GRADING OF KINEMATIC CHAINS ON THE BASIS OF SYMMETRY

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ABSTRACT

An attempt has been made to reveal the characteristics of kinematic chains on the basis of topology. Isomorphism, inversions, symmetry, angular velocity, angular acceleration and jerk can be determined with the help of gradient concept and chains can be graded on the basis of all the above said characteristics of kinematic chains but in this paper chains are graded on the basis of symmetry. Concept is applied in this paper for 6-link one degree of freedom kinematic chains though it is also useful in 8-link, 10-link and 12-link for all known families of kinematic chains i.e. single degree of freedom, multi degree of freedom and graded them on the basis of symmetry. Kinematic chains are directly used for writing the distance matrix. First gradient and second gradient matrices are formed, on the basis of first order gradient matrix first gradient link values and first gradient total value can be determined. Similarly on the basis of second order gradient matrix second gradient link values and second gradient total value can be determined. In this paper second order gradient matrix is used to grade the kinematic chains on the basis of symmetry which discloses flow of magnitude of relative angular acceleration. This concept has great potential and can be used to study symmetry and parallelism which are useful in structural synthesis and platform-type robots respectively.

Keyword: - *Kinematic chains, symmetry, mechanisms, first gradient total value, second order total value*

1. INTRODUCTION

A mechanism is a kinematic chain where at least one link has been "grounded," or fixed, to the frame of reference. A mechanism is a movable closed kinematic chain where one of its link is fixed. A six- bar linkage, if one of its link is fixed to form the frame of the mechanism, as represented in figure-1.

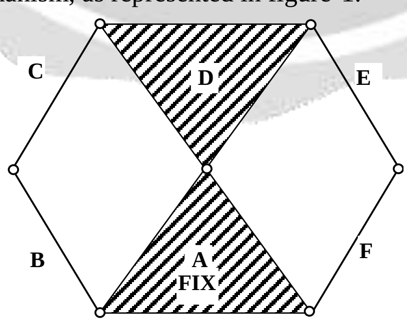
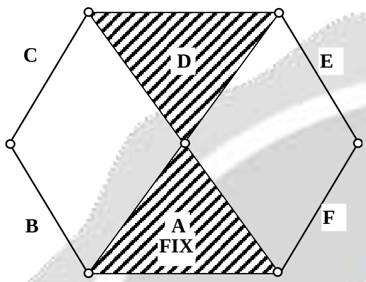


Fig -1: Six Bar Mechanism with Ternary A fix

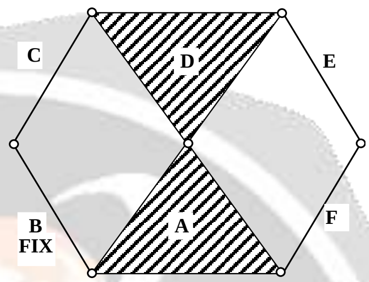
The degree of freedom of an assemblage of links completely predicts its characteristics. There are three possibilities. When the degree of freedom is positive, then it will be a mechanism, and all the links will have the relative motions. If the degree of freedom is exactly zero, it will be a structure, and hence there is no motion is possible. In case when the degree of freedom is negative, then we have to obtain a preloaded structure, which means that there will no rotation is possible and some stresses may also be exist while assembly of links.

1.1 Kinematic Inversion

In any mechanism when different links are grounded one by one then an inversion is found. So there may be as many inversions in a given mechanism. The number of inversions depends upon the number of links a mechanism has. The motions found from each inversion can be different, but some inversions in a mechanism may have motions similar to that of other inversions in the same mechanism. In that cases some inversions may have distinctly different motions. We will represent the inversions which have distinctly different motions as distinct inversions. The Watt's six link chain has two different inversions, and Stephenson's six link chain has three different inversions. The six link Watt’s chain give up two different six link mechanisms as represented in figure-2.



Watt’s Six Link Inversion I



Watt’s Six Link Inversion II

Fig -2: Inversions of the six link Watt chain

1.2 Degrees of Freedom (DOF)

Any type of mechanical system can be categorised according to the number of degrees of freedom (DOF) which it has. The system's DOF is equal to the number of independent parameters (measurements) which are required to uniquely describe their position in the space at any instant of time. Note that the DOF is defined with respect to the selected frame of reference.

Every member of mechanism can move for certain direction or/and rotate about certain axes and constrained in other directions. Thus the Degree of Freedom is the determination of the possible movements of any mechanisms.

The degree of freedom or mobility, D_F, of a kinematic chain or mechanism can be described as the number of independent inputs required to determine the positions of all the links of a chain. One degree of freedom of a mechanism i.e. $D_F = 1$, shows that if any point in the mechanism is moved on a given path, all the other points will have uniquely determined (Constrained) motions. Two degree of freedom of a mechanism i.e. $D_F=2$, that means two independent motions must be presented at the two different points in the mechanism. Corresponding condition put on for more than two degree of freedom. The degree of freedom for the four- bar chain in figure-3 (a) is unity, because only one variable such as Φ_1 is necessary to describe the relative position of all the links. A five- bar chain in figure-3 (b) has two degree of freedom, because two angles such as Φ_1 and Φ_2 are necessary to describe the relative position of all the links.

A kinematic chain is called movable if its number of degree of freedom is one or greater ($D_F \geq 1$); otherwise it is said to be locked ($D_F \leq 1$).

