GLOBAL STABILITY OF A CLASS OF DISCRETE-TIME RECURRENT NEURAL NETWORKS

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ABSTRACT

This paper presents several sufficient conditions and analytical results are established for the global asymptotic stability and global exponential stability of a class of discrete-time recurrent neural network with multiple time varying delays. The linear matrix inequality approach and the globally Lipschitz continuous and monotone non-decreasing activation functions are employed in our investigation. A necessary and sufficient condition is formulated to guarantee the existence and uniqueness of equilibria of such DTRNNS. The obtain results are less restrictive and different from existing ones are also applied to recurrent neural networks with constant time delays.

Keywords: *Asymptotic stability, Exponential stability, discrete-time recurrent neural network, multiple time varying delays, monotone, non-decreasing.*

1.INTRODUCTION:

Neural networks to solve many practical problems in optimization, control, and signal processing, we usually design neural networks to have a unique stable (or attractive) to avoid spurious responses or the problem of local minima. Hence, exploring the global asymptotic stability of neural networks is an important topic. In recent years, stability of neural networks are intensively studied. A global asymptotic stability (GAS) criterion for neural networks and discussed the potential of neural networks with such a stability for signal processing by using the well-known contraction mapping theorem. The global asymptotic stability (GAS) of the equilibrium of a discrete-time dynamical neural network with the saturation activation function using the Lyapunov function method. The results were used to design nth-order fixed point digital filters. They also pointed out that the GAS of neural networks guarentees the nonexistence of limit cycles in such filters. Also emphasized that it is very difficult to search for nondiagonal Lyapunov function. A necessary and sufficient condition for the absolute stability of neural networks with a symmetric connection matrix and discussed the potentials of the neural networks for solving optimization problems. A diagonal Lyapunov function for the Hopfield network with an asymmetric weight matrix, and established some absolute stability conditions which are more relaxed than the previous stability results on dynamical neural networks. More recently, the absolute stability is actually the global asymptotic stability for a given weight matrix and a given input. In this case a neural network has a unique equilibrium and the attractive region of the equilibrium is the whole space . Also discussed such kind of absolute stability for a class of discrete-time recurrent neural network. More recently , a class of continuous-time neural networks were widely applied to solve the linear variational inequalities and some global asymptotic stability and global exponential stability results for the neural networks. Most of the existing results pertain to the stability analysis of continuous-time recurrent neural networks. However, in many applications, we use discrete iteration process rather than continuous version. Generally speaking, the stability analysis of continuous-time recurrent neural networks is not applicable to the discrete version. This paper deals with the global asymptotic stability and global exponential stability of such discrete-time recurrent neural networks with globally Lipschitz continuous and monotone non-decreasing activation function. Several sufficient conditions for the global asymptotic stability and global exponential stability of such discrete-time recurrent neural networks are given. In these conditions, the weight matrix may not be restricted to symmetric. These stability conditions are more relaxed than the existing ones. This paper is organized as, the discrete-time recurrent neural network model, some definitions of stability, global asymptotic stability, and global exponential stability are given. Existence and uniqueness of equilibrium is discussed in existence and uniqueness of equilibrium global asymptotic stability gives a few conditions for global asymptotic stability of discrete-time neural networks. Several global exponential stability results are presented in global exponential stability.

2.Discrete-Time Neural Networks:

Theorem :

The $x(k+1) = Ax(k) + B\sigma(wx(k)+s), x(0) = x_0$ has a unique equilibrium for and$s \in R^n$and and$\sigma \in \mathcal{GL}$ if and only if the matrix $I - A - LBW$ is nonsingular for any diagonal L=diag($\ell_1, \dots \dots \dots \dots, \ell_n$) such that $\underline{L} \leq L \leq \overline{L}$.

