq A$.

Definition 2.5

Let A be an IFS in an IFTS (X, τ). Then the regular interior and the regular closure of A are defined as

$rint(A) = \cup\{G / G \text{ is an IFROS in } X \text{ and } G \subseteq A\}$;

$rcl(A) = \cap\{K / K \text{ is an IFRCS in } X \text{ and } A \subseteq K\}$

Note that for any IFS A in (X, τ), we have $rcl(A^c) = [rint(A)]^c$ and $rint(A^c) = [rcl(A)]^c$

Definition 2.6

Two IFSs A and B in X are said to be q-coincident (AqB for short) if and only if there exist an element $x \in X$ such that $\mu_A(x) > \nu_A(x)$ or $\nu_A(x) < \mu_B(x)$.

Definition 2.7

Two IFSs A and B in X are said to be not q-coincident (Aq^cB for short) if and only if $A \subseteq B^c$.

Definition 2.8

An IFTS (X, τ) is said to be an IF $T_{1/2}$ space if every IFGCS in (X, τ) is an IFCS in (X, τ).

Definition 2.9

An IFTS (X, τ) is said to be an intuitionistic fuzzy C_5-connected (IFC_5-connected for short) space if the only IFSs which are both intuitionistic fuzzy open and intuitionistic fuzzy closed are $0_\sim$ and $1_\sim$.

Definition 2.10

An IFTS (X, τ) is said to be an intuitionistic fuzzy GO-connected (IFGO-connected for short) space if the only IFSs which are both intuitionistic fuzzy generalized open and intuitionistic fuzzy generalized closed are $0_\sim$ and $1_\sim$.

3.INTUITIONISTIC FUZZY GENERALIZED REGULAR CONNECTED SPACE

In this section we introduce intuitionistic fuzzy generalized regular connected space and intuitionistic fuzzy generalized regular super connected space. We investigate some of their properties. Also we provide a characterization theorem for an intuitionistic fuzzy generalized regular super connected space.

Definition 3.1

An IFS A is an intuitionistic fuzzy generalized closed set (IFGCS for short) if $cl(A) \subseteq U$ whenever $A \subseteq U$ and U is an IFOS. The complement of an IFGCS is called an intuitionistic fuzzy generalized open set (IFGOS for short).

Definition 3.2

An IFS A in an IFTS (X, τ) is said to be an intuitionistic fuzzy generalized regular closed set (IFGRCS for short) if $rcl\ (A) \subseteq U$ whenever $(A) \subseteq U$ and U is an IFOS in (X, τ).

Every IFC, IFRCS is an IFGRCS but the separate converses may not be true in general.

Definition 3.3

The complement A^c of an IFGRCS A in a IFTS(X, τ) is called an intuitionistic fuzzy generalized regular open set (IFGROS for short) in X.

Every IFOS, IFROS is an IFGROS but the separate converses may not be true in general.

Definition 3.4

Let A be an IFS in an IFTS (X, τ) . Then the generalized regular interior and the generalized regular closure of A are define as

$grint(A) = \cup \{G\ /G \text{ is an IFGROS in } X \text{ and } G \subseteq A\}$;

$grcl(A) = \cap \{K\ /K \text{ is an IFGRCS in } X \text{ and } A \subseteq K\}$.

Note that for any IFS A in an IFTS (X, τ), we have $grcl(A^c) = [grint(A)]^c$ and $grint(A^c) = [grcl(A)]^c$.

Definition 3.5

If every IFGRCS in an IFTS (X, τ) is IFRCS in (X, τ), then the space can be called as an intuitionistic fuzzy regular $T_{1/2}$ space (IFR$T_{1/2}$ space for short).

Definition 3.6

A mapping $f: (X, \tau) \rightarrow (Y, \sigma)$ is called an intuitionistic fuzzy generalized regular continuous (IFGR continuous for short) mapping if $f^{-1}(V)$ is an IFGRCS in (X, τ) for every IFCS V of (Y, σ).