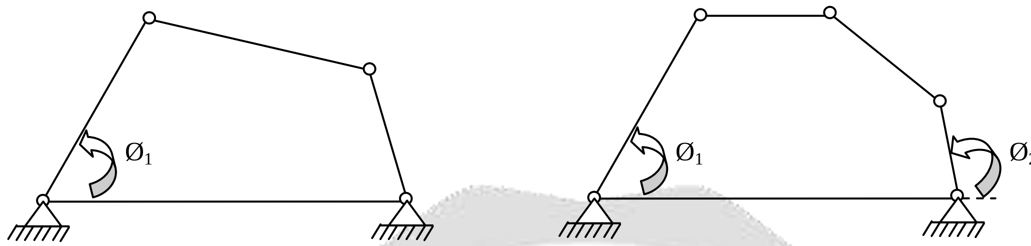
To find out the overall DOF of any type of mechanism, we must have knowledge about the number of links and the number of joints, and the interactions between them. The DOF of any type of assembly of the links can be predicted by an investigation of the Gruebler condition.

$$M = 3L-2J-3G \qquad (1.1)$$

Where:
M = degree of freedom or mobility
L = number of links
J = number of joints
G = number of fixed links

In planar mechanisms, the degree of freedom for an n-link chain connected by J_1 joints which has 1 degree of freedom and J_2 joint which has 2 degree of freedom, with the fixed link considered as one of the links is given by Grubler's criterion as given below:

$$M = 3(L-1) - 2J_1 - J_2 \quad (1.2)$$



(a) Four bar chain, $D_F = 1$ (b) Four bar chain, $D_F = 2$

Fig -3: Mechanism with DOF = 1 and 2

1.3 Grashof Law

The Grashof law is a relationship which tells us about the rotational behaviour of a four bar mechanisms inversions based only on the link lengths.

Let S = Shortest link length.
L = Longest link length.
P = length of one remaining link.
Q = length of another remaining link.

And if $$S+L \leq P+Q \quad (1.3)$$

The linkage is Grashof and at least one link will be able to making a complete revolution with respect to the fixed plane. This is known as a Class I kinematic chain. If an inequality is not satisfied, then the linkage is non-Grashof and none of the link will be able to making a full rotation relative to any other link. This is known as Class II kinematic chain.

In order to assemble a four links the longest link must be shorter than the sum of the remaining three links,

$$L < (S+P+Q) \quad (1.4)$$

1.4 Kinematic Synthesis

Kinematic synthesis is an essential step in the first stage of designing a machine, as it shows the creation of mechanism in order to achieve a desired set of motion characteristics. The overall problem in synthesis is basically approached in three phases.

1. Type synthesis.
2. Number synthesis.
3. Dimensional synthesis.

- Type synthesis refers to the type of mechanism selected, like linkages, a geared system, belts, rack & pinion, pulleys, shaft or a cam and follower system.
- Number synthesis comprises of number of links and the number of joints or pairs of the particular type required to find a given number of independent inputs to the mechanisms.
- Dimensional synthesis deals with the determination of the dimensions of the parts-length, angles etc., necessary to produce a mechanism that results in specified motion requirements.

1.5 Number Synthesis

The term number synthesis has been coined to mean the determination of the number and order of links and joints required to produce motion of a certain DOF. Order in this context means the number of nodes per link, for example: binary, ternary, quaternary, etc. The value of number synthesis is to allow the exhaustive determination of all the possible combinations of links which will yield any selected DOF. This is then prepares the designer with a final catalogue of potential linkages to resolve a variety of motion control problems. Number synthesis of mechanisms gives a thorough and systematic method of discovering, displaying, innovating and evaluating all the possible methods of obtaining a desired motion.

In order to obtain the desired motion a collection of connected links must meet creating requirements i.e. relative motion between the two links should be possible. Crossely has been developed an algorithm and accordingly reported all the possible combinations of polygonal links for a certain number of links with degree of freedom D_F = 1 as represented in Table 1.

Table -1: All possible Combination for links of 1 DOF planar Kinematic Chain upto 10 links

Total links	Link sets				
	Binary	Ternary	Quaternary	Pentagonal	Hexagonal
4	4	0	0	0	0
6	4	2	0	0	0
	5	0	1	0	0
8	7	0	0	0	1
	4	4	0	0	0
	5	2	1	0	0
	6	0	2	0	0
	6	1	0	1	0
10	4	6	0	0	0
	5	4	1	0	0
	6	3	0	1	0
	6	2	2	0	0
	7	1	1	1	0
	7	0	3	0	0
	8	0	0	2	0
	8	0	1	0	1

1.6 Types of Motion

Pure rotation: The body has a one point (centre of rotation) where there is no motion with respect to the "fixed" frame of reference. All the other points on that body describe arcs about that centre. A reference line has drawn on that body through the centre changes only their angular orientation.

Pure translation: All the points on the body describes parallel (curvilinear or rectilinear) paths. A reference line drawn on that body changes their linear position but the same time their angular orientation does not change.

Complex motion: complex motion is nothing but a simultaneous combination of a rotational motion and translational motion. Any reference line drawn on the body will change both their linear position as well as their angular orientation. Points on that body will travel non-parallel paths, and there will be, at each instant, a centre of rotation, which will continuously change location.

2. METHODOLOGY

Numerous distinct and different kinematic chains comprising of same number of links and joints with the same degree of freedom are exists. It is very difficult for a designer to choose the best for the specified objective and has to depend largely on the intuition based on experience, knowledge and information from the available broad choice. The action of chain basically depends upon the number of links, number of joints, kind of loops as well as its topology. Two distinct but different chains with the same degree of freedom and same number of links as well as same design parameters will have different capabilities for creating motion and this fact can be attributed to the difference in the structural topology of the chains. The designer must be able to read a chain and their anticipated performance, in a relative sense, without going through the complete synthesis and analysis.