Proof : (Necessity):

Suppose that$I - A - L_0BW$ is singular for some diagonal L_0 with $\underline{L} \leq L_0 \leq \overline{L}$. Now construct each $\sigma_i(\eta)$ as $\sigma_i(\eta) = \ell_{i0\eta}$ where ℓ_{i0} is the (i,j)th entry of L_0. Then, the equilibrium equation $(I-A)x^* = B\sigma(Wx^* + s)$ becomes $(I-A)x - B\sigma(Wx+s) = (I - A - L_0BW)x - L_0Bs = 0$ which either has infinitely many solutions or no solution depending on whether L_0Bs is in the range space of $I - A - L_0BW$or not. A contradiction occurs.

(Sufficiency) : We first show the existence of the equilibrium given $I - A - LBW$ is nonsingular; that is, there exists x such that $(I-A)x^* = B\sigma(Wx^* + s)$ holds. When $u = 0$, based on For any $u \in R^n$ if $I - A - LBW$ is nonsingular for any diagonal L= diag $(\ell_1, \ell_2, \ldots \ldots \ldots, \ell_n)$ such that $\underline{L} \le L \le \overline{L}$, i.e., $\underline{\ell}_i \le \ell_i \le \overline{\ell}_i$, then there exists x such that $(I-A)x = B\sigma(Wx+s)+u$ where $\underline{L} :=$ diag $(\underline{\ell}_1, \ldots \ldots \ldots, \underline{\ell}_n)$ and $\overline{L} :=$ diag $(\overline{\ell}_1, \ldots \ldots \ldots, \overline{\ell}_n)$, $(I-A)x^* = B\sigma(Wx^*+s)$ has a solution x^* given $I_n - A - LBW$ is nonsingular for any L. Next we show that x^* is unique by contradiction. Suppose that $z + x^*$ is also an equilibrium where $z \ne 0$. Note

$z(k+1) = Az(k) + B\big(\sigma(W(z(k)+x^*)+s) - \sigma(Wx^*+s)\big)$

$\triangleq Az(k) + Bg\big(Wz(k)\big)$ and $z(k+1) = (A + l(k)BW)z(k)$

we may have $z = Az + Bg(z)$ where $g(z) = \sigma(W(z+x^*)+s) - \sigma(Wx^*+s)$ and there exists

a diagonal matrix L such that $\underline{L} \le L \le \overline{L}, g(z) = LWz$, or $(I - A - LBW)z = 0$ which implies that $I - A - LBW$ is singular in terms of $z \ne 0$. On the other hand, $I - A - LBW$ is nonsingular for any L satisfying. A $\underline{L} \le L \le \overline{L}$ contradiction occurs.

Global Asymptotic Stability:

Theorem :

Let $G_i\,(i = 0,1,2, \ldots, n)$

$$A + LBW = \frac{1}{m_0} m_0 A + \frac{\ell_1}{m_1 \overline{\ell}_1} G_1 + \cdots + \frac{\ell_n}{m_n \overline{\ell}_n} G_n \triangleq \frac{1}{m_0} G_0 + \frac{\ell_1}{m_1 \overline{\ell}_1} G_1 + \cdots + \frac{\ell_n}{m_n \overline{\ell}_n} G_n$$

and
$$\begin{cases} F_0 \triangleq \sum_{i=1}^{n} \frac{\ell_i^2}{(m_i \overline{\ell}_i)^2} (G_i^T P G_i - P) + \sum_{i<j}^{n} \frac{\ell_i \ell_j}{m_i m_j \overline{\ell}_i \overline{\ell}_j} \left(G_i^T P G_j + G_j^T P G_i - 2P\right) + \frac{1}{m_0^2} (G_0^T P G_0 - P) \\ F_i \triangleq \frac{1}{m_0 m_i \overline{\ell}_i} (G_0^T P G_i + G_i^T P G_0 - 2P), \quad i = 1,2, \ldots \ldots \ldots, n \end{cases} \qquad (1)$$

The $x(k+1) = Ax(k) + B\sigma(wx(k)+s), x(0) = x_0$ is global asymptotic stability for any s if there exists constants $m_i \ge 1 (i = 0,1,2, \ldots \ldots, n)$ such that $\sum_{i=0}^{n} \frac{1}{m_i} \le 1$ and a matrix P > 0 such that i) $G_i^T P G_i - P < 0, i = 1, \ldots \ldots \ldots, n$;

ii) $F_0/n + \underline{\ell}_i F_i < 0$ and $F_0/n + \overline{\ell}_i F_i < 0, i = 1,2, \ldots \ldots \ldots \ldots, n$.

