Mathematical Analysis on Mild Stenosed Artery with the help of a Two- Phase Non- Newtonian Blood Flow Power Law Model

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Abstract: The aim of the present study is to evaluate a mathematical model of two-phase non-Newtonian blood flow Power law model in the presence of mild stenosed artery. Analytical and numerical methods are used to solve the equations under taken boundary condition. The physical quantities have been expressed into tensorial form. The pressure drop along the length of stenosis has been calculated. A relation acquired between pressure drop and hematocrit which is helpful to anticipate fluctuation in blood flow during stenosis.

Key words: Two Phase Non-Newtonian blood flow, mild stenosed artery, power law model.

Introduction

Stenosis is a type of cardiovascular disease which involves the deposition of plaque on the inner wall of the artery, leading to narrowing of the artery [4]. As the growth projects into the lumen (cavity) of the artery, blood flow is obstructed. The obstruction may damage the internal cells of the wall and may lead to further growth of the stenosis. Thus there is a coupling between the growth of a stenosis and the flow of blood in the artery since each affects the other [12].

The stenosis growth passes through three stages-

The first stage stenosis in blood vessels is 30%, second stage of stenosis increased up to 50% and the third stage of stenosis increased up to 70% i.e. stenosis condition is lie between 30% to 70%. If stenosis in the blood vessels reached above 70% then the case is very serious.

The development of stenosis in an artery can have serious consequences and can disrupt the normal functioning of the circulatory system.

In particular, it may lead to

(i) Increased resistance of flow, with possible sever reduction in blood flow.
(ii) Increased danger of complete obstruction.
(iii) Abnormal cellular growth in the vicinity of the stenosis, which increased the intensity of the stenosis and
(iv) Tissue damage leading to post-stenosis dilation.

2. Mathematical Formulation

According to Upadhyay and Pandey [13], we take the blood flow in coronary arteries remote from heart to be non-Newtonian power law model. We shall consider the steady flow of a Newtonian fluid with an axially-symmetric stenosis whose surface is given by-

$\frac{R(z)}{R_0} = 1 - \frac{\delta}{2R_0}\left(1 + \frac{cos\pi z}{z_0}\right) \quad ; -z_0 \le z \le z_0$ …………. (2.1)

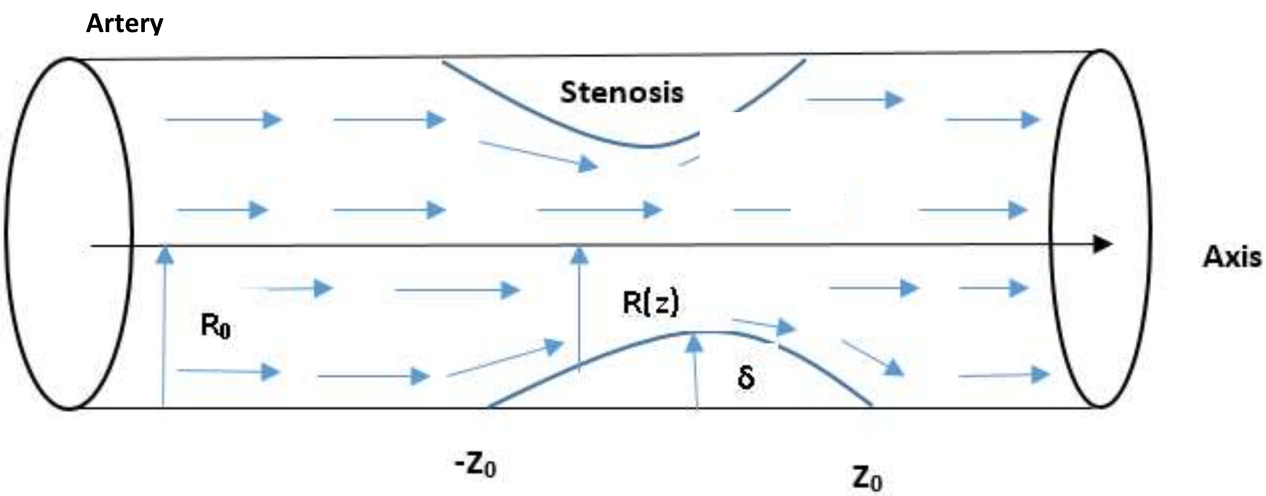


Fig: 2 Mild Stenosis

We shall assume further that $\frac{\delta}{R_0} <<< 1$ and $R_e \frac{\delta}{R_0} <<< 1$,

Where R_e is the Reynold number of blood flow.