In order to accomplish the above, a gradient concept based on distances amongst various links is proposed. This will enable the designer to find out angular acceleration among kinematic chains and compare the chains for various kinematic properties.

2.1 Distance matrix

A distance matrix D can be written for every kinematic chain by incorporating the distances between the various links. The elements of the matrix will be D_{ik} = least number of joints between links I and K with all diagonal elements $D_{ii} = 0$. Sum of node values of the links which are joined together gives us joint value and sum of joint values of the links which are having least distance gives us elements of the matrix. This matrix is called distance matrix.

2.1.1 First order gradient matrix

Gradient Matrix: Taking analogy of water flow between tanks (links), connected by pipelines (joints), it is easy to note that a gradient is necessary for water flow (motion transfer). The quantum of flow is dependent upon the magnitude of the gradient. Gradient can be downward or upward, the former increases the rate and quantum of flow while the other reduces the same. The present analogy belongs to the later case, that is higher the gradient lesser is the flow (motion). First order and second order gradients are defined in terms of distances deal in reality with the relative location of links in a chain. First Order Gradient Matrix F, with elements F_{IK} and Second Order Gradient Matrix S with elements S_{ik} between two links I and K are proposed as

$$F_{IK} = \sum_{I=J=1}^{n} \left(\left| D_{IJ} - D_{KJ} \right| \right) / D_{IK} \qquad (2.1)$$

and

$$S_{IK} = \sum_{I=J=1}^{n} \left(\left| F_{IJ} - F_{KJ} \right| \right) / F_{Ik} \qquad (2.2)$$

where I and K represents the rows and columns of the matrix.

From the definition of the gradient, it is clear that the numerator is the cumulative difference of the two rows representing the respective links and denominator is the distance between the two links I and K. The matrices so developed will be square and symmetric in nature.

According to the definition, element F_{12} for first order gradient matrix F_{IK} for watt's chain will be

$$F_{12} = \sum_{j=i=1}^{n} \left(\frac{|D_{1j} - D_{2j}|}{} \right) / D_{12}$$

$$= \frac{| D_{11} - D_{21} | + | D_{12} - D_{22} | + | D_{13} - D_{23} | + | D_{14} - D_{24} | + | D_{15} - D_{25} | + | D_{16} - D_{26} |}{D_{12}}$$

$$= \frac{|0-4|+|4-0|+|6-6|+|2-6|+|6-10|+|4-8|}{4}$$

$$= 5$$

Similarly, other elements of matrix F_{ik} can be calculated and matrix F_{ik} will be generated for Watt`s chain.

2.1.2 Second order gradient matrix

In a similar way Second Order Gradient Matrix for Watt`s chain can be generated. The element S_{12} for Second Order Gradient Matrix S_{IK} for Watt's chain will be

$$S_{12} = \sum_{j=i=1}^{n} \left(\frac{|F_{1j} - F_{2j}|}{} \right) / F_{12}$$

$$= \frac{|F_{11} - F_{21}| + |F_{12} - F_{22}| + |F_{13} - F_{23}| + |F_{14} - F_{24}| + |F_{15} - F_{25}| + |F_{16} - F_{26}|}{F_{12}}$$

$$= \frac{|0-5|+|5-0|+|4-3.33|+|6-4|+|4-2.80|+|5-3|}{5}$$

$$= 3.17$$

Similarly, other elements of Second Order Gradient Matrix S_{ik} can be calculated and matrix S_{ik} will be generated for watt's Chain.

Distance Matrix, First Order Gradient Matrix and Second Order Gradient Matrix for all the 6-link, 8-link, 10-link, 12-link etc. kinematic chain can be calculated. But methodology is explained here with the help of 6-link kinematic chain.

(1) For 6-link kinematic chain (Watt's Chain):

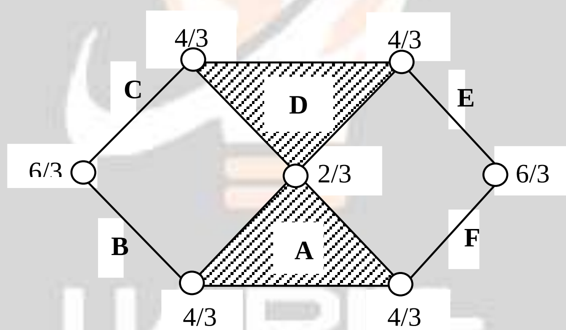


Fig 4: Watt's Chain

Distance Matrix

$$D_1 = (1/3) \begin{array}{c} \textbf{Links} \\ \textbf{A} \\ \textbf{B} \\ \textbf{C} \\ \textbf{D} \\ \textbf{E} \\ \textbf{F} \end{array} \begin{array}{c} \begin{array}{cccccc} \textbf{A} & \textbf{B} & \textbf{C} & \textbf{D} & \textbf{E} & \textbf{F} \end{array} \\ \begin{pmatrix} 0 & 4 & 6 & 2 & 6 & 4 \\ 4 & 0 & 6 & 6 & 10 & 8 \\ 6 & 6 & 0 & 4 & 8 & 10 \\ 2 & 6 & 4 & 0 & 4 & 6 \\ 6 & 10 & 8 & 4 & 0 & 6 \\ 4 & 8 & 10 & 6 & 6 & 0 \end{pmatrix} \end{array}$$

First Order Gradient Matrix

F_1 =

Links	A	B	C	D	E	F	FGLV
A	0.00	5.00	4.00	6.00	4.00	5.00	**24.00**
B	5.00	0.00	3.33	4.00	2.80	3.00	**18.13**
C	4.00	3.33	0.00	5.00	3.00	2.80	**18.13**
D	6.00	4.00	5.00	0.00	5.00	4.00	**24.00**
E	4.00	2.80	3.00	5.00	0.00	3.33	**18.13**
F	5.00	3.00	2.80	4.00	3.33	0.00	**18.13**
						FGTV	**120.52**

For ease of comparison, the sum of each row of the matrix is carried out and is represented as the First Gradient Link Value (FGLV), of individual link, while the sum of all the elements in the matrix is represented as the First Gradient Total Value (FGTV), of the chain. For the Watt's chain, the link FGLVs are respectively 24.00, 18.13, 18.13, 24.00, 18.13 and 18.13 while FGTV of the chain is 120.52.

