

LOAD FLOW ANALYSIS USING NEWTON RAPHSON METHOD – A CASE STUDY OF SOUTH-WEST NIGERIA 330 kV NETWORK

Ikule F. T.¹, Ame-Okoko A.², Idoko E.³

^{1,2,3} *Electrical and Electronics Engineering, University of Agriculture Makurdi, Benue, Nigeria*

ABSTRACT

Power flow study is a fundamental task of power system operation and planning. Among the several methods developed in commercial application software packages for power flow study, the Newton Raphson (NR) method is one of the most successful. The reliability of the NR technique of Load Flow Solution is comparatively better than other load flow methods, as unlike other methods it can solve cases that lead to divergence. The NR method, notwithstanding, has some limitations too. It has been observed that this method tends to fail under some ill-conditioned situations. The distribution systems usually fall into the category of ill-conditioned power systems. Observation of such failures while applying the NR method in distribution systems encourages the research of various ways of improving the reliability of the NR approach. In this paper, the load flow of a small area containing six bus-bars was carried out using the application of NR on MATLAB environment. Data of the case study was used as input to the system. The results obtained shows that the voltage profile of the six buses remain within acceptable limit which implies that there is no divergence in this case using the NR technique. The solution of the load flow could also be used to solve for a reasonable loss coefficient matrix.

KEYWORDS: *Load Flow Studies, Newton Raphson method, active and reactive power flow, convergence, Egbin Thermal Plant, MATLAB.*

1. INTRODUCTION

Power load-flow analysis is executed to find the sensitivity of feeder status with variation of power loading, conductor length, and total capacity of distribution transformers. They are used to ensure that electrical power transfer from generator to consumers through the grid system is stable, reliable and economical. The major aim of a load flow study is to ensure that all bus bars voltage are within specified limits and that the flow of reactive power is reasonable as to ensure that acceptable quality of service could be provided for the consumers [3].

Load flow is a vital tool for power system analysis and design. They are necessary in the planning, operation, economic scheduling, transient stability, contingency studies and control of an existing system as well as planning for its future expansion. The Load flow solution is used to calculate the branch current, bus voltage, real and reactive power flow for the specific generation and load conditions [4].

Based on a specified generating state and transmission network structure, load flow analysis solves the steady operation state with node voltages and branch power flow in the power system.

Load flow analysis can provide a balanced steady operation state of the power system, without considering system transient processes. Hence, the mathematical model of load flow problem is a nonlinear algebraic equation system without differential equations.

2. DATA FOR LOAD FLOW

The simulation for NR is carried out using Matlab. Figure 1 shows the single line diagram of Egbin Plant.