Definition 3.7

A mapping $f: (X, \tau) \rightarrow (Y, \sigma)$ is called an intuitionistic fuzzy generalized regular irresolute (IFGR irresolute for short) mapping if $f^{-1}(V)$ is an IFGRCS in (X, τ) for every IFGCS V of (Y, σ)

Definition 3.8

An IFTS (X, τ) is said to be an intuitionistic fuzzy generalized regular connected space if the only IFSs which are both intuitionistic fuzzy generalized regular open and intuitionistic fuzzy generalized regular closed are $0_\sim$ and $1_\sim$.

Theorem 3.9

. Every intuitionistic fuzzy generalized regular connected space is an intuitionistic fuzzy

C_5- connected space but not conversely.

Proof

Let (X, τ) be an intuitionistic fuzzy generalized regular connected space.

Suppose $(X, \tau)$is not an intuitionistic fuzzy C_5- connected space, then there exists a proper IFS A which both intuitionistic fuzzy open and intuitionistic fuzzy closed in (X, τ).

That is A is both intuitionistic fuzzy generalized regular open and intuitionistic fuzzy generalized regular closed in (X, τ).

This implies that (X, τ) is not intuitionistic fuzzy generalized regular super connected space.

This is contradiction. Therefore (X, τ) is an intuitionistic fuzzy C_5- connected space.

Theorem 3.10

Every intuitionistic fuzzy generalized regular connected space is an intuitionistic fuzzy GO- connected space but not conversely.

Proof

Let (X, τ) be an intuitionistic fuzzy generalized regular connected space.

Suppose (X, τ) is not an intuitionistic fuzzy GO- connected space, then there exists a proper IFS A which both intuitionistic fuzzy g-open and intuitionistic fuzzy g-closed in(X, τ).

That is A is both intuitionistic fuzzy generalized regular open and intuitionistic fuzzy generalized regular closed in (X, τ).

This implies that (X, τ) is not intuitionistic fuzzy generalized regular super connected space.

This is contradiction. Therefore (X,τ) is an intuitionistic fuzzy GO- connected space.

Theorem 3.11

An IFTS (X,τ) is an intuitionistic fuzzy generalized regular connected space if and only if there exist no non-zero intuitionistic fuzzy generalized regular open sets A and B in (X,τ) such that A=B^c.

Proof

Necessity

Let A and B be two intuitionistic fuzzy generalized regular open set in (X,τ) such that $A \neq 0_\sim \neq B$ and $A = B^c$.

Therefore B^c is an intuitionistic fuzzy generalized regular closed set. Since $A \neq 0_\sim, B \neq 1_\sim$.

This implies B is a proper IFS which is both intuitionistic fuzzy generalized regular open and intuitionistic fuzzy generalized regular closed in (X,τ).

Hence (X,τ) is not an intuitionistic fuzzy generalized regular connected space. But this is a contradiction to our hypothesis.

Thus there exist no non-zere intuitionistic fuzzy regular open set A and B in (X,τ) such that $A = B^c$.

Sufficiency

Let A be both intuitionistic fuzzy generalized regular open and intuitionistic fuzzy generalized regular closed in (X,τ)such that $1_\sim \neq A \neq 0_\sim$.

Now let $B = A^c$. Then B is an intuitionistic fuzzy generalized regular open set and $B \neq 1_\sim$.

This implies $B = A^c \neq 0_\sim$, which is a contradiction to our hypothesis. Therefore (X,τ) is an intuitionistic fuzzy generalized regular connected space.

Theorem 3.12

An IFTS (X,τ) is an intuitionistic fuzzy generalized regular connected space if and only if there exist no non-zero intuitionistic fuzzy generalized regular open sets A and B in (X,τ) such that $A = B^c$,$B = (rcl(A))^c$ and $A = (rcl(B))^c$.

