Implementation of Yee's FDTD Algorithm for the Study of the Propagation of Electromagnetic Waves in Vacuum

RANDRIAMAROSON Rivo Mahandrisoa [1], RAKOTOMIRAHO Soloniaina[2], RATSIMBA Mamy Nirina[3]

[1] *PhD, SE-I-MSDE, ED-STII, Antananarivo, Madagascar*
[2] *Laboratory Manager, SE-I-MSDE, ED-STII, Antananarivo, Madagascar*
[3] *PhD, SE-I-MSDE, ED-STII, Antananarivo, Madagascar*

ABSTRACT

The Finite Difference Time Domain (FDTD) method is one of the complete wave techniques used to solve electromagnetic problems. We will see in this paper how to digitally implement the FDTD to simulate the propagation of an electromagnetic (EM) wave in vacuum. After a reminder of the Algorithm of Yee, the centered differences of a function will be recalled in order to be able to define Maxwell's propagation equations with them. From these equations of propagation, the equations allowing the numerical implementation, in order to do a simulation, will be given. Thus an algorithm for the simulation of the propagation of an EM wave will be presented with the results and the related discussions.

Keyword: *FDTD, electromagnetic wave, propagation in vacuum, numerical implementation, algorithm*

---

1. INTRODUCTION

The work presented in this paper is based on the modeling and simulation of an electromagnetic wave propagating in a vacuum who’s modeled by a space in one dimension. We will study the model of Maxwell's equations, the resolution of these equations is reduced to the propagation equations deduced from two equations (Maxwell Ampère and Maxwell Faraday).

The analytical resolution of these equations is not obvious. So it becomes necessary to move to numerical methods. Among these methods we will use FDTD to obtain the solutions in time and space of the Maxwell differential equations. With this method the propagation of EM waves in vacuum will be reproduced numerically.

2. YEE ALGORITHM

The FDTD algorithm proposed by Kane Yee in 1966 uses second-order central differences. The algorithm can be summarized as follows [1]:

a. Replace all derivatives of the Ampere and Faraday laws with finite differences. Discretize space and time so that electric and magnetic fields are shifted in space and time.

b. Solve the resulting difference equations to obtain "update equations" that express future (unknown) fields in terms of past (known) fields.

c. Evaluate magnetic fields a step in the future so that they are known in the present (they actually become past fields).

d. Evaluate electric fields a step in the future so that they are known in the present (they actually become past fields).

e. Repeat the previous two steps (c. and d.) until the fields are obtained for the desired duration.

3. CENTER DIFFERENCES OF A FUNCTION IN A POINT

Consider the Taylor series developments of the function $f\,(x)$ developed around the point x_0 with an offset of $\pm\delta/2$ (Eq.1 and Eq.2) [2].

$$f\left(x_0+\frac{\delta}{2}\right)=f(x_0)+\frac{\delta}{2}f'(x_0)+\frac{1}{2!}\left(\frac{\delta}{2}\right)^2 f''(x_0)+\frac{1}{3!}\left(\frac{\delta}{2}\right)^3 f'''(x_0)+\cdots \quad (1)$$

$$f\left(x_0-\frac{\delta}{2}\right)=f(x_0)-\frac{\delta}{2}f'(x_0)+\frac{1}{2!}\left(\frac{\delta}{2}\right)^2 f''(x_0)-\frac{1}{3!}\left(\frac{\delta}{2}\right)^3 f'''(x_0)+\cdots \quad (2)$$

where 'prime' indicate the differentiation. Subtracting Eq.2 from Eq.1 yields Eq.3.

$$f\left(x_0+\frac{\delta}{2}\right)-f\left(x_0-\frac{\delta}{2}\right)=\delta f'(x_0)+\frac{2}{3!}\left(\frac{\delta}{2}\right)^3 f'''(x_0)+\cdots \quad (3)$$

By dividing Eq.3 by δ, we obtain Eq.4.

$$\frac{f\left(x_0+\frac{\delta}{2}\right)-f\left(x_0-\frac{\delta}{2}\right)}{\delta}=f'(x_0)+\frac{1}{3!}\frac{\delta^2}{2^2}f'''(x_0)+\cdots \quad (4)$$

Thus, the term on the left (of Eq.4) is equal to the derivative of the function $f\,(x)$ at the point x_0 plus a term which depends on δ^2 plus an infinite number of other unrepresented terms. For terms that are not shown, the next depends on δ^4 and all subsequent terms depend on even higher powers of δ. By reorganizing slightly, this relationship is often stated as in Eq.5 [2].

$$\left.\frac{df(x)}{dx}\right|_{x=x_0}=\frac{f\left(x_0+\frac{\delta}{2}\right)-f\left(x_0-\frac{\delta}{2}\right)}{\delta}+O(\delta^2) \quad (5)$$

The term "Big-O" represents all terms that are not explicitly indicated and the value in parentheses, that is, δ^2, indicates the lowest order of these hidden terms. If it is small enough, a reasonable approximation of the derivative can be obtained by simply neglecting all the terms represented by the term "Grand-O". Thus, the central difference approximation is given by Eq.6.

$$\left.\frac{df(x)}{dx}\right|_{x=x_0}=\frac{f\left(x_0+\frac{\delta}{2}\right)-f\left(x_0-\frac{\delta}{2}\right)}{\delta} \quad (6)$$

The centered difference provides an approximation of the derivative of the function at x_0, but the function is not really sampled at x_0. Instead, the function is sampled at the neighboring points $x_0+\delta/2$ and $x_0-\delta/2$. Since the lowest power ignored is the second order, the centered difference is said to have *second-order precision* or *second-order behavior*. This implies that if δ is reduced by a factor of 10, the error in the approximation should be reduced by a factor of 100. In the limit where δ tends to zero, the approximation becomes exact.