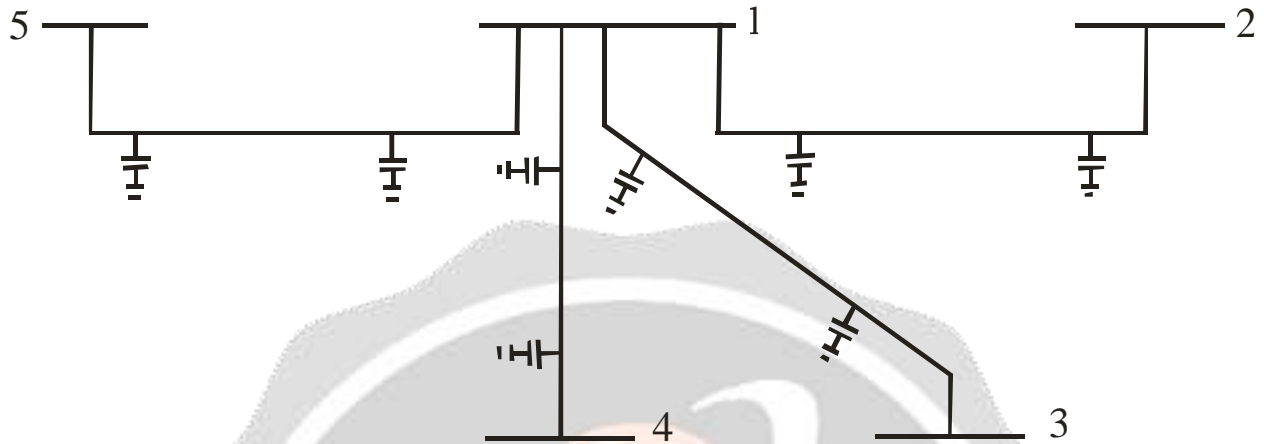


Fig-1: Egbin Single Line Diagram

Table-1: Bus Names and their Types

| Bus – No. | Bus Name | Remark |
|-----------|-------------------|-----------|
| 1 | Egbin | Slack bus |
| 2 | Oke-Aro L1 and L2 | PV bus |
| 3 | Ikeja West L3 | PV bus |
| 4 | Benin L8 | PV bus |
| 5 | Ajah L3 and L4 | PV bus |

[5]

Table-2: Bus Data in per Unit

| Bus Name | Bus No. | Voltage, V (pu) | Angle, δ (degree) | P (pu) | Q (pu) |
|------------|---------|-----------------|--------------------------|--------|--------|
| Egbin | 1 | 1 | 0 | - | - |
| Oke-Aro | 2 | 1 | 0 | 1.86 | 0.43 |
| Ikeja West | 3 | 1 | 0 | 1.52 | 0.21 |
| Benin | 4 | 1 | 0 | 1.81 | 0.49 |
| Ajah 1 | 5 | 1 | 0 | 1.42 | 0.74 |

The values of the load buses are converted to per unit value using equation 1 and the result is shown in Table 2.

$$\text{per unit value} = \frac{\text{actual value}}{\text{base value}} = \frac{(MW + jMVar)}{\text{base value}} \tag{1}$$

Base value = 100 MVA

Slack bus = Egbin GS

The impedances and admittances are converted to their per-unit values according to equations 2 to 7 and results presented in Table 3.

$$Z = R + jX \tag{2}$$

$$\text{per unit impedance} = \frac{\text{actual impedance } (\Omega)}{\text{base impedance}(\Omega)} \tag{3}$$

$$\text{base impedance} = \frac{(\text{base voltage, kV})^2}{\text{base value}} \tag{4}$$

B, is the imaginary part of the admittance Y.

$$Y = G + jB \tag{5}$$

where

G -conductance

B-susceptance

$$\text{per unit admittance} = \frac{\text{actual admittance}}{\text{base admittance}} \tag{6}$$

$$\text{base admittance} = \frac{\text{base value}}{(\text{base voltage, kV})^2} \tag{7}$$

Base value =100 MVA and Base voltage =330 kV

Table 3: Line Data in per Unit

| S/No | Bus – Bus | Impedance (pu) | Shunt Admittance, $\frac{1}{2}$ (pu) |
|------|-----------|-----------------|--------------------------------------|
| 1 | 1-2 | 0.00202+j0.0155 | j0.2316 |
| 2 | 1-3 | 0.00220+j0.0188 | j0.2357 |
| 3 | 1-4 | 0.00781+j0.0663 | j0.8285 |
| 4 | 1-5 | 0.00010+j0.0077 | j0.1142 |

3. FORMATION OF BUS ADMITTANCE MATRIX

The first step in developing the mathematical model describing the power flow in the network is the formulation of the bus admittance matrix.

The Y-bus matrix is formed initially by inspection by considering line data only. Line data consists of the impedances and the shunt admittances. The modified Y-bus matrix is formed by adding the generator and admittances to the respective diagonal elements.