Proof

Necessity

Assume that there exist IFSs A and B such that $A \neq 0_\sim \neq B$, ,$B = (rcl(A))^c$ and $A = (rcl(B))^c$.

Since $(rcl(A))^c$and $(rcl(B))^c$ are intuitionistic fuzzy generalized regular open set in (X,τ), A and B are intuitionistic fuzzy generalized regular open set in (X,τ).

This implies(X,τ) is not an intuitionistic fuzzy generalized regular connected space, which is a contradiction.

Therefore there exist no non-zero intuitionistic fuzzy generalized regular open sets A and B in (X,τ) such that $A = B^c$, $B = (rcl(A))^c$ and $A = (rcl(B))^c$.

Sufficiency

Let A be both intuitionistic fuzzy generalized regular open and intuitionistic fuzzy generalized regular closed in (X,τ) such that $1_\sim \neq A \neq 0_\sim$.

Now by taking $B = A^c$, we obtain a contradiction to our hypothesis .

Hence (X,τ) is an intuitionistic fuzzy generalized regular connected space.

Definition 3.13

An IFTS (X,τ) is said to be an intuitionistic fuzzy regular $T^*{}_{1/2}$ space if every intuitionistic fuzzy generalized regular closed set is an intuitionistic fuzzy closed set in (X,τ).

Remark 3.14

Every intuitionistic fuzzy regular $T^*{}_{1/2}$ space is an intuitionistic fuzzy regular $T_{1/2}$ space but not conversely.

Proof

Let (X,τ) be an intuitionistic fuzzy regular $T^*{}_{1/2}$ space. Let A be an intuitionistic fuzzy generalized regular closed set in (X,τ).

By hypothesis A is an intuitionistic fuzzy closed set.

Since every intuitionistic fuzzy closed set is an intuitionistic fuzzy regular closed set, A is an intuitionistic fuzzy regular closed set in (X,τ).

Hence (X,τ) is an intuitionistic fuzzy regular $T_{1/2}$ space.

Remark 3.15

Every intuitionistic fuzzy regular $T^*{}_{1/2}$ space is an intuitionistic fuzzy $T_{1/2}$ space but not conversely.

Proof

Let (X,τ) be an intuitionistic fuzzy regular $T^*{}_{1/2}$ space. Let A be an intuitionistic fuzzy generalized closed set in (X,τ).

Since every intuitionistic generalized fuzzy closed set is an intuitionistic fuzzy generalized regular closed set, A is an intuitionistic fuzzy generalized regular closed set in (X,τ).

By hypothesis A is an intuitionistic fuzzy closed set.

Hence (X,τ) is an intuitionistic fuzzy $T_{1/2}$ space.

Theorem 3.16

Let (X,τ) be an intuitionistic fuzzy regular $T^*{}_{1/2}$ space, then the following are equivalent .

(i) (X,τ) is an intuitionistic fuzzy GO- connected space,
(ii) (X,τ) is an intuitionistic fuzzy generalized regular connected space,
(iii) (X,τ) is an intuitionistic fuzzy C_5-connected space.

proof

i ⇒ ii is obvious from theorem 3.3

ii ⇒ iii is obvious

iii ⇒ i Let (X,τ) be an intuitionistic fuzzy C_5-connected space. Suppose (X,τ) is not an intuitionistic fuzzy generalized regular connected space, then there exist a proper IFS A in (X,τ) which is both intuitionistic fuzzy generalized regular open and intuitionistic fuzzy generalized regular closed in (X,τ).

But since (X,τ) is an intuitionistic fuzzy regular $T^*{}_{1/2}$ space, A is both intuitionistic fuzzy open and intuitionistic fuzzy closed in (X,τ).

This implies that (X,τ) is not an intuitionistic fuzzy C_5-connected space, which is a contradiction to our hypothesis.

Therefore (X,τ) is intuitionistic fuzzy generalized regular connected space.