4. UPDATE EQUATION IN 1D

Consider a one-dimensional space where there are only variations in the x direction. Assuming that the electric field has only one component z. In this case, Faraday's law can be written as in Eq.7 [3].

$$-\mu\frac{\partial H}{\partial t} = \nabla \times E = \begin{vmatrix} \hat{a}_x & \hat{a}_y & \hat{a}_z \\ \frac{\partial}{\partial x} & 0 & 0 \\ 0 & 0 & E_z \end{vmatrix} = -\hat{a}_y\frac{\partial E_z}{\partial x} \quad (7)$$

Thus, H_y must be the only non-zero component of the magnetic field that varies over time. Ampère's law can be written as in Eq.8 [3].

$$\varepsilon\frac{\partial E}{\partial t} = \nabla \times H = \begin{vmatrix} \hat{a}_x & \hat{a}_y & \hat{a}_z \\ \frac{\partial}{\partial x} & 0 & 0 \\ 0 & H_y & 0 \end{vmatrix} = \hat{a}_z\frac{\partial H_y}{\partial x} \quad (8)$$

The two scalar equations obtained from Eq.7 and Eq.8 are Eq.9 and Eq.10.

$$\mu\frac{\partial H_y}{\partial t} = \frac{\partial E_z}{\partial x} \quad (9)$$

$$\varepsilon\frac{\partial E_z}{\partial t} = \frac{\partial H_y}{\partial x} \quad (10)$$

Equation 9 gives the time derivative of the magnetic field as a function of the spatial derivative of the electric field. Equation 10 gives the time derivative of the electric field as a function of the spatial derivative of the magnetic field. The Eq.9 will serve to advance the magnetic field in time, while the Eq.10 will serve to advance the electric field.

The next step is to replace the derivatives of Eq.9 and Eq.10 with finite differences. To do this, space and time must be discretized. The notation of Eq.11 and Eq.12 will be used to indicate where the fields are sampled in space and time.

$$E_z(x,t) = E_z(m\Delta_x, q\Delta_t) = E_z^q[m] \quad (11)$$

$$H_y(x,t) = H_y(m\Delta_x, q\Delta_t) = H_y^q[m] \quad (12)$$

where Δ_x is the spatial shift between the sampling points and Δ_t is the time shift. The index m corresponds to the spatial step, effectively the spatial location, while the index q corresponds to the time step. When written in exponent, q represents the time step.

4.1. Update equation for magnetic field

The arrangement of the sampling points of electric and magnetic fields, also called nodes, in space and time is illustrated in Fig.1. The electric field nodes are represented by circles and the magnetic field nodes are represented by triangles. Suppose all the fields below the dotted line (red) are known (they are considered past), while the fields above the dotted line are future fields (thus unknown). The FDTD algorithm allow to obtain the future fields from the past fields [4] [5].

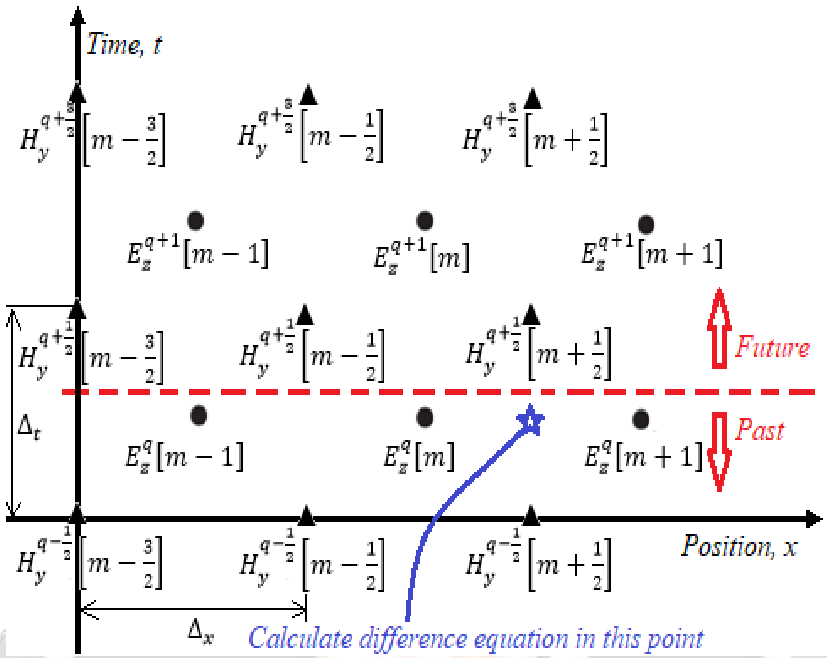

Fig.1: Space-time arrangement of electrical and magnetic field nodes in space and time to obtain an update equation for H_y.