Second Order Gradient Matrix

S_1 =

Links	A	B	C	D	E	F	SGLV
A	0.00	3.17	3.47	2.67	3.47	3.17	**15.95**
B	3.17	0.00	2.72	3.47	2.95	2.36	**14.67**
C	3.47	2.72	0.00	3.17	2.36	2.95	**14.67**
D	2.67	3.47	3.17	0.00	3.17	3.47	**15.95**
E	3.47	2.95	2.36	3.17	0.00	2.72	**14.67**
F	3.17	2.36	2.95	3.47	2.72	0.00	**14.67**
						SGTV	**90.58**

The above matrix is also square and symmetric. The sum of each row of Second Order Gradient Matrix (S_{ik}) is Second Gradient Link Value (SGLV) of the link while the sum of elements of the matrix is Second Gradient Total Value (SGTV) of the chain. The SGLVs are 15.95, 14.67, 14.67, 15.95, 14.67 and 14.67 while SGTV of the chain is 90.58.

(2) For 6-link kinematic chain (Stephenson's Chain):

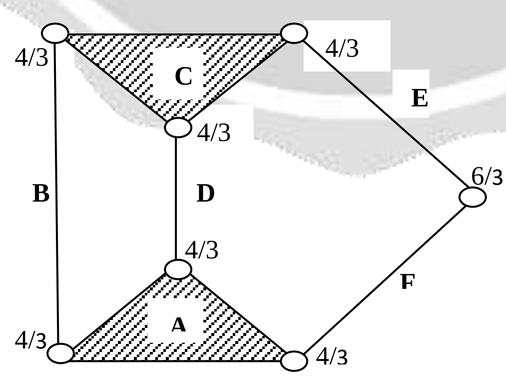


Fig -5: Stephenson's Chain

Distance Matrix

$$D_2 = (1/3)\ \begin{array}{c|cccccc} \textbf{Links} & \textbf{A} & \textbf{B} & \textbf{C} & \textbf{D} & \textbf{E} & \textbf{F} \\ \begin{array}{c}\textbf{A}\\\textbf{B}\\\textbf{C}\\\textbf{D}\\\textbf{E}\\\textbf{F}\end{array} & \multicolumn{6}{c}{\begin{pmatrix} 0 & 4 & 8 & 4 & 10 & 4 \\ 4 & 0 & 4 & 8 & 8 & 8 \\ 8 & 4 & 0 & 4 & 4 & 10 \\ 4 & 8 & 4 & 0 & 8 & 8 \\ 10 & 8 & 4 & 8 & 0 & 6 \\ 4 & 8 & 10 & 8 & 6 & 0 \end{pmatrix}} \end{array}$$

First Order Gradient Matrix

F_2 =

Links	A	B	C	D	E	F	FGLV
A	0.00	5.50	3.50	5.50	3.40	5.50	**23.40**
B	5.50	0.00	5.50	2.00	3.00	3.00	**19.00**
C	3.50	5.50	0.00	5.50	5.50	3.40	**23.40**
D	5.50	2.00	5.50	0.00	3.00	3.00	**19.00**
E	3.40	3.00	5.50	3.00	0.00	4.00	**18.90**
F	5.50	3.00	3.40	3.00	4.00	0.00	**18.90**
						FGTV	**122.60**

FGTV on the basis of FGLVs is 122.60, 2(23.40), 2(19.00), 2(18.90).

Second Order Gradient Matrix

S_2 =

Links	A	B	C	D	E	F	SGLV
A	0.00	3.53	3.20	3.53	4.50	3.04	**17.80**
B	3.53	0.00	3.53	2.00	3.37	3.37	**15.80**
C	3.20	3.53	0.00	3.53	3.04	4.50	**17.80**
D	3.53	2.00	3.53	0.00	3.37	3.37	**15.80**
E	4.50	3.37	3.04	3.37	0.00	3.05	**17.33**
F	3.04	3.37	4.50	3.37	3.05	0.00	**17.33**
						SGTV	**101.86**

SGTV on the basis of SGLVs will be 101.86, 2(17.80), 2(15.80), 2(17.33).

3. ISOMORPHISM

Based on the developed first gradient total value and second gradient total value invariants, a two level isomorphism detection test rule is proposed as following.

"If FGTV and SGTV of any two kinematic chains are similar, then they are isomorphic otherwise not."

Detection of probable isomorphism between the chains is an important problem during structural synthesis of the chains. A number of attempts have been made in the past to create methods for detecting isomorphism. Suggested method is also an attempt in this trend. The proposed method is based on link connectivity. Joints are important element of kinematic chains. Very less work is reported with respect to joint based methods for detection of isomorphism. The proposed method is simple and involves little mathematical calculations. The method is successfully applied to planar kinematic chains of single degree of freedom with six, eight, ten and twelve bar links. The examples reported in the earlier works are also tested and the method is found to be working well.

