MATHEMATICAL ASPECTS FOR ELECTRICAL NETWORK CONNECTIONS

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ABSTRACT

In this paper, we survey the role of mathematics in electrical network connections. We discuss the behavior of current flows, voltages and impedances mainly for series-parallel networks. In both one-port and multiport electrical networks, the currents through these networks are governed by Maxwell's power principle. The joint impedances of the networks, given in terms of series sum and parallel sum, satisfy the series-parallel inequality. An abstract idea can be formulated in functional analysis aspect in which any network connection is viewed to be a binary operation for positive operators satisfying certain algebraic, order and topological properties.

Keyword: - *Electrical network connection, Maxwell's power principle, Parallel sum, Positive definite matrix, Positive operator etc….*

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1. INTRODUCTION

In electrical engineering, an electrical network is an interconnection of physically electrical components (e.g. batteries, resistors, capacitors, inductors, switches) or a model of such an interconnection, consisting of electrical elements (e.g. voltage/current sources, resistances, inductances). This paper provides a discussion of the flows of currents through an electrical network obeying Ohm's law and Kirchhoff's voltage/current laws:

Ohm's law: the current through an electrical device is the ratio between the voltages (electrical potential difference) dropped on this device and its impedance.

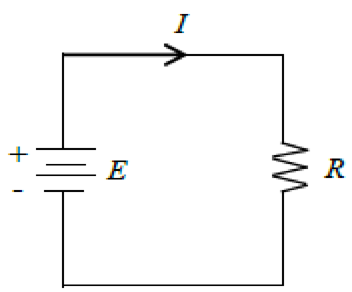
Kirchhoff's current law: the sum of currents meeting at a node is zero.

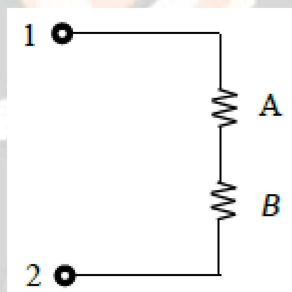
Kirchhoff's voltage law: the sum of voltages in a closed electrical circuit is zero. The main concern here is the joint impedance of series-parallel network. Various properties of series and parallel additions and their physical interpretations are investigated.

For one-port network, the impedance of the network can be described by the notion of parallel sum for scalars. Algebraic properties of this operation were investigated in. The current flow in the network is governed by the so called Maxwell's power principle. Elementary algebra and calculus shows that the joint impedance of the network satisfies the series-parallel inequality. The analysis will become more complicated in the case of multiport electrical networks. Here, the joint impedance of the network is represented in terms of matrix. Many authors discussed the role of linear algebra and matrix theory for network synthesis, focused on series-parallel connections (see e.g. [3-5]). The main tool for analyzing multiport electrical networks is the notion of parallel sum for positive definite matrices. It turns out that the flows of electrical currents satisfy Maxwell's principle and series-parallel inequality as in one-port case. The theory of parallel sums was then discussed by many authors (see e.g. [7-8]). This motivated the study of mathematical operations derived from electrical networks, such as parallel subtraction (see e.g. [9-10]), hybrid connection (e.g. [11-12]), Wheatstone bridge connection, shorted operator. To extend the idea of network connections, the perspective of functional analysis is an appropriate framework. The joint impedance of the network can be viewed as a positive operator acting on a Hilbert space. Currents and power dissipations are described by vectors and inner products on that Hilbert space. The notion of parallel sum for positive operators was considered in. Applications of parallel sum also go to the area of matrix/operator inequalities and equations (see e.g. [21-23]). A general connection is a binary operation for positive operators satisfying certain algebraic, order and topological properties. Series connection and parallel connection are typical examples of this concept. Every connection can be realized as a weighted series connection of weighted parallel connections. This paper is organized as follows. The next section is an analysis of one-port electrical networks. The third section deals with the role of linear algebra in multiport network connections. Analysis of electrical connections is presented in the language of functional analysis in the fourth section. A general setting for network connection is settled in the fifth section. Finally, we summarize the role of mathematics for electrical network connections.

2. ONE-PORT ELECTRICAL NETWORKS

Consider a simple electrical circuit consisting of a battery of fixed voltage and a resistor of resistance as shown in Figure 1.By Ohm's law, the current flowing in the circuit is given by. For the case of alternative current circuits, the voltage source generates sinusoidal waves and electrical components in the circuit may be not pure resistors (e.g. capacitors, inductors). In this case, resistances are replaced by impedances, which are complex numbers, and the Ohm's law still holds. I=E/R. A one-port network is a "black box" with a single pair of input/output terminals. Consider two resistors connected in series as in Figure 2: By Kirchhoff's voltage law and Ohm's law, the joint resistance R between terminals 1 and 2 is determined by $R = A + B$. Circuit equivalently (using Figure 1), two resistors together act as if they were a single resistor whose resistance is given by the *series* sum R. Next, consider the parallel connection shown in Figure 3.