As shown in Fig.1, consider the Faraday law at the spatio-temporal point $\left(\left(m+\frac{1}{2}\right)\Delta_x, q\Delta_t\right)$ described in Eq.13.

$$\mu \frac{\partial H_y}{\partial t}\bigg|_{\left(m+\frac{1}{2}\right)\Delta_x, q\Delta_t} = \frac{\partial E_z}{\partial x}\bigg|_{\left(m+\frac{1}{2}\right)\Delta_x, q\Delta_t} \tag{13}$$

The temporal derivative is replaced by a finite difference involving $H_y^{q+\frac{1}{2}}\left[m+\frac{1}{2}\right]$ and $H_y^{q-\frac{1}{2}}\left[m+\frac{1}{2}\right]$ (i.e. the magnetic field at a fixed location but two different instants), while the spatial derivative is replaced by a finite difference involving $E_z^q[m+1]$ and $E_z^q[m]$ (i.e. electric field at two different locations but at the same time). This gives Eq.14.

$$\mu \frac{H_y^{q+\frac{1}{2}}\left[m+\frac{1}{2}\right] - H_y^{q-\frac{1}{2}}\left[m+\frac{1}{2}\right]}{\Delta_t} = \frac{E_z^q[m+1] - E_z^q[m]}{\Delta_x} \tag{14}$$

Solve Eq.14, for $H_y^{q+\frac{1}{2}}\left[m+\frac{1}{2}\right]$ give the Eq.15.

$$H_y^{q+\frac{1}{2}}\left[m+\frac{1}{2}\right] = H_y^{q-\frac{1}{2}}\left[m+\frac{1}{2}\right] + \frac{\Delta_t}{\mu\Delta_x}\left(E_z^q[m+1] - E_z^q[m]\right) \tag{15}$$

The Eq.15 is the *update equation for* H_y *field*. It is a generic equation that can be applied to any node of the magnetic field. It shows that the future value of H_y depends on only its previous value and the neighboring electric fields.

4.2. Update equation for electric field

After applying Eq.15 to all magnetic field nodes, the dividing line between future values and past values can progress by half step. The space-time grid is, as shown in Fig.2, identical to Fig.1, with the exception of the advance of the dividing line past / future [4] [5].

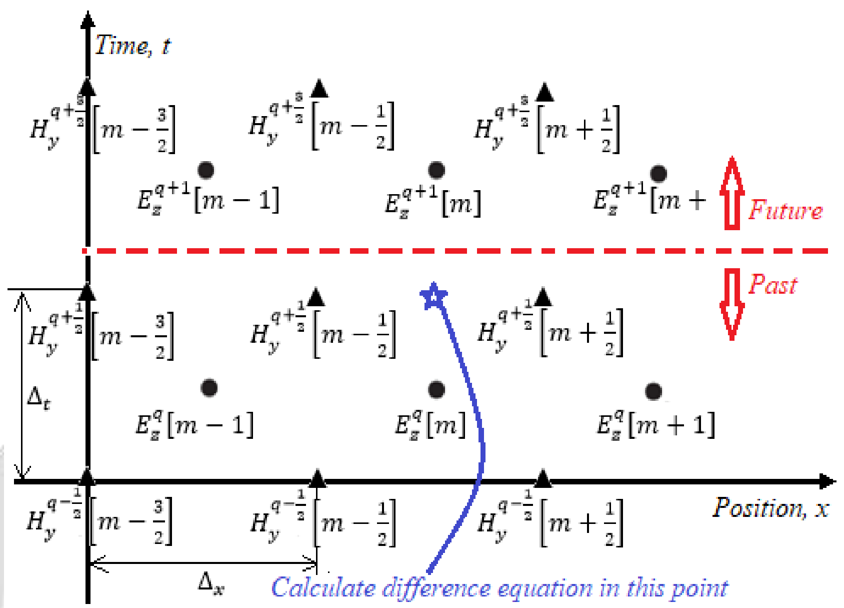

Fig.2 : Space-time arrangement after updating the magnetic field, to obtain the update equation for E_z.

Consider now the Ampere law (Eq.10) applied to the spatio-temporal point $\left(m\Delta_x, \left(q+\frac{1}{2}\right)\Delta_t\right)$ indicated in Fig.2, this gives Eq.16.

$$\varepsilon \frac{\partial E_z}{\partial t}\bigg|_{m\Delta_x,\left(q+\frac{1}{2}\right)\Delta_t} = \frac{\partial H_y}{\partial x}\bigg|_{m\Delta_x,\left(q+\frac{1}{2}\right)\Delta_t} \tag{16}$$

By replacing the temporal derivative on the left with a finite difference involving $E_z^{q+1}[m]$ and $E_z^q[m]$, and replacing the spatial derivative on the right with a finite difference involving $H_y^{q+\frac{1}{2}}\left[m+\frac{1}{2}\right]$ and $H_y^{q+\frac{1}{2}}\left[m-\frac{1}{2}\right]$, we obtain Eq.17.

$$\varepsilon \frac{E_z^{q+1}[m]-E_z^q[m]}{\Delta_t} = \frac{H_y^{q+\frac{1}{2}}\left[m+\frac{1}{2}\right]-H_y^{q+\frac{1}{2}}\left[m-\frac{1}{2}\right]}{\Delta_x} \tag{17}$$

The resolution of Eq.17 for $E_z^{q+1}[m]$ gives Eq.18.

$$E_z^{q+1}[m] = E_z^q[m] + \frac{\Delta_t}{\varepsilon\Delta_x}\left(H_y^{q+\frac{1}{2}}\left[m+\frac{1}{2}\right] - H_y^{q+\frac{1}{2}}\left[m-\frac{1}{2}\right]\right) \tag{18}$$

Equation 18 is the *update equation for* E_z. The indices of this equation are generic, so that the same equation is valid for each node E_z. The future value of E_z depends on its past value and the value of the neighboring magnetic fields.