Proof:

In view of $\sum_{i=0}^{n} \frac{1}{m_i} \le 1$, we then have $\frac{1}{m_0} + \sum_{i=1}^{n} \frac{\ell_i}{m_i \overline{\ell}_i} \le 1$. Given any $\ell_i \in [\underline{\ell}_i, \overline{\ell}_i]$ one can always find $a_i, b_i \ge 0$ such that $a_i + b_i = 1$ and $\ell_i = a_i \underline{\ell}_i + b_i \overline{\ell}_i$, $i = 1,2, \ldots \ldots, n$.

Then ,

$F_0 + \sum_{i=1}^{n} \ell_i F_i = \sum_{i=1}^{n} (a_i + b_i) {}^{F_0}/_n + \sum_{i=1}^{n} (a_i \underline{\ell}_i + b_i \overline{\ell}_i)$

$F_i = \sum_{i=1}^{n} (a_i ({}^{F_0}/_n + \underline{\ell}_i F_i) + b_i ({}^{F_0}/_n + \overline{\ell}_i F_i)) < 0$ in view of ${}^{F_0}/_n + \underline{\ell}_i F_i < 0$ and ${}^{F_0}/_n + \overline{\ell}_i F_i < 0, i = 1,2, \dots \dots \dots, n.$

Hence,

$$(A + LBW)^T P(A - LBW) - P = (\frac{1}{m_0} G_0 + \sum_{i=1}^{n} \frac{\ell_i}{m_i \overline{\ell}_i} G_i)^T . P(\frac{1}{m_0} G_0 + \sum_{i=0}^{n} \frac{\ell_i}{m_i \overline{\ell}_i} G_i) - P \leq (\frac{1}{m_0} G_0 + \sum_{i=1}^{n} \frac{\ell_i}{m_i \overline{\ell}_i} G_i)^T . P(\frac{1}{m_0} G_0 - \sum_{i=1}^{n} \frac{\ell_i}{m_i \overline{\ell}_i} G_i) - (\frac{1}{m_0} + \sum_{i=1}^{n} \frac{\ell_i}{m_i \overline{\ell}_i}) . P\left(\frac{1}{m_0} + \sum_{i=1}^{n} \frac{\ell_i}{m_i \overline{\ell}_i}\right)$$

$$= \sum_{i=1}^{n} \frac{\ell_i^2}{m_i \overline{\ell}_i} (G_i^T P G_i - P) + \sum_{i<j}^{n} \frac{\ell_i \ell_j}{m_i m_j \overline{\ell}_i \overline{\ell}_j} . (G_i^T P G_j + G_j^T P G_i - 2P) + \frac{1}{m_0^2} (G_0^T P G_0 - P) + \sum_{i=1}^{n} \frac{\ell_i}{m_0 m_i \overline{\ell}_i} (G_0^T P G_i + G_i^T P G_0 - 2P) \leq \sum_{i=1}^{n} \frac{\ell_i^2}{(m_i \overline{\ell}_i)^2} (G_i^T P G_i - P) + \sum_{i<j}^{n} \frac{\ell_i \ell_j}{m_i m_j \overline{\ell}_i \overline{\ell}_j} . (G_i^T P G_j + G_j^T P G_i - 2P) + \frac{1}{m_0^2} (G_0^T P G_0 - P) + \sum_{i=1}^{n} \frac{\ell_i}{m_0 m_i \overline{\ell}_i} (G_i^T P G_i + G_i^T P G_0 - 2P) = F_0 + \sum_{i=1}^{n} \ell_i F_i < 0$$

