Lost Sales Reduction due to investing on Periodic Review Inventory Model with a Service Level Constraint and Varying Inventory Holding Cost

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ABSTRACT

Previous papers have investigated stochastic periodic review inventory model with optimal lost sales caused by investment strategy. A service level constraint is added and the review period and the lost sales are treated as decision variables. In this study the above investigation is extended to include variable inventory holding cost per item per year. The results of previous study are obtained as a particular case of the present study. A sensitivity study is conducted to examine the effect on the expected annual cost to the variability in the average annual demand and the length of the lead time.

Keywords: *Periodic Review Inventory Model, Service Level Constraint, Inventory Holding Cost*

INTRODUCTION

The periodic review systems in which unfilled demands are treated as lost sales are of importance as building blocks for inventory control and coordination, particularly in the retail sector. In practical stochastic inventory control situations it is often not possible to specify shortage costs. In many practices the stock out cost includes such as loss of goodwill and potential delay to the other parts of inventory system, and hence it is difficult to determine an exact value for the stock out cost. Oyang and Chang (2001) studied the continuous review inventory model and proposed the lost sales rate can be reduced by capital investment. Most of the inventory models discuss two extreme situations regarding the demand process when the items are stock out. They are: (1) all of the demand within shortage period is backordered and (2) all of demand within shortage period is lost sales. There are several research papers which discuss the back order situations (e.g., Nick T. Thomopouls (2004); Chandershekhar Das (1983); Bore-Ren Chouang et al (2004)). Some research workers investigated the partial back order situations with lead time reduction (e.g., M.Hariga and M. Ben-Daya (1999); S.P. Lan etal (1999).

Some investigators suggested the following factors which increase the product cost and investigated the methods to reduce the effect of these factors.

i) Setup cost
ii) Variability in lead time
iii) Lost sales due to stockouts

D.Gross and A. Soriano (1969) and C.E. Vinson (1972) demonstrated that the lead time variation has a major impact on inventory cost. Inventory depends a great deal on what happens to demand when the system is out of stock. Hung-Chi Chang (2001) developed a mixture inventory model involving variable lead time with lost sales reduction. In above referred papers in which investigators tried to reduce setup costs, lead time variations and lost sales, an option of capital investment is considered. The investment is assumed o be a logarithmic function.

The effect of varying inventory holding cost on total expected annual cost is considered by some investigators.Hala A. Fergany (2005) developed a periodic review probabilistic multi-item inventory system with zero lead time under constraints and varying order cost. Kotab Abd-EL-Hamid Mahamoud Kotab and Huda Mahamed Hamid AL-Sabare (2011) studied a constrained probabilistic EOQ model under varying order cost and zero lead time. Hala A. Fergany and Nagla Hassan EL-Sodany (2011) investigated a probabilistic periodic review backorder and lost sales inventory models under constraints and varying holding cost.

In this paper a stochastic periodic review inventory model with partial lost sales (or back orders) rate, subject to a service level constraints is extended to include the variable inventory holding cost. The effect of increasing investment to reduce the lost sales is analyzed. The form of the probability of protection interval is unknown, but only the first and second moments are given. This inventory model is solved by using the minimax distribution free approach. A numerical example is provided to illustrate to illustrate the model.

2. The model with partial lost sale under service level constraints and varying inventory holding cost

The following assumptions are taken while developing the model.

i) The protection interval, T+L and demand x has a probability density function f_x with finite mean D(T+L) and standard deviation $\sigma\sqrt{(T+L)}$,where σ denotes the standard deviation of the demand per year, T the length of the review period, D the average demand per year and L the length of the lead time.

ii) The target level $R=D(T+L)+k\sigma\sqrt{(T+L)}$, where k is the safety factor and satisfy p(x>R) =q, where q represents the allowable stock out probability during the protection interval.

iii) During the stock out period, a fraction α of the demand will be lost, and the remaining fraction (1-α) will be back logged.

iv) I (α) denote the capital investment which reduces the lost sales fraction α. The function I(α) is given below.