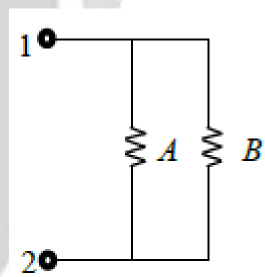


Fig -1 Simple electrical circuit **Fig -2** Series connection of two resistors **Fig -3** Parallel connection of two resistors

Using Kirchhoff's current law and Ohm's law, the joint resistance R between terminals 1 and 2 satisfies the relation

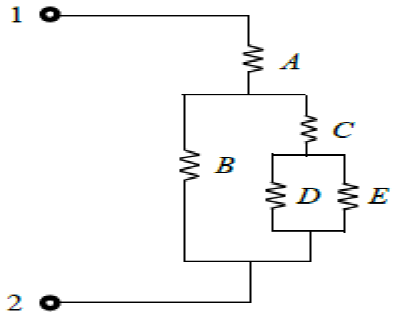
$$\frac{1}{R} = \frac{1}{A} + \frac{1}{B} \quad \text{OR } R = (A^{-1}+B^{-1})^{-1} = \frac{AB}{A+B} \qquad (1)$$

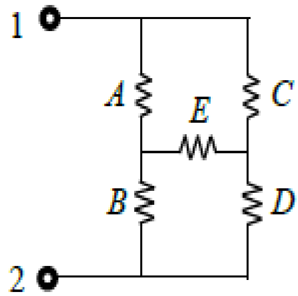
More precisely, the resistors together act as if they were a single resistor whose resistance is given by the parallel sum R, denoted by A: B. The algebraic operation: is termed the parallel addition. The network model shows that the parallel addition is commutative and associative. Moreover, multiplication is distributive over this operation. Consider a series-parallel connection as in Figure 4: The joint resistance of this network is given in terms of series addition and parallel addition as follows:

$$R = A + [B :(C + (D: E))] \qquad (2)$$

Every series-parallel connection network can be interpreted in terms of series addition and parallel addition. However, not every network is a series-parallel connection, for example, the Wheatstone bridge connection in Figure 5. In fact, a network is a series-parallel connection if and only if there is no embedded network having the Wheatstone bridge connection. Recall that the flow of currents through electrical circuits is governed by Maxwell's power principle: the current will take flow paths in such the way that the power dissipation is minimized. This principle, also known as Rayleigh's principle, is equivalent to a variational description of the parallel sum A: B as follows:

$$A: B = \min(x+y=1) = Ax^2 + By^2 \qquad (3)$$





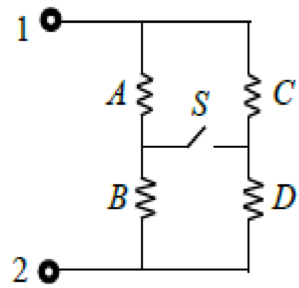


Fig -4 Series-parallel network **Fig -5** Wheatstone bridge connection **Fig -6** An electrical network for proving the series-parallel inequality

The extremal characterization (3) can be derived using optimization technique in multivariable calculus. This serves an easy proof of Lehman's series-parallel inequality (see [3]) as follows. Consider an electrical network as shown in Figure 6. When the switch S is open, the joint resistance is given by

$$R_o = (A+B) :(C+D) \qquad (4)$$

On the other hand, when S is closed, the joint resistance becomes

$$R_c = (A: B) +(C: D) \qquad (5)$$

Since the current takes the path of least resistance (that is, least power) and there is less constraint with the switch close, we arrive at the Lehman's series-parallel inequality:

$$(A: B) +(C: D) \leq (A+B) :(C+D) \qquad (6)$$

Alternatively, the series-parallel inequality can be expressed as

$$(AC/A+C) + (BD/B+D) \leq ((A+B)\ (C+D))/\ (A+B+C+D) \qquad (7)$$

It is worth nothing that the connection in Figure 6 corresponds to replacing the resistor E in the Wheatstone bridge connection in Figure 5 with a switch. Let R_W be the joint resistance of Wheatstone bridge. By Rayleigh's principle, we obtain:

$$R_c \leq R_W \leq R_o \qquad (8)$$

3. MULTIPORT ELECTRICAL NETWORKS

A multiport network consists of several pairs of input/output terminals. In this section, we analyze multiport electrical networks using linear algebra. Consider an electrical network with two pairs of terminals as in Figure 7: The first pair of terminals is in circuit 1 and the second one is in circuit 2. Then the currents and voltages in these circuits are related by the following system of linear equations:

$$E_1=R_{11}I_1+R_{12}I_2$$
$$E_2=R_{21}I_1+R_{22}I_2 \qquad (9)$$

Rewrite these equations in vector/matrix form as

E=RI

Where

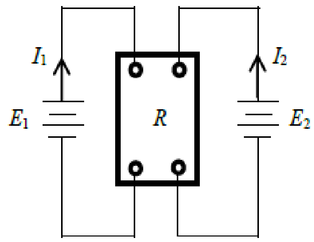
$$E=\begin{bmatrix} E1 \\ E2 \end{bmatrix},\ R=\begin{bmatrix} R11 & R12 \\ R21 & R22 \end{bmatrix},\ I=\begin{bmatrix} I1 \\ I2 \end{bmatrix} \qquad (10)$$

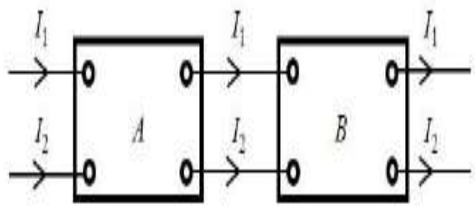
If the resistor box R contains interconnected resistors, then $R_{12=}$ $R_{21,}$ that is R Moreover, the conservation law of energy implies that R is a positive semidefinite matrix, more precisely $x^T Rx \geq 0$ for all $x \epsilon R^2$. In what follows, a positive semidefinite symmetric matrix will be called a resistance matrix. Resistor boxes can be added in series as is show in Figure 8. Here, we assume that the current I_1 in the first circuit of box A is the same as the current in the first circuit of the box B It is similar for the current $I_{2.}$ This can be achieved via use of isolation transformers.

If A and B are the resistance matrices R of these networks, then the joint resistance matrix is given by R= A+B. In other words, series connection of resistance boxes corresponds to addition of their resistance matrices. Figure 9 gives a symbolic meaning of the series addition of resistor boxes. From the relation (10), any current vector I can be an input of a resistor box. However, not every voltage vector E can be an input if the resistance matrix R is not invertible. In any cases, the following fact relates the range spaces of the series connection of the networks.

$$\text{Range } (A+B) = \text{Range } (A) + \text{Range } (B) \qquad (11)$$

Now, consider the case when we connect the resistor boxes in parallel as in Figures 10 and 11.





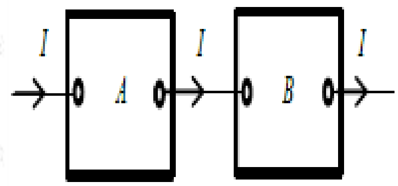
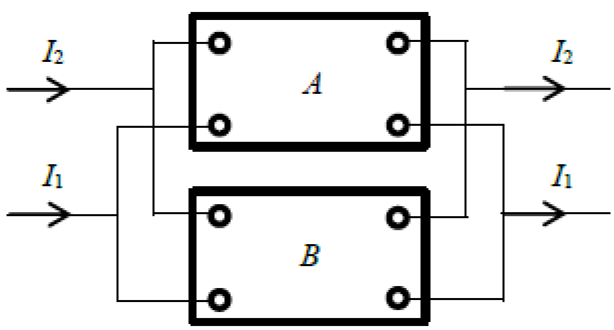


Fig -7 Two-port electrical network **Fig -8** Series connection of two-port networks **Fig -9** symbolic meaning of the series addition



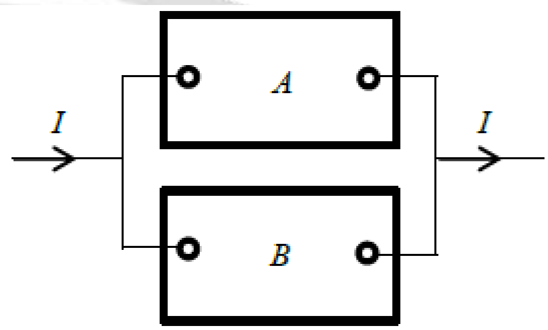


Fig -10 Parallel connection of two-port networks **Fig -11** Symbolic form of a parallel connection