We see that FGTV for watt's (figure-4) and stephenson's (figure-5) kinematic chain is different, therefore these two 6-link kinematic chains are distinctly different and we can say that these kinematic chains are non-isomorphic. The same conclusion we are getting when we compare the SGTV of both the 6-link kinematic chain.

4. SYMMETRY

The number of inversion in any kinematic chain is equal to the no. of links it has, but the number of distinct inversion may differ. The symmetry of any kinematic chain is related to the distinct inversion. When any one of the link of a kinematic chain is fixed and then distance matrix is prepared accordingly, it is observed from the first order gradient matrix and second order gradient matrix, that the link value of all the links may be entirely different or some of the links may have identical (same) link value. Then we can say that the links which have same link value are in symmetry. The links which are in symmetry gives the same result and have similar behaviour.

4.1 Symmetry for Watt chain (Figure - 4)

Now we know that for Watt chain, link A & D will have the same inversions and link B, C, E & F will have the same inversions. For finding symmetry among kinematic links, link A or D is fixed and link B, or C, or E or F is fixed simultaneously.

As shown in figure-6, Firstly the link A of the Watt chain is fixed. Therefore its contribution to the node connecting links B and F is neglected. A matrix will be formed showing the distance between links. The DISTANCE MATRIX (DM) for Watt chain when link A is fixed can be given as explained in methodology.

(1) Watt's chain when link A is fixed –

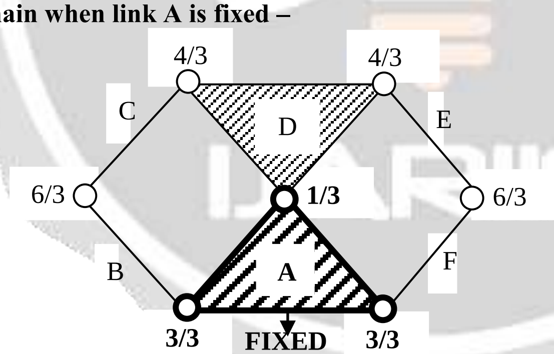


Fig -6: Watt's chain when link A is fixed

Distance Matrix

$$D_3 = (1/3)\quad \begin{array}{c} \textbf{Links} \\ \textbf{A} \\ \textbf{B} \\ \textbf{C} \\ \textbf{D} \\ \textbf{E} \\ \textbf{F} \end{array} \begin{array}{c} \begin{array}{cccccc} \textbf{A} & \textbf{B} & \textbf{C} & \textbf{D} & \textbf{E} & \textbf{F} \end{array} \\ \begin{pmatrix} 0 & 3 & 5 & 1 & 5 & 3 \\ 3 & 0 & 6 & 10 & 14 & 20 \\ 5 & 6 & 0 & 4 & 8 & 14 \\ 1 & 10 & 4 & 0 & 4 & 10 \\ 5 & 14 & 8 & 4 & 0 & 6 \\ 3 & 20 & 14 & 10 & 6 & 0 \end{pmatrix} \end{array}$$

First Order Gradient Matrix

F_3 =

Links	A	B	C	D	E	F	FGLVI
A	0.00	14.0	6.00	18.0	6.00	14.0	**58.00**
B	14.0	0.00	5.33	4.40	3.71	2.80	**30.24**
C	6.00	5.33	0.00	6.00	4.00	3.71	**25.04**
D	18.0	4.40	6.00	0.00	6.00	4.40	**38.80**
E	6.00	3.71	4.00	6.00	0.00	5.33	**25.04**
F	14.0	2.80	3.71	4.40	5.33	0.00	**30.24**
						FGTVI	**207.36**

Second Order Gradient Matrix

S_3 =

Links	A	B	C	D	E	F	SGLVI
A	0.00	3.98	7.49	3.07	7.49	3.98	**26.01**
B	3.98	0.00	4.03	3.94	5.63	3.16	**20.74**
C	7.49	4.03	0.00	4.60	2.81	5.63	**24.56**
D	3.07	3.94	4.60	0.00	4.60	3.94	**20.15**
E	7.49	5.63	2.81	4.60	0.00	4.03	**24.56**
F	3.98	3.16	5.63	3.94	4.03	0.00	**20.74**
						SGTVI	**136.76**

Now it is observed from the first order gradient matrix that –
The link value of link A = 58.00, B & F = 30.24, C & E = 25.04, D = 38.80
And from second order gradient matrix –
The link value of link A =26.01, B & F = 20.74, C & E = 24.56, D = 20.15

From the above link values it is thus clear that when the link A of the Watt chain is fixed, link B & F will have symmetrical behavior because of identical values. Similarly link C & E will have symmetrical behaviour.

Now the link B of the Watt chain is fixed as shown in figure-7. The Distance Matrix for Watt chain when the link B is fixed can be given as explained in methodology.

