# Hexagonal Difference Prime Labeling of Fan Graph, Wheel Graph, Prism Graph, Umbrella Graph and Triangular Belt

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**ABSTRACT**

*The labeling of a graph, we mean assign some integers to the vertices or edges (or both) of the graph. Here the vertices of the graph are labeled with hexagonal numbers and the edges are labeled with absolute difference of the end vertex labels. Here the greatest common incidence number of a vertex of degree greater than one is defined as the g c d of the labels of the incident edges. If the greatest common incidence number of each vertex of degree greater than one is 1, then the graph admits hexagonal difference prime labeling. Here we characterize Fan graph, wheel graph, prism graph, umbrella graph and triangular belt for hexagonal difference prime labeling.*

**Keyword : -** *Graph labeling, hexagonal numbers, greatest common incidence number.*

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## 1. INTRODUCTION

In this paper we deal with graphs that are connected, simple, finite and undirected. The symbol V and E denote the vertex set and edge set of a graph G. The graph whose cardinality of the vertex set is called the order of G, denoted by p and the cardinality of the edge set is called the size of the graph G, denoted by q. A graph with p vertices and q edges is called a (p,q)- graph.

A graph labeling is an assignment of integers to the vertices or edges. Some basic notations and definitions are taken from [2],[3] and [4]. Some basic concepts are taken from [1] and [2]. In this paper we investigated the hexagonal difference prime labeling of Fan graph, wheel graph, prism graph, umbrella graph and triangular belt.

**Definition: 1.1** Let G be a graph with p vertices and q edges. The greatest common incidence number(g c i n) of a vertex of degree greater than or equal to 2, is the g c d of the labels of the incident edges.

**Definition: 1.2** $n^{th}$ hexagonal number is n(2n-1), where n is a positive integer. The hexagonal numbers are 1, 6, 15, 28, 45, 66--------------.

## 2. MAIN RESULTS

**Definition 2.1** Let G be a graph with p vertices and q edges . Define a bijection

f : V(G) → {1,6,15,28 ,------------------,p(2p-1)} by $f(v_i) = i(2i-1)$ , for every i from 1 to p and define a 1-1 mapping $f_{hdpl}^{*}$ : E(G) → set of natural numbers N by $f_{hdpl}^{*}(uv)$ = | f(u)-f(v) |. The induced function $f_{hdpl}^{*}$ is said to be hexagonal difference prime labeling, if the g c i n of each vertex of degree at least 2, is one.

**Definition 2.2** A graph which admits hexagonal difference prime labeling is called hexagonal difference prime graph.

**Theorem: 2.1** Fan graph $F_n$, admits hexagonal difference prime labeling, when n-1 is not a multiple of 5 and $n < 16$.
**Proof :** Let G = $F_n$ and let $v_1,v_2$,----------------,$v_{n+1}$ are the vertices of G.
Here |V(G)| = n+1 and |E(G)| = 2n-1.
Define a function f : V → {1,6,15,28 ,------------------,(n+1)(2n+1)} by
$f(v_i) = i(2i-1)$ , i = 1,2,------,n+1.
For the vertex labeling f, the induced edge labeling $f_{hdpl}^*$ is defined as follows
$f_{hdpl}^*(v_i\, v_{i+1})$ = (4i+1), i = 1,2,------------,n.
$f_{hdpl}^*(v_1\, v_{i+2})$ = $2i^2+7i+5$, i = 1,2,------------,n-1.
Clearly $f_{hdpl}^*$ is an injection.
g c i n of ($v_1$) = g c d of { $f_{hdpl}^*(v_1\, v_2)$ , $f_{hdpl}^*(v_1\, v_3)$ }
= g c d of { 5, 14}= 1.
g c i n of ($v_{i+1}$) = g c d of { $f_{hdpl}^*(v_i\, v_{i+1})$ , $f_{hdpl}^*(v_{i+1}\, v_{i+2})$ }
= g c d of { (4i+1) , (4i+5) } =1, i = 1,2,-------------,n-1.
g c i n of ($v_{n+1}$) = g c d of { $f_{hdpl}^*(v_1\, v_{n+1})$ , $f_{hdpl}^*(v_n\, v_{n+1})$ }
= g c d of {$2n^2+3n$, 4n+1}
= g c d of {2n+3, 4n+1 }
= g c d of {2n-2, 2n+3 }
= g c d of {2n-2, 5 } = 1.
So, g c d of each vertex of degree greater than one is 1.
When n = 16, $f_{hdpl}^*(v_{16}\, v_{17}) = f_{hdpl}^*(v_1\, v_6)$ = 65
So, $F_n$ admits hexagonal difference prime labeling up to n=15.
**Theorem: 2.2** Wheel graph $W_n$, admits hexagonal difference prime labeling, when n-1 is not a multiple of 5 and $n < 16$.
**Proof :** Let G = $W_n$ and let $v_1,v_2$,----------------,$v_{n+1}$ are the vertices of G.
Here |V(G)| = n+1 and |E(G)| = 2n.
Define a function f : V → {1,6,15,28 ,------------------,(n+1)(2n+1)} by
$f(v_i) = i(2i-1)$ , i = 1,2,------,n+1.
For the vertex labeling f, the induced edge labeling $f_{hdpl}^*$ is defined as follows
$f_{hdpl}^*(v_i\, v_{i+1})$ = (4i+1), i = 1,2,------------,n.
$f_{hdpl}^*(v_1\, v_{i+2})$ = $2i^2+7i+5$, i = 1,2,------------,n-1.
$f_{hdpl}^*(v_2\, v_{n+1})$ = $2n^2+3n-5$.
Clearly $f_{hdpl}^*$ is an injection.
From Theorem 1 , it is clear that g c i n of each vertex of degree greater than one is 1.
When n = 16, $f_{hdpl}^*(v_{16}\, v_{17}) = f_{hdpl}^*(v_1\, v_6)$ = 65
So, $W_n$ admits hexagonal difference prime labeling up to n=15.
**Theorem: 2.3** Prism graph $Y_n$, admits hexagonal difference prime labeling, when n is odd.
**Proof :** Let G = $Y_n$ and let $v_1,v_2$,----------------,$v_{2n}$ are the vertices of G.
Here |V(G)| = 2n and |E(G)| = 3n.
Define a function f : V → {1,6,15,28 ,------------------,(2n)(4n+1)} by
$f(v_i) = i(2i-1)$ , i = 1,2,------,2n.
For the vertex labeling f, the induced edge labeling $f_{hdpl}^*$ is defined as follows
$f_{hdpl}^*(v_i\, v_{i+1})$ = (4i+1), i = 1,2,------------,2n-1.
$f_{hdpl}^*(v_i\, v_{2n+1-i})$ = $(8n^2+6n+1) – 2i(4n+1)$, i = 1,2,------------,n-1.
$f_{hdpl}^*(v_1\, v_n)$ = $2n^2-n-1$.
$f_{hdpl}^*(v_{n+1}\, v_{2n})$ = $6n^2-5n-1$.
Clearly $f_{hdpl}^*$ is an injection.
g c i n of ($v_1$) = g c d of { $f_{hdpl}^*(v_1\, v_2)$ , $f_{hdpl}^*(v_1\, v_4)$ }
= g c d of { 5, 27}= 1.
g c i n of ($v_{i+1}$) = g c d of { $f_{hdpl}^*(v_i\, v_{i+1})$ , $f_{hdpl}^*(v_{i+1}\, v_{i+2})$ }
= g c d of { (4i+1) , (4i+5) } =1, i = 1,2,----------------,n-1.
g c i n of ($v_{2n}$) = 1.
So, g c i n of each vertex of degree greater than one is 1.
Hence $Y_n$ admits hexagonal difference prime labeling.