The equation for an n-bus system in terms of admittance bus matrix could be written as:

$$\begin{bmatrix} I_1 \\ I_2 \\ \vdots \\ I_n \end{bmatrix} = \begin{bmatrix} Y_{11} & Y_{12} & \cdots & Y_{1n} \\ Y_{21} & Y_{22} & \cdots & Y_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ Y_{n1} & Y_{n2} & \cdots & Y_{nn} \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \\ \vdots \\ V_n \end{bmatrix}$$

Or

$$I = Y_{bus} \cdot V \quad 8$$

$$Y_{bus} = \begin{bmatrix} Y_{11} & Y_{12} & \cdots & Y_{1n} \\ Y_{21} & Y_{22} & \cdots & Y_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ Y_{n1} & Y_{n2} & \cdots & Y_{nn} \end{bmatrix} \quad 9$$

Denoting the elements in row i and column j as Y_{ij} which is not the admittance but rather the elements of the admittance matrix.

The diagonal element, Y_{ii} , of the bus admittance matrix is equal to the sum of the total admittances values of all the elements connected at bus i, plus the shunt admittance.

$$Y_{ii} = B_{ij} + \sum_{i=1, i \neq j}^n y_{ij} \quad 10$$

where

y_{ij} is the series admittance in Siemens.

B_{ij} is the half line shunt admittance in Siemens.

The off diagonal elements Y_{ij} of the bus admittance matrix is equal to the negative of the admittance value of the connecting element between buses i and j, if any.