(2) Watt's chain when link B is fixed –

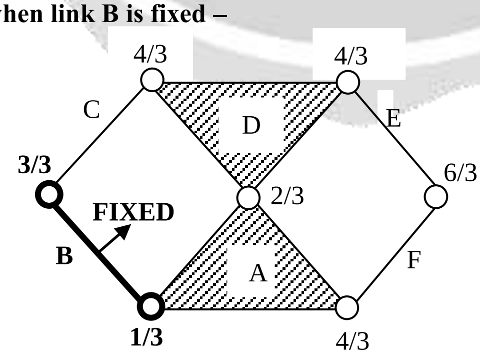


Fig -7: Watt's chain when link B is fixed

Distance Matrix

D_4 = (1/3)

Links	**A**	**B**	**C**	**D**	**E**	**F**
A	0	1	6	2	6	4
B	1	0	3	3	7	5
C	6	3	0	4	8	10
D	2	3	4	0	4	6
E	6	7	8	4	0	6
F	4	5	10	6	6	0

First Order Gradient Matrix

F_4 =

Links	**A**	**B**	**C**	**D**	**E**	**F**	**FGLVI**
A	0.00	8.00	4.00	6.00	4.00	5.00	**27.00**
B	8.00	0.00	6.00	4.00	3.71	4.80	**26.51**
C	4.00	6.00	0.00	5.00	3.00	2.80	**20.80**
D	6.00	4.00	5.00	0.00	5.00	4.00	**24.00**
E	4.00	3.71	3.00	5.00	0.00	3.33	**19.04**
F	5.00	4.80	2.80	4.00	3.33	0.00	**19.93**
						FGTVI	**137.28**

Second Order Gradient Matrix

S_4 =

Links	**A**	**B**	**C**	**D**	**E**	**F**	**SGLVI**
A	0.00	2.56	3.55	3.17	3.99	3.41	**16.68**
B	2.56	0.00	3.29	3.27	4.55	3.37	**17.04**
C	3.55	3.29	0.00	3.44	2.94	3.26	**16.48**
D	3.17	3.27	3.44	0.00	2.99	3.42	**16.29**
E	3.99	4.55	2.94	2.99	0.00	2.99	**17.46**
F	3.41	3.37	3.26	3.42	2.99	0.00	**16.45**
						SGTVI	**100.04**

Now it is observed from the first order gradient matrix that –
The link value of link A =27.00, B = 26.51, C = 20.80, D = 24.00, E = 19.04 & F = 19.93
And from second order gradient matrix –
The link value of link A =16.68, B = 17.04, C = 16.48, D = 16.29, E = 17.46 & F = 16.45

From the above link values it is thus clear that when the link B of the Watt chain is fixed, No links are having symmetrical behaviour. Other links are not required to be fixed as we know that link A & D will have the same inversions and link B, C, E & F will also have the same inversions.

4.2 Symmetry for Stephenson chain (Figure - 5)

Now we know that for Stephenson chain, link A & C, link B & D and link E & F will have the same inversions. For finding symmetry among kinematic links, link A or C is fixed, link B or D is fixed and link E or F is fixed simultaneously.

As shown in figure-8, firstly the link A of the Stephenson chain is fixed. A matrix will be formed showing the distance between links. The DISTANCE MATRIX (DM) for Stephenson chain when link A is fixed can be given as explained in methodology.

(1) Stephenson's chain when link A is fixed –

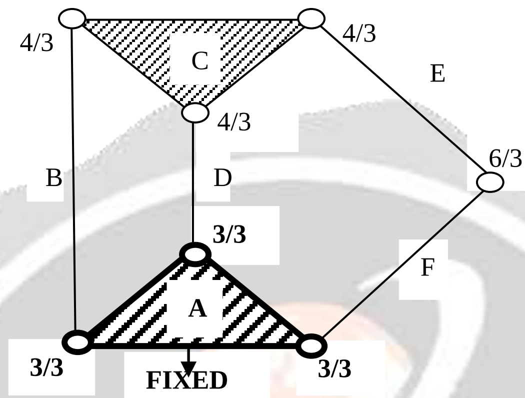


Fig -8: Stephenson's chain when link A is fixed

Distance Matrix

$$D_5 = (1/3)\ \begin{array}{c|cccccc} \text{Links} & A & B & C & D & E & F \\ \hline A & 0 & 3 & 7 & 3 & 9 & 3 \\ B & 3 & 0 & 4 & 8 & 8 & 14 \\ C & 7 & 4 & 0 & 4 & 4 & 10 \\ D & 3 & 8 & 4 & 0 & 8 & 14 \\ E & 9 & 8 & 4 & 8 & 0 & 6 \\ F & 3 & 14 & 10 & 14 & 6 & 0 \end{array}$$

First Order Gradient Matrix

F_5 =

Links	A	B	C	D	E	F	FGLVI
A	0.00	8.67	4.00	8.67	3.78	11.33	**36.45**
B	8.67	0.00	6.00	2.00	3.75	3.00	**23.42**
C	4.00	6.00	0.00	6.00	5.50	4.60	**26.10**
D	8.67	2.00	6.00	0.00	3.75	3.00	**23.42**
E	3.78	3.75	5.50	3.75	0.00	6.00	**22.78**
F	11.33	3.00	4.60	3.00	6.00	0.00	**27.93**
						FGTVI	**160.10**

Second Order Gradient Matrix

$$S_5 = \begin{array}{c|cccccc|c} \textbf{Links} & \textbf{A} & \textbf{B} & \textbf{C} & \textbf{D} & \textbf{E} & \textbf{F} & \textbf{SGLVI} \\ \textbf{A} & 0.00 & 3.96 & 5.45 & 3.96 & 6.41 & 3.25 & \textbf{23.03} \\ \textbf{B} & 3.96 & 0.00 & 4.00 & 2.00 & 4.70 & 4.44 & \textbf{19.10} \\ \textbf{C} & 5.45 & 4.00 & 0.00 & 4.00 & 3.11 & 5.01 & \textbf{21.57} \\ \textbf{D} & 3.96 & 2.00 & 4.00 & 0.00 & 4.70 & 4.44 & \textbf{19.10} \\ \textbf{E} & 6.41 & 4.70 & 3.11 & 4.70 & 0.00 & 3.66 & \textbf{22.58} \\ \textbf{F} & 3.25 & 4.44 & 5.01 & 4.44 & 3.66 & 0.00 & \textbf{20.80} \\ & & & & & & \textbf{SGTVI} & \textbf{126.18} \end{array}$$

Now it is observed from the first order gradient matrix that –
The link value of link A = 36.45, B & D = 23.42, C = 26.10, E = 22.78 & F = 27.93
And from second order gradient matrix –
The link value of link A = 23.03, B & D = 19.10, C = 21.57, E = 22.58 & F = 20.80

From the above link values it is thus clear that when the link A of the Stephenson chain is fixed, links B & D are having symmetrical behaviour.