……………………(2) In this first inequality of $F_0 + \sum_{i=1}^{n} \ell_i F_i < 0$, we use the inequality by nothing

$$\frac{1}{m_0} + \sum_{i=1}^{n} \frac{\ell_i}{m_i \overline{\ell}_i} \leq 1 - P \leq -(\frac{1}{m_0} + \sum_{i=1}^{n} \frac{\ell_i}{m_i \overline{\ell}_i}) P (\frac{1}{m_0} + \sum_{i=1}^{n} \frac{\ell_i}{m_i \overline{\ell}_i}) .$$

According to If there exists a matrix P > 0 such that

$(A + LBW)^T P(A + LBW) - P < 0$ for any L satisfying $\underline{L} \leq L \leq \overline{L}$, then $x(k+1) = Ax(k) + B\sigma(wx(k) + s), x(0) = x_0$ is global asymptotic stability for any s. it is easy to see that the

$$x(k+1) = Ax(k) + B\sigma(wx(k) + s), x(0) = x_0$$

is global asymptotic stability for any s.

Global Exponential Stability :

Theorem :

The $x(k+1) = Ax(k) + B\sigma(wx(k) + s), x(0) = x_0$ is global exponential stability for any s if $\widehat{M}$ in

$$\widehat{M} = [\widehat{m}_{ij}]_{n \times n} \triangleq \begin{bmatrix} |a_1 + \hat{\ell}_1 b_1 w_{11}| & \overline{\ell}_1 |b_1 w_{12}| & \overline{\ell}_1 |b_1 w_{1n}| \\ \overline{\ell}_2 |b_2 w_{21}| & |a_2 + \hat{\ell}_2 b_2 w_{22}| & \overline{\ell}_2 |b_2 w_{2n}| \\ \vdots & \vdots \ \ddots & \vdots \\ \overline{\ell}_n |b_n w_{n1}| & \overline{\ell}_n |b_n w_{n2}| & |a_n + \hat{\ell}_n b_n w_{nn}| \end{bmatrix}$$

is a stable matrix.

Proof :

We first show that $z(K+1) = (A + L(k)BW)z(k)$ and the following system :

$$y(k+1) = \widehat{M}y(k) \quad \ldots\ldots\ldots\ldots\ldots\ldots \ (3)$$

where $y(k) = \left(y_1(k), y_2(k), \ldots \ldots \ldots \ldots \ldots, y_n(k)\right)^T$. By induction, we will prove

$$|z_i(k)| \leq y_i(k) \quad \ldots\ldots\ldots\ldots\ldots\ldots \ (4)$$

$\forall k = 0,1,2, \ldots \ldots, i = 1, \ldots \ldots \ldots, n. \forall z(0) \in R^n$,

let $y(0) \triangleq \left(y_1(0), y_2(0), \ldots \ldots \ldots \ldots \ldots, y_n(0)\right)^T$

$= (|z_1(0)|, |z_2(0)|, \ldots \ldots \ldots, |z_n(0)|)^T$.

Obviously, $y_i(0) \geq 0$ and $\|z(0)\| = \|y(0)\|$. When $k = 1$, it follows from $y(k+1) = \widehat{M}y(k)$ that $y_i(1) = \sum_{j=1}^{n} \widehat{m}_{ij} y_j(0) \geq 0, 1 \leq i \leq n$.On the other hand, applying absolute value into the $z(K+1) = (A + L(k)BW)z(k)$ yields