$$Y_{ij} = Y_{ji} = -y_{ij} = -\frac{1}{z_{ij}} \quad 11$$

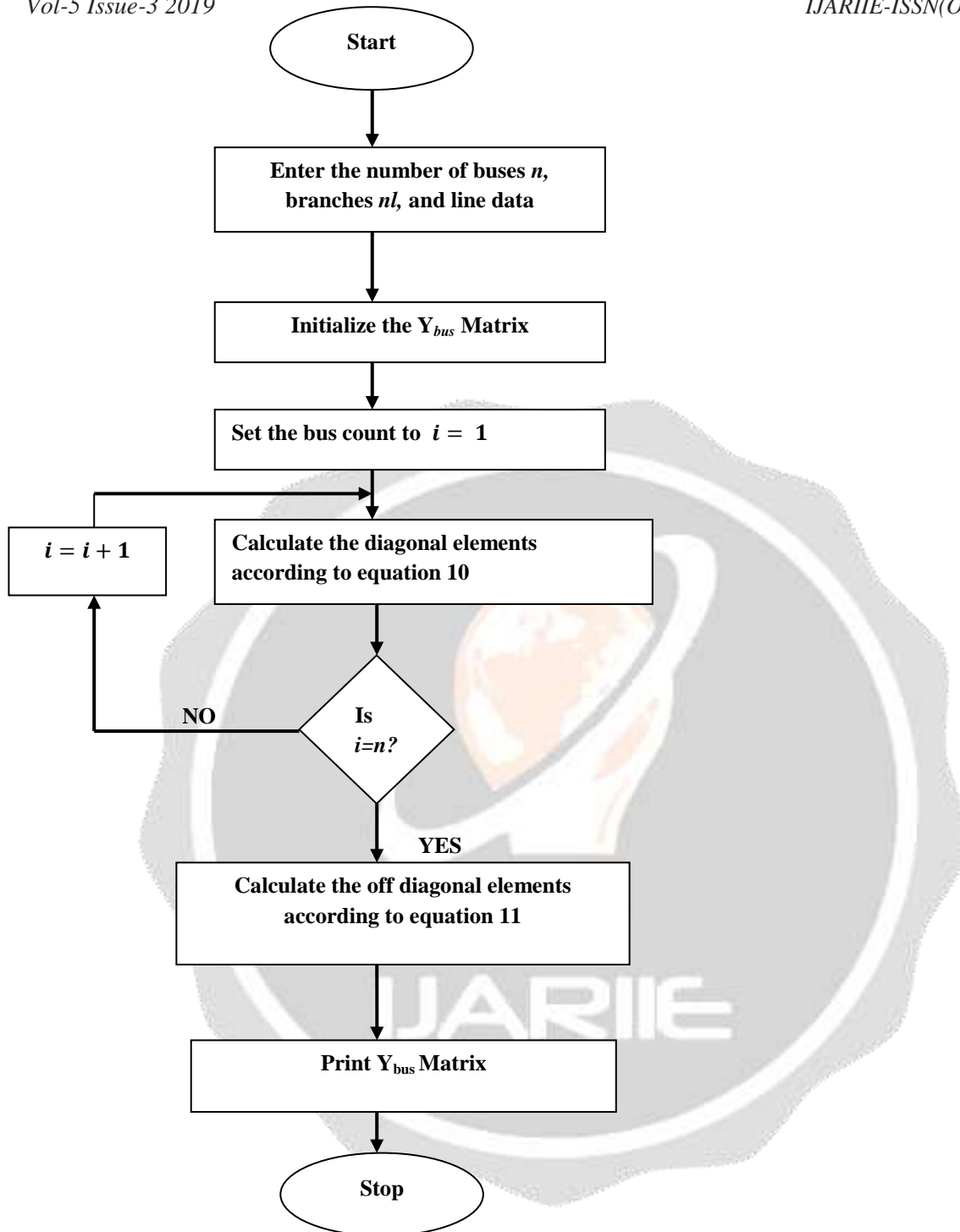


Fig-2: Flow Chart of Bus Admittance Matrix

3.1. Algorithm of Bus Admittance

- **STEP 1:** Read the values of number of buses, number of lines or branches and line data of the given system.
- **STEP 2:** Read the self-admittance of each bus and the mutual admittance between the buses.
- **STEP 3:** Initialize the Y-bus matrix
- **STEP 4:** Calculate the diagonal element term called the driving point admittance, Y_{ii} , using equation (10).
- **STEP 5:** Compute the off-diagonal element term called the transfer admittance, Y_{ij} , using equation (11).
- **STEP 6:** Check for the end of bus count and print the computed Y-bus matrix.

- **STEP 7:** Stop the program and print the results.

4. NEWTON RAPHSON METHOD

This method was named after Isaac Newton and Joseph Raphson. The origin and formulation of NR method was dated back to late 1960s [6]. It is the most iterative method used for the load flow because its convergence characteristics are relatively powerful compared to other alternative processes and the reliability of NR approach is comparatively good since it can solve cases that lead to divergence. If the assumed value is near the solution, then the result is obtained very quickly, but if the assumed value is farther away from the solution then the method may take longer to converge [7].

4.1. Inclusion of Slack Bus in Newton Raphson Load Flow Study

Load flow solution is an unavoidable tool for power system planning and operation. NR and its derivatives are the most popular and widely used ones among the various methods of load flow solution techniques. In NR method one of the generator buses is treated as slack bus which is kept out of calculation during iterations. This is because of the fact that the slack bus is to carry entire loss of the system and total loss cannot be calculated before the end of iterations [8]. The exclusion of slack bus from the load flow iteration restricts many analyses to make. Consideration of distributed slack bus, load flow for Micro grids, study on minimization of system losses etc. cannot be performed when the slack bus is excluded from the iterations.

4.2. Load Flow Analysis by Newton Raphson Method

The results obtained from this analysis comprises the real and reactive power flow on transmission lines, reactive powers on generator buses, magnitudes and phase angles of load bus voltages. NR method of load flow analysis is an iterative method which approximates the set of non linear simultaneous equations to a set of linear simultaneous equations using Taylor's series expansion and the terms are limited to first order approximation [9].

The non-linear equations governing the power system network are:

$$S_i = V_i I_i^* = P_i + jQ_i \quad 12$$

where

S_i is the complex power in bus i

V_i is the bus voltage at bus i given as

$$V_i = |V_i| e^{j\sigma_i} \quad 13$$

and

$$V_i^* = |V_i| e^{-j\sigma_i} \quad 14$$

σ_i is the bus angle at node i

e is Euler's constant

V_i^* is the complex bus voltage at node i

also

$$V_k = |V_k| e^{j\sigma_k} \quad 15$$

V_k is the voltage at bus k

σ_k is the bus angle at node k

I_i is the current injected into bus i, given by

$$I_i = \sum_{k=1}^n Y_{ik} V_k; i = 1, 2, \dots, n \quad 16$$

$$Y_{ik} = |Y_{ik}| e^{-j\theta_{ik}} \quad 17$$

Y_{ik} is the admittance at bus i and k

θ_{ik} is the admittance angle at nodes i and k

P_i, Q_i is the real and reactive power respectively at bus i

$$S_i^* = V_i^* I_i = P_i - jQ_i = \sum_{k=1}^n |V_i| |V_k| |Y_{ik}| e^{-j(\theta_{ik} + \sigma_i - \sigma_k)} \quad 18$$