Theorem 3.17

If $f:(X,\tau)\to(Y,\sigma)$ is an intuitionistic fuzzy regular continuous surjection and (X,τ) is an intuitionistic fuzzy generalized regular connected space, then (Y,σ) is an intuitionistic fuzzy C_5 −connected space.

Proof

Let (X,τ) be an intuitionistic fuzzy generalized regular connected space.

Suppose (Y,σ) is not an intuitionistic fuzzy C_5-connected space, then there exists a proper IFS A which is both intuitionistic fuzzy open and intuitionistic fuzzy closed in (Y,σ).

Since f is an intuitionistic fuzzy regular continuous mapping $f^{-1}(A)$ is both intuitionistic fuzzy generalized regular open and intuitionistic fuzzy generalized regular closed in (X,τ).

But this is a contradiction to hypothesis.

Hence (Y,σ) is an intuitionistic fuzzy C_5 −connected space.

Theorem 3.18

If $f:(X,\tau)\to(Y,\sigma)$ is an intuitionistic fuzzy regular irresolute surjection and (X,τ) is an intuitionistic fuzzy generalized regular connected space, then (Y,σ) is also an intuitionistic fuzzy generalized regular connected space.

Proof

Suppose (Y,σ) is not an intuitionistic fuzzy generalized regular connected space, then there exists a proper IFS A which is both intuitionistic fuzzy generalized regular open and intuitionistic fuzzy generalized regular closed in (Y,σ).

Since f is an intuitionistic fuzzy regular irresolute mapping, $f^{-1}(A)$ is both intuitionistic fuzzy generalized regular open and intuitionistic fuzzy generalized regular closed in (X,τ). But this is a contradiction to hypothesis.

Hence (Y,σ) is an intuitionistic fuzzy generalized regular connected space.

Definition 3.19

An IFTS (X,τ) is called intuitionistic fuzzy C_5- connected between two IFSs A and B if there is no intuitionistic fuzzy open set E in (X,τ) such that $A\subseteq E$ and Eq^cB.

Definition 3.20

An IFTS (X,τ) is called intuitionistic fuzzy generalized regular connected between two IFSs A and B if there is no intuitionistic fuzzy generalized regular open set E in (X,τ) such that $A\subseteq E$ and Eq^cB.

Theorem 3.21

If an IFTS (X,τ) is intuitionistic fuzzy generalized regular connected between two IFSs A and B, then it is intuitionistic fuzzy C_5- connected between two IFSs A and B but the converse my not be true in general.

Proof

Suppose (X,τ) is not intuitionistic fuzzy C_5- connected between A and B, then there exists an intuitionistic fuzzy open set E in (X,τ) such that $A \subseteq E$ and Eq^cB.

Since every intuitionistic fuzzy open set is intuitionistic fuzzy generalized regular open set, there exists an intuitionistic fuzzy generalized regular open set E in (X,τ) such that $A \subseteq E$ and Eq^cB.

This implies (X,τ) is not intuitionistic fuzzy generalized regular connected between A and B, a contradiction to our hypothesis.

Therefore (X,τ) is intuitionistic fuzzy C_5- connected between A and B.

4.CONCLUSION

We have discussed about the intuitionistic fuzzy generalized regular connected space, intuitionistic fuzzy generalized regular super connected space and intuitionistic fuzzy generalized regular extremally disconnected space. In this definition, theorem and properties use the application. And also this application use the noise reduction by fuzzy Image filtering. This is my future research work fuzzy rules are fired to consider every direction around the processed pixel. Additionally, the shape of the membership function is adapted according to the remaining amount of noise after each iteration. Experimental result show the feasibility oh the new filter and a simple stop criterion, although its relative simplicity and the straightforward implementation of the fuzzy operators, the fuzzy filter is able to complement with state of the art filter techniques for noise reduction. A numerical measure, such as MSE, and visual observation show convincing results. Finally, the fuzzy filter scheme is sufficiently simple to enable fast hardware implementations.

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