Now the link B of the Stephenson chain is fixed as shown in figure - 9. A matrix will be formed showing the distance between links. The DISTANCE MATRIX (DM) for Stephenson chain when link B is fixed can be given as explained in methodology.

(2) Stephenson's chain when link B is fixed –

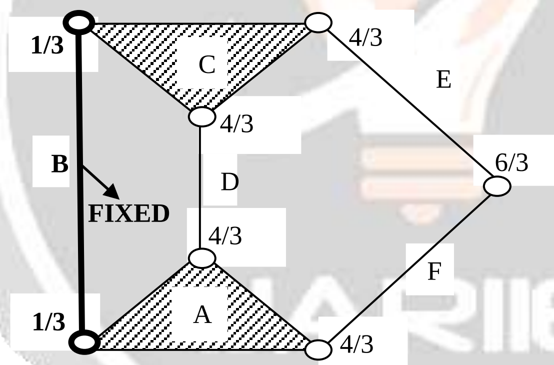


Fig -9: Stephenson's chain when link B is fixed

Distance Matrix

$$D_6 = (1/3) \begin{array}{c|cccccc|} \textbf{Links} & \textbf{A} & \textbf{B} & \textbf{C} & \textbf{D} & \textbf{E} & \textbf{F} \\ \textbf{A} & 0 & 1 & 8 & 4 & 10 & 4 \\ \textbf{B} & 1 & 0 & 1 & 5 & 5 & 5 \\ \textbf{C} & 8 & 1 & 0 & 4 & 4 & 10 \\ \textbf{D} & 4 & 5 & 4 & 0 & 8 & 8 \\ \textbf{E} & 10 & 5 & 4 & 8 & 0 & 6 \\ \textbf{F} & 4 & 5 & 10 & 8 & 6 & 0 \end{array}$$

First Order Gradient Matrix

F_6 =

Links	A	B	C	D	E	F	FGLVI
A	0.00	16.0	3.50	5.50	3.40	5.50	**33.90**
B	16.0	0.00	16.0	4.40	5.20	5.20	**46.80**
C	3.50	16.0	0.00	5.50	5.50	3.40	**33.90**
D	5.50	4.40	5.50	0.00	3.00	3.00	**21.40**
E	3.40	5.20	5.50	3.00	0.00	4.00	**21.10**
F	5.50	5.20	3.40	3.00	4.00	0.00	**21.10**
						FGTVI	**178.20**

Second Order Gradient Matrix

S_6 =

Links	A	B	C	D	E	F	SGLVI
A	0.00	2.98	3.20	5.00	6.94	4.55	**22.67**
B	2.98	0.00	2.98	7.77	6.94	6.94	**27.61**
C	3.20	2.98	0.00	5.00	4.55	6.94	**22.67**
D	5.00	7.77	5.00	0.00	3.30	3.30	**24.37**
E	6.94	6.94	4.55	3.30	0.00	3.05	**24.78**
F	4.55	6.94	6.94	3.30	3.05	0.00	**24.78**
						SGTVI	**146.88**

Now it is observed from the first order gradient matrix that –
The link value of link A & C = 33.90, B = 46.80, D = 21.40, E & F = 21.10
And from second order gradient matrix –
The link value of link A & C = 22.67, B = 27.61, D = 24.37, E & F = 24.78

By analyzing the first order gradient matrix and second order gradient matrix, it is thus clear that when the link B of the Stephenson chain is fixed, links A & C will have symmetrical behavior and the links E & F will also have symmetrical behaviour because of identical values.

Now the link F of the Stephenson chain is fixed as shown in figure - 10. A matrix will be formed showing the distance between links. The DISTANCE MATRIX (DM) for Stephenson chain when link F is fixed can be given as explained in methodology.

(3) Stephenson's chain when link F is fixed –

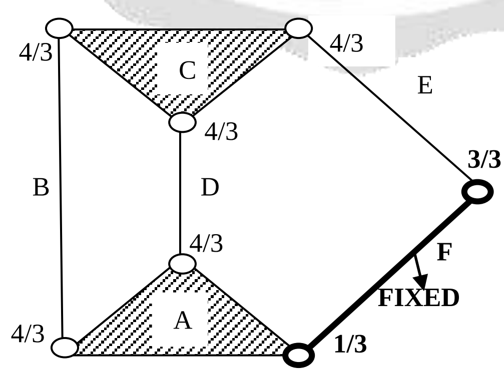


Fig -10: Stephenson's chain when link F is fixed

Distance Matrix

$$
D_7 = (1/3)\;
\begin{array}{cc}
\textbf{Links} & \begin{array}{cccccc} \mathbf{A} & \mathbf{B} & \mathbf{C} & \mathbf{D} & \mathbf{E} & \mathbf{F} \end{array} \\
\begin{array}{c} \mathbf{A} \\ \mathbf{B} \\ \mathbf{C} \\ \mathbf{D} \\ \mathbf{E} \\ \mathbf{F} \end{array} &
\begin{pmatrix}
0 & 4 & 8 & 4 & 12 & 1 \\
4 & 0 & 4 & 8 & 8 & 5 \\
8 & 4 & 0 & 4 & 4 & 7 \\
4 & 8 & 4 & 0 & 8 & 5 \\
12 & 8 & 4 & 8 & 0 & 3 \\
1 & 5 & 7 & 5 & 3 & 0
\end{pmatrix}
\end{array}
$$