After applying Eq.18 to each electric field node of the grid, the line of separation between what is known and what is unknown advances by another half-step. We return essentially to the situation described in Fig.1: the future fields closest to the demarcation line between the future and the past are the magnetic fields. They would be updated again, then the electric fields would be updated, and so on.

The *update coefficients* $\Delta_t/\varepsilon\Delta_x$ and $\Delta_t/\mu\Delta_x$ will be represented in terms of the distance ratio between the energy that can propagate in a time step to the spatial step. The maximum speed of electromagnetic energy is the speed of light in free space $c = 1/\sqrt{\varepsilon_0\mu_0}$ and therefore the maximum distance that energy can travel in a time step is equal to $c\Delta_t$. The ratio $c\Delta_t/\Delta_x$ is called *Courant number* noted S_c. By putting $\mu = \mu_0\mu_r$ and $\varepsilon = \varepsilon_0\varepsilon_r$, the coefficients of Eq.18 and Eq.15 become Eq.19 and Eq.20.

$$\frac{1}{\varepsilon}\frac{\Delta_t}{\Delta_x} = \frac{1}{\varepsilon_0\varepsilon_r}\frac{\sqrt{\varepsilon_0\mu_0}}{\sqrt{\varepsilon_0\mu_0}}\frac{\Delta_t}{\Delta_x} = \frac{\sqrt{\varepsilon_0\mu_0}}{\varepsilon_0\varepsilon_r}\frac{c\Delta_t}{\Delta_x} = \frac{1}{\varepsilon_r}\sqrt{\frac{\mu_0}{\varepsilon_0}}\frac{c\Delta_t}{\Delta_x} = \frac{\eta_0}{\varepsilon_r}\frac{c\Delta_t}{\Delta_x} = \frac{\eta_0}{\varepsilon_r}S_c \tag{19}$$

$$\frac{1}{\mu}\frac{\Delta_t}{\Delta_x} = \frac{1}{\mu_0\mu_r}\frac{\sqrt{\varepsilon_0\mu_0}}{\sqrt{\varepsilon_0\mu_0}}\frac{\Delta_t}{\Delta_x} = \frac{\sqrt{\varepsilon_0\mu_0}}{\mu_0\mu_r}\frac{c\Delta_t}{\Delta_x} = \frac{1}{\mu_r}\sqrt{\frac{\varepsilon_0}{\mu_0}}\frac{c\Delta_t}{\Delta_x} = \frac{1}{\mu_r\eta_0}\frac{c\Delta_t}{\Delta_x} = \frac{1}{\mu_r\eta_0}S_c \tag{20}$$

where $\eta_0 = \sqrt{\mu_0/\varepsilon_0}$ is the characteristic impedance of free space.

In FDTD simulations, there are restrictions on the size of a time step. If it is too large, the algorithm produces unstable results. However, the energy cannot propagate further than one of the steps of space, ie $c\Delta_t \leq \Delta_x$. Indeed, in the FDTD algorithm, each node only affects its closest neighbors. In a complete cycle of updating the fields, the propagation of a furthest disturbance is a spatial step. It turns out that the optimum ratio for the current number is also the maximum ratio. Thus, for the one-dimensional simulations considered initially, we will use the Eq.21.

$$S_c = \frac{c\Delta_t}{\Delta_x} = 1 \tag{21}$$

When the first obtaining of the update equations for the FDTD algorithm, it is useful to think in space-time terms. It is more practical to think in terms of the unique spatial dimension where the electric and magnetic fields are shifted by half a step in space. This is described in Fig.3. The time difference between the electric and magnetic fields is always included, whether explicitly shown or not.

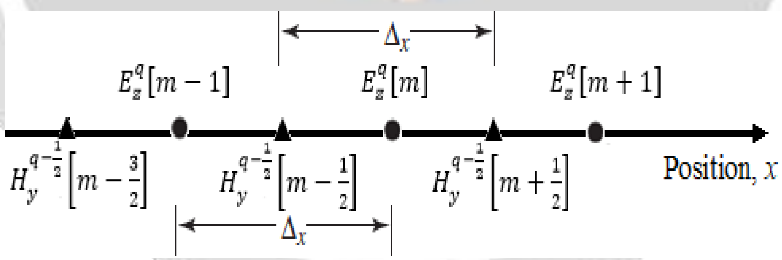

Fig.3 : A 1D FDTD space showing the spatial shift between magnetic and electric fields.

5. COMPUTER IMPLEMENTATION OF A FDTD 1D SIMULATION

5.1. Principles

The goal is to translate Eq.15 and Eq.18 update equations into a usable computer program. The first step is to eliminate, at least to some extent, the upper indices: time is a global parameter and will be recorded in a single integer variable. Time is not something that every node has to be concerned about.

When writing a computer program to implement the FDTD algorithm, it is worthwhile trying to build a program that explicitly uses half offsets. Nodes are stored in arrays and individual array elements are specified with integer indexes. Thus, the computer program implicitly incorporates the fact that the electric and magnetic fields are shifted while using only integer indices to specify the location. The spatial location and the index of the table will be practically synonymous. Let us denote two tables, *ez* and *hy*, they contain the fields E_z and H_y on 200 nodes.