Separate the conjugate complex power S_i^* , into real and imaginary power

$$P_i = \sum_{k=1}^n |V_i||V_k||Y_{ik}|\cos(\theta_{ik} + \sigma_i - \sigma_k) \tag{19}$$

Compute and add P_i for $i=k$ and for $i \neq k$

$$P_i = V_i^2 Y_{ii} \cos \theta_{ii} + \sum_{k=1, k \neq i}^n |V_i||V_k||Y_{ik}|\cos(\theta_{ik} + \sigma_i - \sigma_k) \tag{20}$$

Similarly,

$$Q_i = V_i^2 Y_{ii} \sin \theta_{ii} + \sum_{k=1, k \neq i}^n |V_i||V_k||Y_{ik}|\sin(\theta_{ik} + \sigma_i - \sigma_k) \tag{21}$$

Equations 20 and 21 constitute the polar form of the power flow equations that provide the calculated values for the net real power P_i , and reactive power Q_i , entering the network at bus i .

Denoting the calculated values of P_i by $P_{i\text{ cal}}$ and Q_i by $Q_{i\text{ cal}}$ leads to the definition of mismatches ΔP_i and ΔQ_i .

$$\Delta P_i = P_{i\text{ sch}} - P_{i\text{ cal}} \tag{22}$$

$$\Delta Q_i = Q_{i\text{ sch}} - Q_{i\text{ cal}} \tag{23}$$

$$P_{i\text{ sch}} = P_{gi} - P_{di} \tag{24}$$

$$Q_{i\text{ sch}} = Q_{gi} - Q_{di} \tag{25}$$

where

$P_{i\text{ sch}}$ and $Q_{i\text{ sch}}$ are the net real and reactive scheduled power injected into bus i

P_{gi} and Q_{gi} denotes the scheduled real and reactive generated power at bus i .

P_{di} and Q_{di} denotes the scheduled real and reactive power demand at load bus i .

Mismatches occur when $P_{i\text{ cal}}$ and $Q_{i\text{ cal}}$ do not coincide with the scheduled values.

The set of linear equations formed expressing the relationship between changes in real and reactive power components of the bus voltages can be written in compact form as:

$$\begin{bmatrix} \Delta P_i \\ \Delta Q_i \end{bmatrix} = \begin{bmatrix} J_{11} & J_{12} \\ J_{21} & J_{22} \end{bmatrix} \begin{bmatrix} \Delta \sigma_i \\ \Delta V_i \end{bmatrix} \tag{26}$$

The Jacobian matrix J_{11}, J_{12}, \dots is set up as a partitioned matrix by convention in the form

$$\begin{bmatrix} \frac{\partial P}{\partial \sigma} & \frac{\partial P}{\partial |V|} \\ \frac{\partial Q}{\partial \sigma} & \frac{\partial Q}{\partial |V|} \end{bmatrix} \tag{27}$$

The off diagonal and diagonal term in the sub-matrices are real numbers calculated by partial differentiation of equations 20 and 21 with respect to σ_i and $|V_i|$ as shown by the following relations.