**Theorem: 2.4** Umbrella graph U(n,n), admits hexagonal difference prime labeling, when n+1≡ 0(mod4) and n ≡ 0(mod4).

**Proof :** Let G = U(n,n) and let $v_1,v_2$,----------------,$v_{2n}$ are the vertices of G.

Here |V(G)| = 2n and |E(G)| = 3n-2.

Define a function f : V → {1,6,15,28 ,------------------,(2n)(4n+1)} by

$f(v_i)$ = i(2i-1) , i = 1,2,------,2n.

For the vertex labeling f, the induced edge labeling $f_{hdpl}^*$ is defined as follows

$f_{hdpl}^*(v_i\ v_{i+1})$ = (4i+1), i = 1,2,------------,2n-1.

$f_{hdpl}^*(v_{n+1}\ v_i)$ = $(2n^2+3n+1)$ – i(2i-1), i = 1,2,------------,n-1.

Clearly $f_{hdpl}^*$ is an injection.

g c i n of ($v_1$) = 1.

g c i n of ($v_{i+1}$) = 1, i = 1,2,------------,2n-2.

So, g c i n of each vertex of degree greater than one is 1.

Hence U(n,n), admits hexagonal difference prime labeling.

**Theorem: 2.5** Triangular belt TB($\alpha$), admits hexagonal difference prime labeling.

**Proof :** Let G = TB($\alpha$), where $\alpha$ =(↑↑↑ – – – – – –↑) and let $v_1,v_2$,----------------,$v_{2n}$ are the vertices of G.

Here |V(G)| = 2n and |E(G)| = 4n-3.

Define a function f : V → {1,6,15,28 ,------------------,(2n)(4n+1)} by

$f(v_i)$ = i(2i-1) , i = 1,2,------,2n.

For the vertex labeling f, the induced edge labeling $f_{hdpl}^*$ is defined as follows

$f_{hdpl}^*(v_i\ v_{i+1})$ = (4i+1), i = 1,2,------------,2n-1.

$f_{hdpl}^*(v_{2i-1}\ v_{2i+1})$ = 16i-2, i = 1,2,------------,n-1.

$f_{hdpl}^*(v_{2i}\ v_{2i+2})$ = 16i+6, i = 1,2,------------,n-1.

Clearly $f_{hdpl}^*$ is an injection.

g c i n of ($v_1$) = g c d of { $f_{hdpl}^*(v_1\ v_2)$ , $f_{hdpl}^*(v_1\ v_3)$ }

= g c d of { 5, 14}= 1.

g c i n of ($v_{i+1}$) =1, i = 1,2,----------------,2n-2.

g c i n of ($v_{2n}$) = g c d of { $f_{hdpl}^*(v_{2n-1}\ v_{2n})$ , $f_{hdpl}^*(v_{2n-2}\ v_{2n})$ }

= g c d of {8n-3, 16n-10}

= g c d of {8n-7, 8n-3 }

= g c d of {4, 8n-7 }

= g c d of {1, 4 } = 1.

So, g c i n of each vertex of degree greater than one is 1.

Hence TB($\alpha$), admits hexagonal difference prime labeling.

## 4. CONCLUSIONS

In this paper we used the concept "greatest common incidence number" of a vertex introduced by us to develop the concept hexagonal difference prime labeling. Here we proved that some graphs admit hexagonal difference prime labeling.

## 5. REFERENCES

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