The variable imp0, denotes η_0 the characteristic impedance of the free space (it is initialized to a value of 377.0 ohms). The elements of the *ez* and *hy* arrays are offset from one another by half a step in space, even if the values of the array will be accessible using an integer index.

We will assume here that the *ez* nodes are to the left of the nodes *hy* having the same index. This is illustrated in Fig.4 where *ez [1]* is to the left of *hy [1]*, *ez [2]* to the left of *hy [2]*, etc. In general, when the *hy [m]* representation is used, we consider a table and all half-step offsets associated with this table are understood implicitly. When the representation is $H_y^{q+\frac{1}{2}}\left[m+\frac{1}{2}\right]$, we discuss the field itself and offsets are given explicitly.

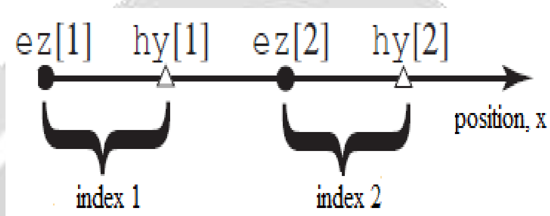

Fig.4 : A one-dimensional FDTD space showing the assumed spatial arrangement of electrical and magnetic field nodes in the *ez* and *hy* arrays.

Assuming a unit Courant number (*Sc = 1*), the *hy [m]* node could be updated with an instruction such as:

hy[m] = hy[m] + (ez[m + 1] – ez[m]) / imp0 ;

For electric field nodes, the update equation can be written:

*ez[m] = ez[m] + (hy[m] – hy[m – 1]) * imp0 ;*

The two update equations, placed in appropriate loops, are the engines that drive a FDTD simulation.

5.2. Simulation of the propagation of a pulse in the vacuum

Consider a simulation of a wave propagating in a free space where there are 200 nodes of electric and magnetic fields. A Gaussian pulse with Amplitude 1 is introduced at the source node, with a phase shift of -30 and a width of 10. The logic diagram is shown in Fig.5.

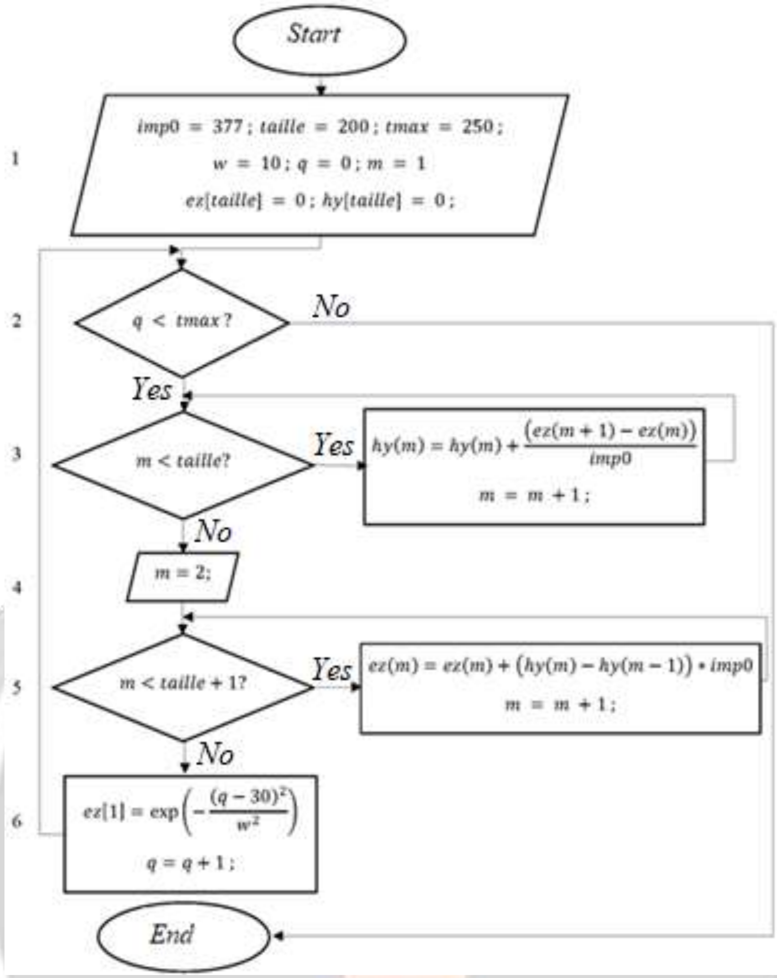

Fig.5 : Calculation algorithm FDTD_1D for propagation in vacuum

5.3. Description of the algorithm

In block 1 (Fig.5) the tables are initialized to zero. The variable q is an integer counter serving as a time index or time step. The total number of time steps in the simulation is dictated by the variable $tmax$ which is set to 250.

The stepwise step is accomplished with the loop from block 2 to block 6. Two additional (spatial) loops are included in this temporal step loop. One to update the magnetic field and the other to update the electric field. The update loop of the magnetic field (block 3 of Fig. 5) excludes the last magnetic field node of the matrix, $hy(200)$, because this node has no electric field on the right. This node will be left to zero in this algorithm. The electric field update loop (block 4 and block 5) begins with a spatial index m of 2, i.e. it does not include *ez (1)*, which is the first node *Ez* of the grid.