$$J_{11} = \frac{\partial P}{\partial \sigma} = \begin{bmatrix} \frac{\partial P_i}{\partial \sigma_i} & \dots & \frac{\partial P_i}{\partial \sigma_n} \\ \vdots & \ddots & \vdots \\ \frac{\partial P_n}{\partial \sigma_i} & \dots & \frac{\partial P_n}{\partial \sigma_n} \end{bmatrix} \tag{28}$$

$$J_{12} = \frac{\partial P}{\partial |V|} = \begin{bmatrix} \frac{\partial P_i}{\partial |V_i|} & \dots & \frac{\partial P_i}{\partial |V_n|} \\ \vdots & \ddots & \vdots \\ \frac{\partial P_n}{\partial |V_i|} & \dots & \frac{\partial P_n}{\partial |V_n|} \end{bmatrix} \tag{29}$$

$$J_{21} = \frac{\partial Q}{\partial \sigma} = \begin{bmatrix} \frac{\partial Q_i}{\partial \sigma_i} & \dots & \frac{\partial Q_i}{\partial \sigma_n} \\ \vdots & \ddots & \vdots \\ \frac{\partial Q_n}{\partial \sigma_i} & \dots & \frac{\partial Q_n}{\partial \sigma_n} \end{bmatrix} \tag{30}$$

$$J_{22} = \frac{\partial Q}{\partial |V|} = \begin{bmatrix} \frac{\partial Q_i}{\partial |V_i|} & \dots & \frac{\partial Q_i}{\partial |V_n|} \\ \vdots & \ddots & \vdots \\ \frac{\partial Q_n}{\partial |V_i|} & \dots & \frac{\partial Q_n}{\partial |V_n|} \end{bmatrix} \tag{31}$$

The solution for $\Delta \sigma_i, \Delta V_i$ according to equation 51 gives the correction to be applied to $|V_i|$ and σ_i .

$$|V_i|^{(r+1)} = |V_i|^{(r)} + \Delta |V_i|^{(r)} \tag{32}$$

$$\sigma_i^{(r+1)} = \sigma_i^{(r)} + \Delta \sigma_i^{(r)} \tag{33}$$

The Newton-Raphson method solves the polar form of the power flow equations until the mismatches at all buses fall within specified tolerances.