$I(\alpha)=1/\delta \ln(\alpha_0/\alpha)$, where $0<\alpha \le \alpha_0$, α_0 is the original fraction of shortage that will be lost and δ is the percentage decrease in α per dollar increase in I(α) The logarithmic function I(α) has widely been used in literature to formulate various inventory options(see e.g.,Porteus(1985);Paknejad et al (1992)

; and Bor-Ren chaung et al (2004).

v) The inventory holding cost h per item per year is assumed to be of the following form.

$h= c_h T^{\beta, \text{ where}}$ c_h and β are constant.

Ben-Ren chuang et al (2004) presented the stochastic periodic review model with partial lost sales (back orders) rate under service level constraints. The expected annual total cost denoted by EAC (T, γ) is given by

$$EAC (T, \alpha hy) = \theta I (\alpha) +A/T+h[R-DL-DT/2+\alpha E(x-R)^{+}] =\theta/\delta \ln (\alpha_0 /\alpha) + A/T + h[R-DL-DT/2 +\alpha E(X-R)^{+}] \quad (1)$$

$$\text{Subject to } E(X-R)^{+/} D (T+L) \le \varepsilon \quad (2)$$

Where ε is the proportion of demand that is not met, hence (1-ε) is the service level.

$E(X-R)^{+}$ = maximum value of E(X-R) and 0, i.e.

$E(X-R)^{+} = \text{Max}\{E(X-R), 0\}$.

The form of probability distribution of protection interval demands x is unknown, only first and second moments are given. Hence exact value of $E(X-R)^{+}$ cannot be found .Bor-Ren Chuang et al(2004) used distribution free approach to solve this problem. The same approach is used in the present study. They showed that the condition $E(x-R)^{+} \le \varepsilon$ can be reduced to the following form.

$(\sigma\sqrt{(T+L)}(\sqrt{(1+k^2)}-k))/ (2D (T+L)) \le \varepsilon$.

The final form of Bor-Ren et al model takes the following form

$$\text{Min } EAC^{w} (T, \alpha) =.\theta/\delta \ln (\alpha_0 /\alpha) +A/T+h [DT/2+k\sigma\sqrt{(T+L)} + \tfrac{1}{2} \alpha\sigma\sqrt{(T+L)} (\sqrt{(1+k^2} -k)] \quad (3)$$

$$\text{Subject to } (\sigma\sqrt{(T+L)} (\sqrt{(1+k^2)}-k))/ (2D (T+L)) \le\varepsilon \quad (4)$$

Where EAC^{w} (T+L) is the least upper bound of EAC (T, α) and A is the fixed ordering cost per year.

3. The model with varying inventory holding cost

The BOr-REn Chung et al (2004) model is extended to include varying inventory holding cost. It reduces to the following form.

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$$\text{Min } EAC^{w} (T, \alpha) =\theta/\delta \ln (\alpha_0 /\alpha) +A/T +c_h T^{\beta} [DT/2+k\sigma\sqrt{(T+L)} + 1/2\alpha\sigma\sqrt{(T+L)} (\sqrt{(1+k^2)}-k)] \quad (5)$$

$$\text{Subject to } (\sigma\sqrt{(T+L)}(\sqrt{(1+k^2)}-k))/(2D(T+L)) \le\varepsilon \quad (6)$$

This is a nonlinear programming problem. It is verified that Kuhn-T%ucker condition is not satisfied. A method is devised by Bor-Ren chuang

Et al (2004) to solve this problem. First ignoring the constraint it can be

Shown that Eq. (5) is a convex curve. The proof is given below.

Differtiating eq.(3) partially with respect to T we get

$$\partial /\partial T(EAC(T+L)) \quad \text{`q } DT/2+\sigma\sqrt{T+L}\{k+\alpha/2(\sqrt{1+k^2})\}] \quad (7)$$

$$\partial^2 /\partial T^{2(} \; EAC(T,\alpha)) = 2A/T^3 -C_h T^{\beta} [\sigma/2(T+L)^{\beta/2} \{k/2+\alpha/4(\sqrt{1+k^2} -k)\}] + C_h\beta T^{\beta-1} [D+\sigma/\sqrt{T+L}\{k+\alpha/2(\sqrt{1+k^2} -k)\}] +C_h\beta(\beta-1)T^{\beta-2}[DT/2 +\sigma\sqrt{(T+L)}\{k+\alpha/2(\sqrt{(1+k^2)} -k)\}] \quad (8)$$