In blood flow Reynold number can up to 2000.

$$R_e = \frac{Inertial\ force}{Viscous\ force}$$

Due to high Reynold number blood flow cannot turbulent.

Now the equation of blood flow in tensorial form is given as follows-

The equation of continuity

$\frac{1}{\sqrt{g}(\sqrt{g v^i})_{,i}} = 0$ ………………. (2.2)

Again the equation of motion

$\rho_m \frac{\partial v^i}{\partial t} + \rho_m v^j v^i_{,j} = T^{ij}_{,j}$ ………………. (2.3)

The constitutive equation for the power law non-Newtonian blood flow is as follows-

$T^{ij} = -pg^{ij} + \eta_m (e^{ij})^n$

$T^{ij} = -pg^{ij} + T'^{ij}$ ……………….. (2.4)

By caring out the order of magnitude analysis on these basic equations of flow transferred into cylindrical form, it can be observed that the radial velocity can be neglected in relation to axial velocity v which is determined by

$-\frac{\partial p}{\partial r} = 0$ ………….......... (2.5)

$-\frac{\partial p}{\partial z} + \frac{\eta_m}{r}\frac{\partial}{\partial r}\left[r\left(\frac{\partial v}{\partial r}\right)^n\right]$ ………………… (2.6)

Using no- slip condition on the stenosis surface is given as-

$v = 0 \;\; at \;\; r = R(z) \;\;, \qquad -z_0 \le z \le z_0$

$v = 0 \;\; at \;\; r = R_0 \;\;, \qquad |z| \ge z_0$ …………… (2.7)

3. Solution

For a mild stenosis,

The pressure gradient $P = -\frac{\partial p}{\partial z}$

and axial velocity are functions of z also.

Hence equation (2.5) & (2.6) can be combined into the following equations

$-P(z) = \frac{\eta_m}{r}\frac{\partial}{\partial r}\left\{r\left(\frac{\partial v}{\partial r}\right)^n\right\}$ …………………. (3.1)

On integration, we find

$r\left(\frac{\partial v}{\partial r}\right)^n = -\frac{P(z)r^2}{2\eta_m} + A(z)$ ………………….. (3.2)

Applying the boundary condition, we obtain

$$A(z) = 0$$

And equation (3.2) transformed into the following form:

$r\left(\frac{\partial v}{\partial r}\right)^n = -\frac{P(z)r^2}{2\eta_m}$

$\frac{\partial v}{\partial r} = \left(\frac{P(z)r}{2\eta_m}\right)^{1/n}$ …………………… (3.3)

On integration, we obtain

$v = -\left(\frac{P(z)}{2\eta_m}\right)^{1/n}\frac{n}{n+1}\; r^{1/n+1} + B(z)$ ..………………. (3.4)

Applying the boundary condition we find the value of arbitrary constant

$B(z) = \left(\frac{P(z)}{2\eta_m}\right)^{1/n}\frac{n}{n+1}\; R(z)^{1/n+1}$ ……………………… (3.5)

Hence the equation (3.4) reduced into the following form-

$v = \frac{n}{n+1}\left(\frac{P(z)}{2\eta_m}\right)^{1/n}\left[\,R(z)^{1/n+1} - r^{1/n+1}\,\right]$ …………… (3.6)

The equation (3.6) gives the velocity of blood flow passing through stenosis.

4. Bio-Physical Interpretation

The flow flux of blood is given by

$$Q = \int_0^{R(z)} v \, . 2\pi r \, dr$$

$$= \int_0^{R(z)} \frac{n}{n+1}\left(\frac{P(z)}{2\eta_m}\right)^{1/n} \left[R(z)^{1/n+1} - r^{1/n+1} \right] 2\pi r dr$$

$$Q = \frac{n\pi}{3n+1}\left(\frac{P(z)}{2\eta_m}\right)^{1/n} R(z)^{1/n+3} \quad \text{……………….. (4.1)}$$

Now according to Gupta et.al. [5], we have

$$Q = 250 \, ml/\min = 0.004166 \, m^3/sec$$

And radius of coronary artery R = 0.2 cm = 0.002 m [7]

Length of coronary artery $\Delta z = 50 \, cm = 0.5 \, m$

According to Gustafson and Daniel $\eta_p = 0.0015 \, pascal \; sec$ [6]

According to Glenn Elert $\eta_m = 0.035 \, pascal \; sec$ [4]

Patient case history:

Name – Mrs. Renu Bind , Age – 27 Y , Sex – F

Diagnosis – Mild Stenosis

Date	Hemoglobin (gm /dl)	Hematocrit (kg/m^3)	Blood pressure (mmHg)	Blood Pressure Drop	Arteries Pressure Drop in Pascal-sec $\Delta P = \frac{S+D}{2} - S$
5/11/22	11.8	0.033396	100/60	20	2666.44
6/11/22	11.2	0.031698	110/70	20	2666.44
7/11/22	10.8	0.030566	110/70	20	2666.44
8/11/22	10.6	0.03	120/70	25	3333.05
9/11/22	10.3	0.029150	120/80	20	2666.44

Table - 1

$1 \, mmhg \; at \; 0°c = 133.322 \, pascal/sec$ [15].