The value of $ez(1)$, defined in block 6, is a Gaussian function whose maximum value is unitary when the time counter q is equal to 30.

5.4. Results of the algorithm

A graph of the output generated by the algorithm for calculating the propagation of a Gaussian pulse in a vacuum (FIG. 5) is presented in Fig.6.

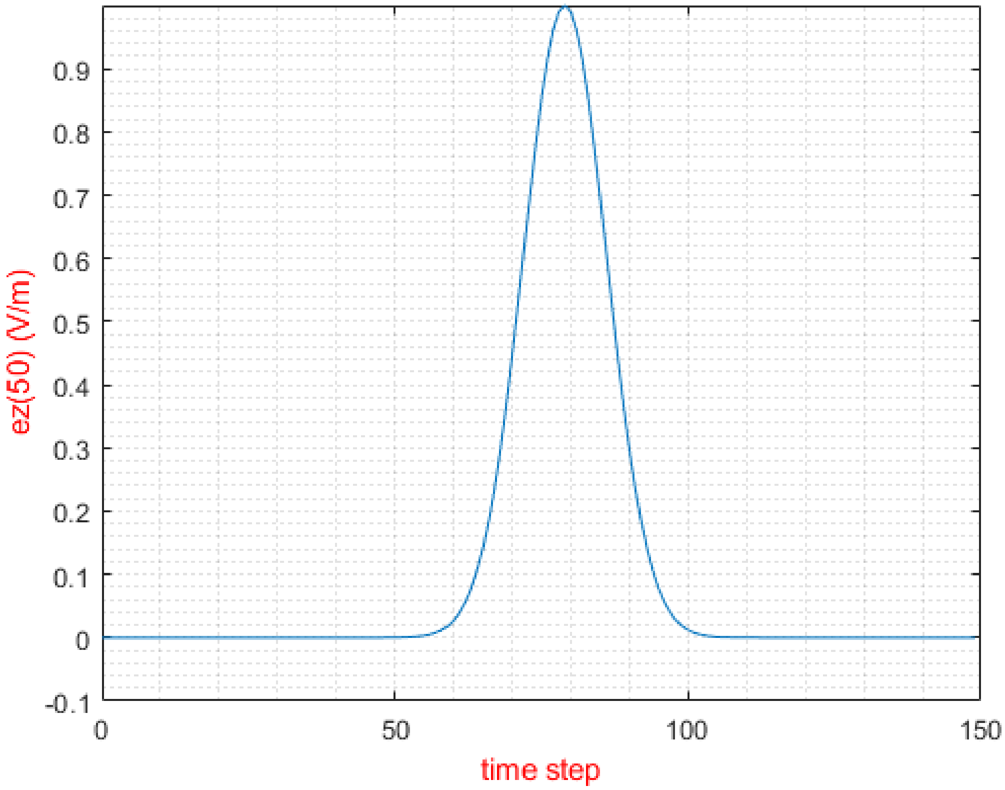

Fig.6 : Results generated by the calculation algorithm FDTD_1D (observation node 50)

In Fig.6 the output is a Gaussian. The excitation is introduced at *ez (1)* but the displayed field is that recorded at ez (50), (at the 50th spatial node). Since $c\Delta_t = \Delta_x$ in this simulation (that is, the current number is unity), the field moves one spatial step for each time step. The separation between the source point and the observation point causes the delay of the observed signal by 50 increments of time compared to what it was at the source. The source function has a peak at 30 time steps but, as can be seen in Fig. 6, the field at the observation point is maximum at time step 80.

Fig.7 shows snapshots of the field at time 20, 30, 40 and 50 using the calculation algorithm FDTD_1D. In these snapshots, we can see the field entering the computational domain from left to right and propagating to the right.

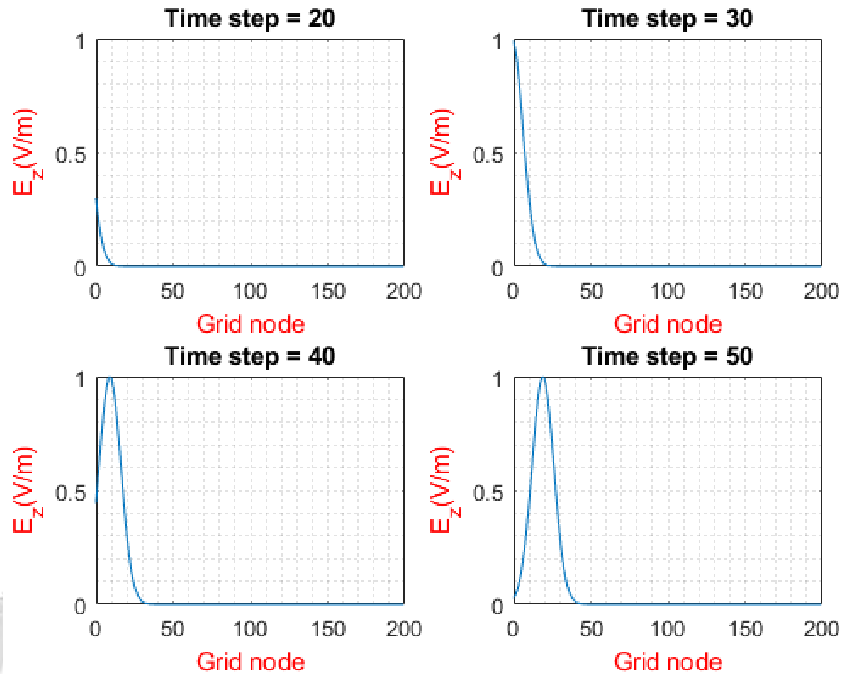

Fig.7 : Snapshots taken at time 20, 30, 40 and 50 of the field Ez generated by the calculation algorithm FDTD_1D.