$$\partial^2/\partial T\partial\alpha (EAC (T, \alpha)) =C_h T^{\beta} [\sigma (\sqrt{(1+k)}-k)]/(4\sqrt{(T+L)}) + C_h \beta T^{\beta-1}[\sigma\sqrt{(T+L)}(\sqrt{(1+k^2)}-k)/2] \quad (9)$$

$$\partial^2/\partial\alpha\partial T(EAC(T,\alpha))=C_h T^{\beta} [\sigma[\{(\sqrt{(1+k^2)}-k\}/(4\sqrt{(T+L)})] + C_h \beta T^{\beta-1} [\sigma/2\sqrt{(T+L)} \{\sqrt{(1+k^2)}-k\}] \quad (10)$$

$$\partial^2/\partial\alpha^2 (E) = \theta/\delta\alpha^2 \quad (11)$$

For the values of all the parameters taken in the numerical example given in section 5, it is numerically verified that

$\partial^2 /\partial T^{2(}EAC (T, \alpha))$ and the

Hessian

$\partial^2 / \partial T^2 (EAC\ (T, \alpha))$ $\quad \partial^2 / \partial T \partial \alpha\ (EAC\ (T, \alpha))$

$\partial^2 / \partial \alpha \partial T\ (EAC\ (T, \alpha))$ $\quad \partial^2 / \partial \alpha^2\ (EAC\ (T, \alpha))$

are greater than zero.

The optimal values of T and α are obtained by equating the first derivatives of EAC (T, α) with respect to T and α to zero.

$\partial/\partial T\ (EAC\ (T, \alpha)) = 0$ (12)

$\partial/\partial \alpha\ (EAC\ (T, \alpha) = 0$ (13)

From equations (12) -13) we obtain respectively

$A/T^2 = c_h\ T^\beta\ [D/2 + k\sigma/2\sqrt{}\ (T+L) + \alpha\sigma/4\sqrt{}\ (T+L)\ (\sqrt{}\ (1+k^2) - k)] +$

$c_h\ \beta\ T^{\beta-1}\ [DT/2 + k\ \sigma\sqrt{}(T+L) + \alpha\sigma\sqrt{}(T+L)(\sqrt{}(1+k^2\ ) - k)/2]$ (14)

$\alpha = 2\theta/[\ \delta\ c_h\ T^\beta\ \sigma\ \sqrt{}(T+L)(\sqrt{}(1+k^2\ ) - k)]$ (15)

Substituting (12) into (14) we obtain

$A/T^2 = c_h\ T^\beta\ [D/2 + \sigma\{k + \theta/(\delta c_h \sigma T^\beta\ \sqrt{}(T+L)\}] +$

$c_h\ \beta T^{(\beta-1)}\ [DT/2 + k\sigma\sqrt{}(T+L) + \ \theta/(\delta\ c_h\ T^\beta)\ ]$ (16)

For given values of β, A, L, ch, σ, δ, θ and k the optimal values of T and α can be obtained from equations (15) and (16). The total expected optimal annual cost may be obtained from equations (5) and (6).

Since f_x is unknown one cannot get the exact value of k. The optimal values K, T and α are obtained by the method developed by Bon –Ren chaung (2004) and are denoted by k* ,T* and α * respectively.

The optimal value of annual cost is denoted by $EAC^w\ (T^*, \alpha^*)$ and is given Below

$EAC^w\ (T^*, \alpha^*) = \theta\ I(\alpha^*) + A/T^* + \ln(\ \alpha_0/\alpha^*) + c_h\ T^{*\beta}\ [DT^*/2 + k\sigma\sqrt{}(T^*+L) +$

$\alpha^*\sigma\sqrt{}\ (T^*+L)(\sqrt{}(1+k^2) - k^*)]$ (17)

METHODOLOGY

We $EAC^w\ (T, \alpha)$ has a smooth curve for k belonging to the interval [0, √ (1/q-1)].

The following method is used to obtain optimal values of k, T and α.