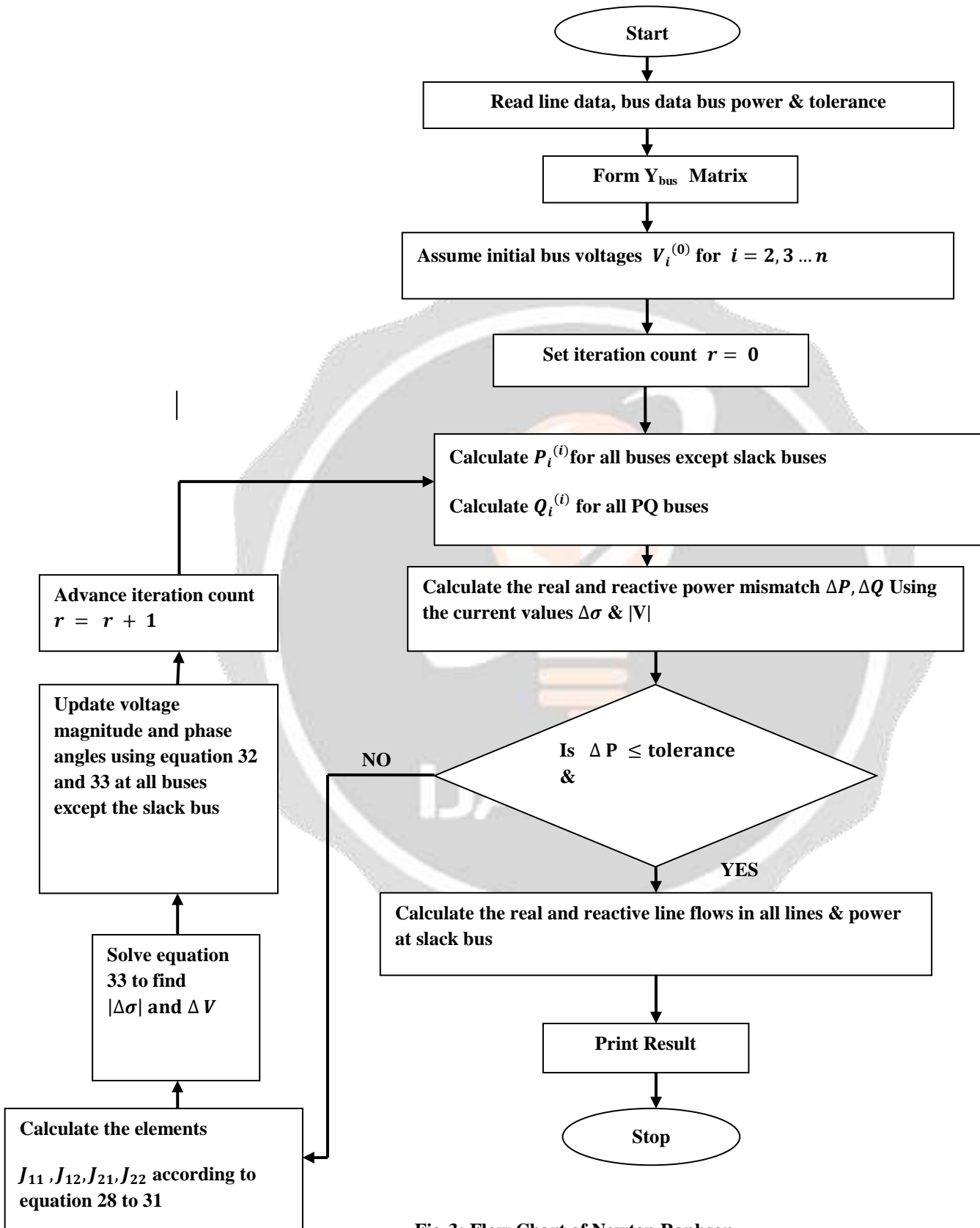


Fig-3: Flow Chart of Newton Raphson

4.3. Algorithm of Newton Raphson

- **STEP 1:** Form Y-bus matrix
- **STEP 2:** Assume initial value of bus voltage magnitude, $|V_i|$ and its phase angle, σ_i equal to slack quantities. Normally $|V_1|=1.00$ pu and $\sigma_1=0$ radians
- **STEP 3:** Set iteration count $r=0$
- **STEP 4:** Calculate the real and reactive power for each load bus according to equation 20 and 21
- **STEP 5:** Compute scheduled errors for each load bus using equations 22 to 25. For generator (PV) buses, if the value of reactive power is within its limits, calculate change in real power only. If it violates the limits, then equate the violated limit as reactive power and treat it as PQ bus.
- **STEP 6:** Calculate the Jacobian matrix elements using estimated $|V_i|$ and σ_i from step 2
- **STEP 7:** Obtain the value of $\Delta|V_i|$ and $\Delta\sigma_i$ from equations 26.
- **STEP 8:** Using the values of $\Delta|V_i|$ and $\Delta\sigma_i$ obtained in step 7, modify $|V_i|$ and σ_i at all loads using equations 32 and 33. Start the next iteration cycle at step 2 with the modified values of $|V_i|$ and σ_i .
- **STEP 9:** Continue until scheduled errors for all load buses are within a specified error tolerance i.e. $\Delta P_i^{(r)} < \varepsilon, \Delta Q_i^{(r)} < \varepsilon$, where ε is the tolerance level for load buses.
- **STEP 10:** Calculate line flows and power at the slack bus.

5.RESULTS AND DISCUSSION

The Egbin thermal power plant is a gas-fired plant with six 220 MW independent boiler turbine units with an operational capacity of 1320 MW. Power generated is sent to the national grid by three main transmission lines namely; Ikeja West (330 kV), Ajah (330 kV) and Ikorodu (132 kV) lines. The complex diagram of Egbin thermal plant is simplified to a single line diagram as shown in Figure 1. These consist of four load buses and a slack bus as shown in Table 1.

The bus admittance matrix is formulated, which is the first step required in developing mathematical models, describing the power flow in the network. This is computed from the line impedances and shunt admittances of the equivalent circuit making up the power system network.