By modifying the program FDTD_1D, taking 1000 steps instead of 250 (that is, $tmax$ is set to 1000). The result obtained in this case is illustrated in Fig.8.

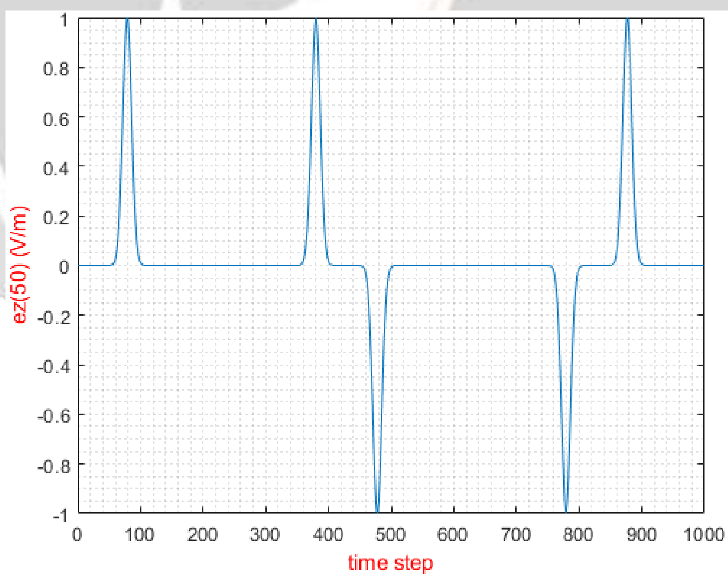

Fig.8 : Results generated by the FDTD_1D calculation algorithm for *tmax* = *1000* (observation node 50).

Fig.8 shows several peaks that are either positive or negative. The last magnetic field node of the grid is initially equal to zero and the remainder throughout the simulation. When the field meets this node, it essentially sees a perfect magnetic conductor (PMC). To satisfy the boundary condition at this node, i.e. the total magnetic field goes to zero, a reflected wave is created that reverses the magnetic field sign but preserves the sign of the electric field. The second peak of Fig.7 is this reflected wave. The reflected wave continues to travel in the negative direction until it encounters the first electric field node $ez(1)$. This node has its value defined by the source function and is unaware of what is happening inside the grid. In this particular case, when the reflected field reaches the left end of the gate, the source function goes to zero. Thus, the node $ez(1)$ behaves like a perfect electric conductor (PEC). To satisfy the boundary conditions on this node, the wave is again reflected, but this time the electric field changes sign while the sign of the magnetic field is preserved. In this way, the field introduced into the grid continues to bounce until the end of the simulation. The simulation consists of a resonator with a PMC wall and a PEC wall.

6. ADDITIVE SOURCE

The wiring of the source, as has been done previously, has the major disadvantage that no energy can pass through the source node. This problem can be corrected by using an additive source. Take the Ampère Law in Eq.22.

$$\nabla \times H = J + \varepsilon \frac{\partial E}{\partial t} \tag{22}$$

The current density J can represent both the conduction current due to the charge flow in a material under the influence of the electric field, that is to say the current given by σE, as well as the current associated with any source, that is to say a "imposed current". At this point, we are simply interested in the source aspect of J. The rearrangement of Eq.22 gives Eq.23.

$$\frac{\partial E}{\partial t} = \frac{1}{\varepsilon}(\nabla \times H) - \frac{1}{\varepsilon}J \tag{23}$$

This equation gives the temporal derivative of the electric field as a function of the spatial derivative of the magnetic field and an additional term that can be considered as a forcing function of the system. This current can be specified to be what we want.

To translate Eq.23 into a form suitable for the FDTD algorithm, spatial derivatives are again expressed in terms of finite differences and then resolved for future fields in terms of past fields. Recall that for the Ampère law, the update equation for $E_z^q[m]$ was obtained by applying finite differences at the spatio-temporal point $(m\Delta_x, (q + 1/2)\Delta_t)$. By following exactly the same procedure but adding the source term, Eq.24 is obtained.

$$E_z^{q+1}[m] = E_z^q[m] + \frac{\Delta_t}{\varepsilon\Delta_x}\left(H_y^{q+\frac{1}{2}}\left[m+\frac{1}{2}\right] - H_y^{q+\frac{1}{2}}\left[m-\frac{1}{2}\right]\right) - \frac{\Delta_t}{\varepsilon}J_z^{q+\frac{1}{2}}[m] \qquad \text{(prg.24)}$$

The source current could possibly be distributed over a certain number of nodes, but to introduce energy into the network, it is sufficient to apply it to a single node. In order to preserve the original update equation, Eq.24 can be separated into two steps: the usual update is applied (Eq.25) and then the source term is added (Eq.26).

$$E_z^{q+1}[m] = E_z^q[m] + \frac{\Delta_t}{\varepsilon\Delta_x}\left(H_y^{q+\frac{1}{2}}\left[m+\frac{1}{2}\right] - H_y^{q+\frac{1}{2}}\left[m-\frac{1}{2}\right]\right) \qquad \text{(prg.25)}$$

$$E_z^{q+1}[m] = E_z^{q+1}[m] - \frac{\Delta_t}{\varepsilon}J_z^{q+\frac{1}{2}}[m] \qquad \text{(prg.26)}$$

The source current can only exist on a single node of the 1D grid. Thus, the Eq.prg.26 would be applied only to the node where the source current is non-zero.