Divide the interval [0, √ (1/q-1)] into N equal subintervals. N is taken to be

Large. Define $k_j = k_{j-1} + k_N - k_0\ /N$ [j=1,2 N-1] where $k_N = \sqrt{}(1/q-1)$,k_0=0.Corresponding to each k_j find T_{kj} from equation (12). Using numerical search technique compute the value of α_{kj} correspond ding to T_{kj} with help of equation (13). Compare α_{kj} with α_0. Complete the following steps:

i) $\alpha_{kj} \le \alpha_0$, than for each (T_{kj}, α_{kj}) compute the corresponding total expected annual cost $EAC^w\ (T_{kj}, \alpha_{kj}\ )$

ii) If $\alpha_{kj} \rangle \alpha_0$, set $\alpha_{kj} = \alpha_0$ and compute the corresponding value of T_{kj} from equation (12) and then compute $EAC^w\ (T_{kj}, \alpha_{kj})$.

iii) Find Min $EAC^w\ (T_{kj,\ \alpha kj})$ corresponding to some T_{kj}, α_{kj} which satisfies the

Constraint of the model. The values of T_{kj} and α_{kj} thus obtained is the optimal

Optimal solution. The corresponding values are denoted by k*, T* and α* respectively.

Numerical Example

In order to see the effect of variable inventory holding cost on expected

Annual cost the following numerical example is solved using following parameters.

β [0-0.1],D=1000 units per year, A=$200per year, c_h =25, σ=7, L=3,

(1-ε)=0.98(the service level), θ=0.1 per $ per year=0.005%, α_0=0.08 and

q=0, 2

Results of Numerical Example and Sensitivity Analysis

Applying the algorithm described in section 4, the results of the numerical Example are computed and presented through Table 1. β =0 represents the results of BOr-Ren chuang et al (2004). It is found that as β increases, the length of the inventory review period and the fraction of shortage that will be lost increases. The optimal value of k and the total expected annual cost decreases. For β >=0.05, the total expected annual cost increases. The values of investment required reducing the lost sales fraction and the lost sales rate reduction both decrease, but the decrease in investment is more rapid in compare to lost sales reduction.

Sensitivity analysis

The sensitivity analysis is conducted and it may be interpreted as follows.

i) Two different firms, each identical except for their different values of parameters are discussed.

ii) A single firm with several products, differing only in one parameter value. We cannot speak about a single firm whose parameter value for a given product has changed at some point in time, because a correct model of that situation would have the firm's decisions depend on the prospects for change in the parameters in the future.

iii) A single firm with different possible starting values of the parameters. In the sensitivity analysis, it is found that in all the experiments there is a common trend that as the value of β increases from zero t0 0.05, the value of the least upper bound of expected annual cost decreases and it increases for β >=0.05.

Table 1 Results of numerical example (σ=7, L=3)

	T*	α*	k*	I(α*)	EAC^w (T*,α*)	Lost sales rate reduction (%)
0	6.37	0.193	1.336	284.04	3926.91	75.83
0.01	6.44	0.198	1.33	278.51	3925.5	75.15
.02	6.51	0.204	1.322	272.52	3923.29	74.4
0.03	6.58	0.21	1.316	267.05	3922.51	73.69
0.04	6.64	0.216	1.31	261.61	3922.03	72.96
0.05	6.71	0.222	1.305	256.45	3922.41	72.25
0.06	6.78	0.228	1.299	251.08	3922.5	71.5
0.07	6.85	0.234	1.293	245.	3922.86	70.73
0.08	6.92	240	1.288	240.67	3924.07	69.98
0.09	6.98	0.25	1.282	235.38	3924.95	69.17
0.1	5-Jul	0.25	1.278	230.63	3927.24	68.43

Table 2 Results of numerical example (σ=7, L=2)

B	T*	α*	k*	I(α*)	EAC^w (T*,α*)	Lost sale rate reduction (%)
0	6.37	0.21	1.431	267.26	3933.77	73.72
0.01	6.43	0.216	1.424	261.64	3932.32	72.96
0.02	6.5	0.222	1.417	256.05	3931.18	72.2
0.03	6.57	0.229	1.41	250.49	3930.35	71.42
0.04	6.64	0.235	1.403	244.97	3929.82	70.61
0.05	6.78	0.241	1.397	239.7	3930.11	69.83
0.06	6.77	0.248	1.39	234.24	3930.13	69
0.07	6.84	0.255	1.383	228.8	3930.43	68.15
0.08	6.91	0.261	1.377	223.62	3931.52	67.31
0.09	6.98	0.268	1.37	218.25	3932.33	66.41
0.1	7.05	0.276	1.364	213.13	3933.92	65.55