Suppose that both A and B are represented by invertible positive semidefinite matrices (that is, positive definite matrices). Then the joint resistance matrix R of the parallel connection and the resistance matrices A, B are related by

$$R^{-1} = A^{-1} + B^{-1} \qquad (12)$$

We obtain that $R=A\,(A+B)^{-1}\,B$ For simplicity, we write A: B for R and call it the parallel addition of A and B that is

$$A: B = A\,(A+B)^{-1}\,B \qquad (13)$$

Note that the relation (11) shows that Range (A+B) ⊇Range (B).This means that A+B is invertible on its range. Hence, $(A + B)^{-1}\, B$ is a well-defined matrix. Thus, the parallel addition is a well-defined operation for any pair of positive semidefinite matrices. Various algebraic, order and analytic properties of this operation were investigated in [10- 13]. We will discuss some of these properties. By virtue of the network model, the parallel addition is expected to be commutative and associative. Here, we give a direct proof of commutativity:

$$A: B = (A+B-B)\,(A+B)^{-1}\,B = B-B)\,(A+B)^{-1}\,B$$

$$B: A = B \quad (A+B)^{-1}(A+B-B) = B-B)\,(A+B)^{-1}\,B \qquad (14)$$

This implies that

$$A: B = B\,(A+B)^{-1}A \qquad (15)$$

From the definition (13) of parallel sum, we clearly have Range ($A + B$) ⊆ Range (A). Similarly, the relation (15) shows that Range ($A + B$) ⊆ Range (B). Further analysis gives

$$\text{Range } (A+B) = \text{Range } (A) \cap \text{Range } (B) \qquad (16)$$

The relations (11) and (16) reveal a remarkable duality between series addition and parallel addition. To give an application of the above duality principle, we will analyze the networks in Figures 12 and 13. Clearly the joint resistance matrix of the first network is given by

$$R_1 = (A+B) :(B+C) :(C+A) \qquad (17)$$

Let, X, Y, Z be the range spaces A, B, C of respectively. From (11) and (17), we have,

$$\text{Range } (R_1) = (X+Y) \cap (Y+Z) \cap (Z+X) \qquad (18)$$

On the other hand, the joint resistance matrix of the second network is

$$R_2 = [A :(B+C)] + [B :(A+C)] \qquad (19)$$

Hence, the range of is R_2 is

$$\text{Range}\,(R_2) = (X\cap(Y+Z)) + [Y\cap (Z+X)] \quad (20)$$

Now, recall that the collection of subspaces of a vector space form a modular lattice. From modular identity, we have

$$(X+Y) \cap (Y+Z) \cap (Z+X) = (X\cap(Y+Z)) + [Y\cap (Z+X)] \quad (21)$$

This means that

$$\text{Range}R_1=\text{Range}R_2 \quad (22)$$

Thus, various analogous procedures for constructing networks with the same range can be obtained by the duality principle.

The network connection used by Lehman [3] to obtain the series-parallel inequality for positive reals can be extended to resistor boxes. More precisely, for positive semidefinite matrices A, B, C, D of the same size, we have

$$(A: C)+ (B: D) \leq (A+B): C+D) \quad (23)$$

Here, the partial order $X\leq Y$ means B-A is positive semidefinite whenever X and Y are Hermitian or real symmetric matrices. The series-parallel inequality means that the joint impedance of the network in Figure 14 is not greater than that in Figure 15.

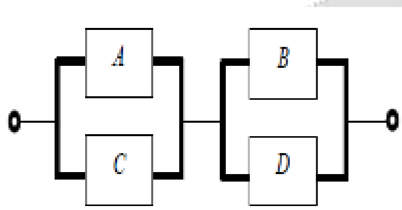


Fig -14 The joint impedance of the network with parallel first and Series last

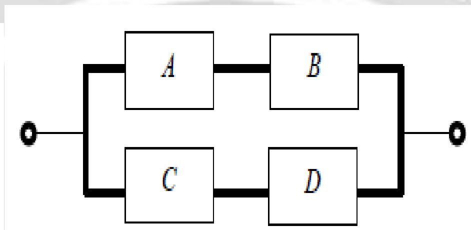


Fig -15 The joint impedance of the network with series' first and. parallel last

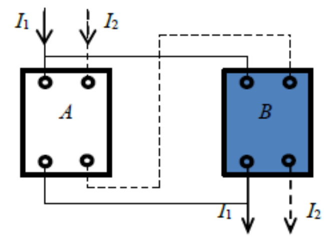


Fig -16 A network hybrid connection

Consider the network connections in which some circuits are put in series and some circuits are put in parallel. Such a connection is called the hybrid connection as shown in Figure 16. An elegant network synthesis of the hybrid connection can be found in [6]. The joint resistance of the hybrid connection is called the hybrid sum. In this type of connections, the series-hybrid inequality is valid.

4. CONCLUSIONS

The treatment of calculus, linear algebra and functional analysis, applied to electrical network connections, results in the interpretations of Maxwell's power principle, the series-parallel inequality and many physical phenomena. An abstract network connection can be viewed as a binary operation for positive operators satisfying certain order, algebraic and topological properties

5. ACKNOWLEDGEMENT

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