We do not need to explicitly specify the value of Δ_t/ε in the Eq.prg.26, it suffices to simply treat this coefficient as being contained in the source function itself. The source function is exactly the same as before except that instead of setting the value of $ez\,(1)$ to the value of this function, the source function is added to $ez\,(50)$. The source is introduced after the update equations that are unchanged.

Snapshots of Ez taken at time steps 20, 30, 40 and 50 are shown in Fig.9. The field comes from the node 50 and it propagates on both sides of this node. The maximum amplitude is half of what it was when the source function was implemented as a cable source.

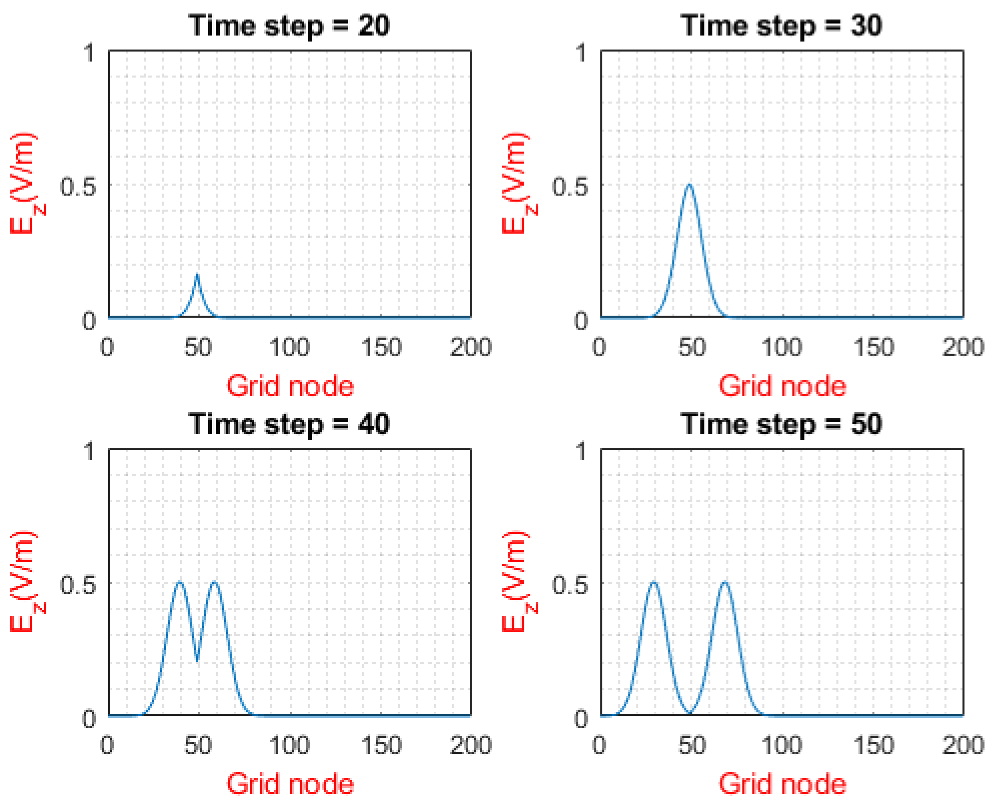

Fig.9 : Snapshots taken at the time intervals 20, 30, 40 and 50 of the field E_z generated by the calculation algorithm FDTD_1D with an additive source applied to node 50

7. CONCLUSION

The work gives an insight into the behavior of an electromagnetic wave propagating in a vacuum, modeled by a 1D space. The implementation of the calculation algorithm FDTD_1D makes possible to observe the electric field (E_z) as a function of time on a selected observation node, but also the shape of the field on the whole of the grid of calculation for a defined time. Although we have considered vacuum as a homogeneous material in our discussions, inhomogeneous materials can be taken for calculations using an impedance of the medium as being spatially varying.

But in order to be able to perform a simulation over a wider range of scenarios, ABC (Absorbing Boundary Condition) should be added to this algorithm so that the grid behaves like an infinite space. Moreover, when using an additive source, the wave can propagates only in one direction, if a TFSF limit (Total-Field / Scattered-Field) is added.

8. REFERENCES

[1]. K.S. Yee, « Numerical solution of initial boundary value problems involving Maxwell's equations in isotropic media », *IEEE Trans. Antenn. Propag.*, vol. 17, 1966, pp. 585–589.

[2]. M. Lefebvre, « Équations différentielles»,2nd Ed, Les Presses de l'Université de Montréal, 2016.

[3]. B. Nogarede,« Electrodynamique appliquée: Fondements et principes physiques de l'électrotechnique», Dunod, 2005

[4]. A. Taflove, « Computational electrodynamics – The Finite-Difference Time-Domain Method », 3rg Ed, Artech House, 2005

[5]. S. Narayan, « FDTD Modeling of Em Field Inside Microwave Cavities », Springer, 2017