First Order Gradient Matrix

$$
F_7 =
\begin{array}{ccc}
\textbf{Links} & \begin{array}{cccccc} \mathbf{A} & \mathbf{B} & \mathbf{C} & \mathbf{D} & \mathbf{E} & \mathbf{F} \end{array} & \textbf{FGLVI} \\
\begin{array}{c} \mathbf{A} \\ \mathbf{B} \\ \mathbf{C} \\ \mathbf{D} \\ \mathbf{E} \\ \mathbf{F} \end{array} &
\begin{pmatrix}
0.00 & 6.00 & 3.75 & 6.00 & 3.17 & 14.0 \\
6.00 & 0.00 & 5.50 & 2.00 & 3.25 & 4.80 \\
3.75 & 5.50 & 0.00 & 5.50 & 6.00 & 3.43 \\
6.00 & 2.00 & 5.50 & 0.00 & 3.25 & 4.80 \\
3.17 & 3.25 & 6.00 & 3.25 & 0.00 & 8.67 \\
14.0 & 4.80 & 3.43 & 4.80 & 8.67 & 0.00
\end{pmatrix} &
\begin{array}{c} \mathbf{32.92} \\ \mathbf{21.55} \\ \mathbf{24.18} \\ \mathbf{21.55} \\ \mathbf{24.34} \\ \mathbf{35.70} \end{array} \\
 & \textbf{FGTVI} & \mathbf{160.23}
\end{array}
$$

Second Order Gradient Matrix

$$
S_7 =
\begin{array}{ccc}
\textbf{Links} & \begin{array}{cccccc} \mathbf{A} & \mathbf{B} & \mathbf{C} & \mathbf{D} & \mathbf{E} & \mathbf{F} \end{array} & \textbf{SGLVI} \\
\begin{array}{c} \mathbf{A} \\ \mathbf{B} \\ \mathbf{C} \\ \mathbf{D} \\ \mathbf{E} \\ \mathbf{F} \end{array} &
\begin{pmatrix}
0.00 & 4.51 & 5.84 & 4.51 & 6.13 & 2.59 \\
4.51 & 0.00 & 3.79 & 2.00 & 4.60 & 5.81 \\
5.84 & 3.79 & 0.00 & 3.79 & 3.72 & 6.18 \\
4.51 & 2.00 & 3.79 & 0.00 & 4.60 & 5.81 \\
6.13 & 4.60 & 3.72 & 4.60 & 0.00 & 3.90 \\
2.59 & 5.81 & 6.18 & 5.81 & 3.90 & 0.00
\end{pmatrix} &
\begin{array}{c} \mathbf{23.58} \\ \mathbf{20.71} \\ \mathbf{23.32} \\ \mathbf{20.71} \\ \mathbf{22.95} \\ \mathbf{24.29} \end{array} \\
 & \textbf{SGTVI} & \mathbf{135.56}
\end{array}
$$

Now it is observed from the first order gradient matrix that –
The link value of link A = 32.92, B & D = 21.55, C = 24.18, E = 24.34 & F = 35.70
And from second order gradient matrix –
The link value of link A = 23.58, B & D = 20.71, C = 23.32, E = 22.95 & F = 24.29

By analysing the first order gradient matrix and second order gradient matrix, it is thus clear that when the link F of the Stephenson chain is fixed, links B & D will have symmetrical behaviour because of identical values.

In the similar way the symmetry of 8 - link chain is obtained. The symmetry for 6 - link kinematic chain and 8 - link kinematic chain is shown in the table below:

Table -2: Symmetry in various 6 link kinematic Chains

6 link kinematic Chains	Fixed Link	Symmetrical links	Total symmetry
Watt Chain	A	(B, F), (C, E)	2
	B	No Symmetrical links	0
Stephenson chain	A	B & D	1
	B	(A, C), (E, F)	2
	F	B & D	1

5. RESULT

According to proposed method we can see that both the 6 – link kinematic chains i.e. watt's chain and stephenson's chain is non isomorphic as the SGTV of both the kinematic chains is different. On the basis of symmetry by using second order gradient matrix which discloses flow of magnitude of relative angular acceleration, we can grade that the stephenson's chain is more efficient than the watt's chain as we have found more symmetrical links in stephenson's chain. In the similar manner we can grade 8-link, 10-link and 12-link kinematic chains.

6. CONCLUSION

In this report, a very simple, efficient and reliable method is proposed in well organised manner to grade the kinematic chains according to their kinematic characteristics. The work performed in the present report deals with kinematic synthesis and analysis of mechanisms. Kinematic synthesis generally deals with structural synthesis of kinematic chain/ mechanisms. By this method, the grading of mechanisms/ kinematic chains can easily be done. This work shows a universal methodology to find out best possible inversion of a structure of simple jointed planar kinematic chain. This method can be easily implemented whenever the number of links and degree of freedom increases, as well as with some modification it can be suitable for n-number of link and f- number of degree of freedom. Such method can also computerize and coding can be developed to reduce time and acquire results faster.

The structural synthesis of kinematic chains involves studies pertaining to enumeration of all possible kinematic chains, distinct inversions and determination of characteristics of kinematic chains based on their topology.

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