$|z_i(1)| = \left|(a_i + \ell_i(1)b_i w_{ii})z_i(0) + \sum_{j=1, j\neq i}^{n} \ell_i(1) b_i w_{ij} z_j(0)\right| \leq \left|a_i + \hat{\ell}_i b_i w_{ii}\right| |z_i(0)| + \sum_{j=1, j\neq i}^{n} \overline{\ell}_i |b_i w_{ij}| |z_j(0)| = \sum_{j=1}^{n} \widehat{m}_{ij} y_j(0) = y_i(1)$

by recalling $\left|a_i + \hat{\ell}_i b_i w_{ii}\right| = \min_{\underline{\ell}_i \leq \hat{\ell}_i \leq \overline{\ell}_i} \left|a_i + \hat{\ell}_i b_i w_{ii}\right|, 1 \leq i \leq n$. Now assume that $|z_i(k)| \leq y_i(k)$ for $k = K, 1 \leq i \leq n$. Our next task is prove $|z_i(k)| \leq y_i(k)$ for $k = K + 1$.

According to $y(k+1) = \widehat{M}y(k)$, we may have

$$y_i(k+1) = \sum_{j=1}^{n} \widehat{m}_i y_j(k) \geq 0 \quad \ldots\ldots\ldots\ldots\ldots \ (5)$$

Moreover, from $z(K+1) = (A + L(qk)BW)z(k)$ it follows that :

$|z_i(k+1)| = \left|(a_i + \ell_i(k)b_i w_{ii})z_i(k) + \sum_{j=1, j\neq i}^{n} \ell_i(k) b_i w_{ij} z_j(k)\right| \leq \left|a_i + \hat{\ell}_i b_i w_{ii}\right| |z_i(k)| + \sum_{j=1, j\neq i}^{n} \overline{\ell}_i |b_i w_{ij}| |z_j(k)| = \sum_{j=1}^{n} \widehat{m}_{ij} |z_j(k)| \leq \sum_{j=1}^{n} \widehat{m}_{ij} y_i(k) = y_i(k+1)$

. Thus, we have proved $|z_i(k)| \leq y_i(k)$ by mathematical induction.

According to $|z_i(k)| \leq y_i(k)$, we may immediately obtain

$$\|z(k)\| \leq \|y(k)\|, \quad k = 0,1,2, \ldots \ldots \quad \ldots\ldots\ldots\ldots\ldots\ldots \ (6)$$

Since $\widehat{M}$ is a stable matrix, $\forall y(0) \in R^n$ and $y_i(0) \geq 0$ the solution $y(k)$ of the linear time – invariant system $y(k+1) = \widehat{M}y(k)$ will converge exponentially to the equilibrium $y^* = 0$, i.e., there exist two constant $\xi, \eta > 0$ such that, $\|y(k)\| \leq \xi \exp(-\eta k)$, for $k \geq 0$. Therefore,$\forall z(0) \in R^n$, in view of $\|z(k)\| \leq \|y(k)\|$, $k = 0,1,2, \ldots \ldots, n$ the solution $z(k)$ of system $z(K+1) = (A + L(k)BW)z(k)$ satisfy $\|z(k)\| \leq \xi \exp(-\eta k)$, for $k \geq 0$.So, system $z(K+1) = (A + L(k)BW)z(k)$ at $z^* = 0$ is globally exponentially convergent. When then show that system $z(K+1)=(A+L(k)BW)z(k)$ is stable at $z^*=0$. Since $\widehat{M}$ is a stable matrix, system $y(k+1)=\widehat{M}y(k)$ at $y^*=0$ is stable.

Thus, any given $\in > 0, \exists\delta=\delta(\varepsilon)>0$ such that $\|y(k)\|<\varepsilon\forall k=0,1,2,\dots$ whenever $\|y(o)\|<\delta\varepsilon$ where each $y_i(0)\geq 0$. consider, any $z(0)\in R^n$ and $\|z(o)\|<\delta\varepsilon$.