Table3 Results of sample example (σ=7, L=4)

B	T*	α*	k*	I(α*)	EAC^w (T*,α*)	Lost sales rate reduction (%)
0	6.38	0.187		290.96	3916.48	76.58
0.01	6.45	0.192		284.95	3915.24	75.94
0.02	6.52	0.197		279.58	3914.32	75.29
0.03	6.58	0.203		274.25	3913.72	74.62
0.04	6.65	0.208		268.94	3913.42	73.93
0.05	6.72	0.214		263.67	3913.42	73.24
0.06	6.79	0.218		258.97	3914.9	72.18
0.07	6.85	0.223		255.11	3914.9	72.07
0.08	6.93	0.231		248.06	3915.09	71.07
0.09	6.99	0.237		242.92	3916.18	70.32
0.1	7	0.243		238.06	3917.74	69.58

Table 1 is taken a sample Table and sensitivity of various parameters to model results is examined by comparing the results with Tables1. Table1 through 3 shows the sensitivity of the model results to length of the lead time .It is found that an increase in the length of lead time results in a small increase in review period and decrease in the ratio of investment to sales rate reduction.

Table 4 The results of sample example (σ=5, L=3)

B	T*	α*	k*	I(α*)	EAC^{w} (T*,α*)	Lost sales reduction rate (%)
0	6.42	0.591	0.84	60.38	3510.61	26.06
0.01	6.48	0.615	0.831	52.49	3507.51	23.08
0.02	6.55	0.63	0.827	46.75	3506.86	20.48
0.03	6.62	0.653	0.822	40.59	3506.1	18.17
0.04	6.68	0.671	0.818	34.91	3506.08	16.015
0.05	6.75	0.692	0.813	28.81	3505.93	13.418
0.06	6.82	0.711	0.81	23.61	3506.92	11.137
0.07	6.89	0.731	0.806	18.01	3507.77	8.61
0.08	6.95	0.753	0.801	11.99	3508.46	6.95
0.09	7.02	0.756	0.799	11.29	3510.82	5.49
0.1	7.09	0.798	0.792	0.464	3511.05	0.23

Table 5 The results of sample example (σ=9, L=3)

B	T*	α*	k*	I(α*)	EAC^{w} (T*,α*)	Lost sales rate reduction (%)
0	6.32	0.095	1.805	427.04	4466.01	88.17
0.01	6.39	0.097	1.798	421.77	4464.62	87.86
0.02	6.46	0.099	1.79	416.38	4462.85	87.53
0.03	6.53	0.102	1.783	411.18	4462.07	87.2
0.04	6.59	0.105	1.776	406.03	4461.58	86.8
0.05	6.66	0.107	1.769	400.9	4461.38	86.52
0.06	6.73	0.117	1.762	395.98	4461.45	86.19
0.07	6.79	0.113	1.755	390.75	4461.78	85.83
0.08	6.86	0.116	1.748	385.72	4462.37	85.46
0.09	6.93	0.118	1.745	381.42	4466.09	85.14
0.1	7	0.121	1.74	376.8	4468.62	84.84

Tables 1, 4 and 5 show that as the value of σ decreases , the value of expected cost decreases and the length of the review period increases. Hence the less variability in average demand per year reduces the annual cost and increases the review period. The tables also show the investment and corresponding lost sales reduction. It is found that as the variability in annual demand decreases, the ratio of the investment to lost sales rate reduction increases.

CONCLUSIONS

A model with partial lost sales under service level constraint and varying inventory holding cost has been presented in this paper. The effect of varying inventory holding cost on the length of review period, fraction of shortage that will be ,lost sales reduction and the investment required to reduce the lost sales fraction has been examined. The sensitivity analysis has been conducted and the sensitivity of the expected annual cost to the variability in average demand per year and the length of lead time has been examined.

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