The impedance and shunt admittance of the line data are converted to their respective per unit value according to equations 2 and 7. The load bus of the bus data is also converted to its per unit value using equation 1. For a given load demand, power flow study can be used to calculate active and reactive power generations, line flows and losses and also furnishes some control parameters such as the magnitude of voltage and voltage phase differences.

Before applying the non conventional power flow method, load flow analysis using NR method is used to obtain the unknown bus voltages, bus angles and the generator reactive powers as follows:

active and reactive power of generator units

$$Pg1=2.2000; \quad Qg1=0.0i;$$

$$Pg2=1.6308; \quad Qg2=0.1481i;$$

$$Pg3=2.0908; \quad Qg3=0.4469i;$$

$$Pg4=2.1292; \quad Qg4=0.4391i;$$

$$Pg5=2.1683; \quad Qg5=0.4243i;$$

$$Pg6=2.2000; \quad Qg6=0.0i;$$

Complex voltages

$$v1=1.0+0i; v2=0.9923-0.02842i; v3=0.9963-0.02863i; v4=0.9934-0.1227i; v5=0.9944-0.01087i;$$

6. CONCLUSION

In this paper, the power flow problem, also known as load flow problem, has been dealt with. The load flow solution gives the complex voltages at all the buses and the complex power flows in the lines [10].

This paper reports the development of a method of including slack bus into the Jacobians for NR load flow study. The agreement of the results confirms the accuracy of the method used.

Though the NR technique has found vast applications in the field of power systems, it does not perform adequately for systems with high R/X ratios. The application of the NR method in such situations either results in drastic increase of the number of iterations required for the system to converge or the maximum power mismatch of the system increases.

7. REFERENCES

- [1] Salgar, S. A. and Mallareddy, CH. (2015). Load Flow Analysis for a 220kV Line-Case Study. *International Journal of Engineering Research and Technology*, vol 2, no. 5, pp 1-12.
- [2] Aroop, B., Satyajit, B. and Sanjib, H. (2014) Power Flow Analysis on IEEE 57 bus System Using Matlab. *International Journal of Engineering Research & Technology (IJERT)*, 3.
- [3] Abdulkareem, A. Awosope, C. O. A., Orovwode, H. E. and Adelokun A. A. (2014). Power Flow Analysis of Abule-Egba 33-kV Distribution Grid System with real network Simulations. *IOSR Journal of Electrical and Electronics Engineering*, vol 6, no. 2, pp. 67-80.
- [4] Onojo, O. J., Ononiwu, G. C. & Okozi, S. O. (2013). Analysis of power flow of Nigerian 330kv grid system (pre and post) using Matlab. *European Journal of Natural and Applied Sciences*, 1(2), 59-66.
- [5] Onah, C. O., Agber, J. U. and Ikule, F. T. (2016). "Optimal Dispatch of Real Power Generation using Particle Swarm Optimization: a case study of Egbin Thermal Station" *International Journal of Modern Trends in Engineering and Research*, vol. 3, no. 12, pp. 1-9,
- [6] A. Oudalov, R. Cherkaoui, A.J. Germond, "Coordinated control of multiple series FACTS devices", *Proceedings of the 1999 Power System Automation and Control Conference (PSAC'99), Bled, Slovenia, ISBN 961-6210-68-8, pp. 71-77;*
- [7] Afolabi, O. A., Ali, W. H., Cofie, P., Fuller, J., Obiomon, P. and Kolawole, S. E. (2015). Analysis of the Load Flow Problem in Power System Planning Studies. *Energy and Power Engineering*, vol.7, 509-523
- [8] S. K. Goswami and S. K. Basu, "Direct solution of distribution systems," *IEE Proc., pt. C, vol. 188, no. 1, pp. 78-88, 1999.*
- [9] Glover, J.D. and Sarma, M.S. (2002) Power System Analysis and Design. 3rd Edition, Brooks/Cole, Pacific Grove.
- [10] Mander, D.K., and Virdi, GS. (2017). Result Analysis on Load Flow by Using Newton Raphson Method. *International Journal of Advanced Research in Electrical, Electronics and Instrumentation Engineering*, vol. 6, no. 7, pp. 5835-5844.