Let $y_i(0)=|z_i(0)|>0$, then $\|y(o)\|= \|z(o)\|<\delta\varepsilon$. As a result, $\|y(k)\|<\varepsilon$ $\forall k=0,1,2,\dots$ note $\|z(k)\|\leq\|y(k)\|, k=0,1,2,\dots\dots$ we may have $\|z(k)\|<\varepsilon$ $\forall k=0,1,2,\dots$ this shows that system $z(K+1)=(A+L(k)BW)z(k)$ is stable according to the $x(k+1)=Ax(k)+B\sigma(wx(k)+s), x(0)=x_0$ is said to be stable at x^* if $\forall\varepsilon>0$ there exists a $\delta(\varepsilon)>0$ such that $\|x(k)-x^*\|<\varepsilon, \forall k=0,1,\dots\dots\dots$ whenever

$$\|x_0-x^*\|<\delta(\varepsilon).$$

The $x(k+1)=Ax(k)+B\sigma(wx(k)+s), x(0)=x_0$ is said to be global asymptotic stability if it has a unique stable equilibrium x^* and the attractive region of x^* is the whole space or the state solution $\phi(k,x_0,s)$ for any initial state $x_0\in R^n$ satisfie s $x^*=\lim_{k\to+\infty}\phi(k,x_0,s)$.

The $x(k+1)=Ax(k)+B\sigma(wx(k)+s), x(0)=x_0$ is said to be global exponential stability if for any initial state x_0 the state solution $\phi(k,x_0,s)$ approaches this unique equilibrium in terms of norm ; that is , there exist two positive constants $\xi,\eta>0$ such that $\|x(k)-x^*\|\leq\xi\exp(-\eta k)$, for$k\geq 0$.Therefore, we have proved the DTRNN, $z(K+1)=(A+L(k)BW)z(k)$ is global exponential stability. Now given any $L=diag(l_1,l_2,\dots\dots l_n)$ satisfying $\underline{L}\leq L\leq\overline{L}$ repeating the above procedure can always lead to the following:$z(k+1)=(A+LBW)z(k)$.Is global exponential stability;that is, $A+LBW$ is stable for any L. In view of the $x(k+1)=Ax(k)+B\sigma(wx(k)+s), x(0)=x_0$ has a unique equilibrium for any s if the matrix $A+LBW$ is stable for any L satisfying $\underline{L}\leq L\leq\overline{L}$ shows that once $A+LBW$ is stable for any L, the $x(k+1)=Ax(k)+B\sigma(wx(k)+s), x(0)=x_0$ has a unique equilibrium for any s. In this case, global asymptotic stability of the $x(k+1)=Ax(k)+B\sigma(wx(k)+s), x(0)=x_0$

at x^* is equivalent to that ofthe $z(k+1) = (A + l(k)BW)z(k)$ at $z = 0$, the DTRRN's $x(k+1) = Ax(k) + B\sigma(wx(k) + s), x(0) = x_0$ is global asymptotic stability for any s.

$\dot{V}(x(t)) = 0$ if and only if $f(x(t)) = x(t) = f(x(t - \mathcal{T}_i(t)) = 0$. Otherwise, $\dot{V}(x(t)) < 0$. By Lyapunov stability theory, it follows that the origin of system $\dot{x}(t) = -Ax(t) + Wf(x(t)) + \sum_{k=1}^{N} W_k f\left(x\left(t - \mathcal{T}_k(t)\right)\right)$ is globally asymptotically stable.

Hence the proof

CONCLUSION:

In this paper several sufficient conditions and analytical results are established on the global asymptotic stability and global exponential stability of discrete time recurrent neural networks with multiple time-varying delays. The Lyapunov-Krasovskii stability theory for functional differential equations and linear matrices inequality approach are employed with existence and uniqueness of an equilibrium is also given as a matrix determinant in this study. Global asymptotic stability and global exponential stability results are also established for discrete-time recurrent neural networks with multiple time-varying delays. The obtained results extend the existing ones on global asymptotic stability and global exponential stability of neural networks with special classes of activation functions. Finally, global asymptotic stability and global exponential stability results are established for discrete time recurrent neural networks with constant time delays.

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