If 30% Stenosis

$\delta = 30\%$ of radius of coronary artery = 0.002 * 0.2 m = 0.0004 m

$R(z) = R_0 - \delta = 0.002 - 0.0004 = 0.002m$

According to used pathological data (table 1)

Hematocrit **H** = 0.033396 and

Pressure drop $(P_f - P_i) = 2666.44 \, pascal \; sec$

$X = \frac{H}{100} = 0.00033396$

Using relation $\eta_m = \eta_c X + \eta_p(1 - X)$

$$0.035 = 0.00033396\eta_c + 0.0015 * 0.99966604$$

$$0.00033396\eta_c = 0.03350050094$$

$$\eta_c = 100.312914541\ pascal\ sec$$

Again using the relation and change into the hematocrit form, we get

$$\eta_m = 1.00312914541H + 0.00149949906$$

Now putting all the values in equation (4.1)

$$0.004166 = \frac{n\pi}{3n+1}\left(\frac{2666.44}{2 * 0.035 * 0.5}\right)^{1/n}(0.002)^{1/n+3}$$

On solving by numerical method, we get

$$\boldsymbol{n = 0.3655}$$

Now applying n = 0.3655 in equation (4.1), we get

$$0.004166 = \frac{0.3655 * 3.14}{(3 * 0.3655) + 1}\left(\frac{P(z)}{2\eta_m}\right)^{1/n}(0.002)^{1/0.3655+3}$$

$$\Delta P = 2\eta_m(z_f - z_i) * 2.30468028$$

$$\Delta P = 2\eta_m * 0.5 * 2.30468028$$

$$\Delta P = \eta_m * 2.30468028$$

$$\Delta P = (1.00312914541H + 0.00149949906) * (2.30468020)$$

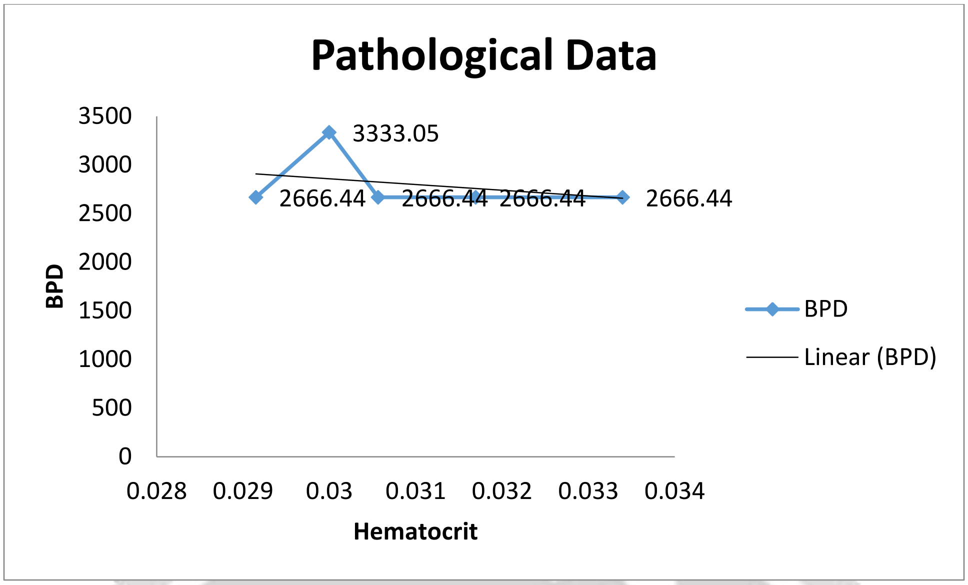
$$\Delta P = 2.31189195972H + 0.00345586591$$

Date	**Hematocrit (kg/m^3)**	**Blood Pressure Drop in Pascal second**
5/11/22	0.033396	0.0806638098
6/11/22	0.031698	0.07673821725
7/11/22	0.030566	0.07412115555
8/11/22	0.03	0.0728126247
9/11/22	0.029150	0.07084751654

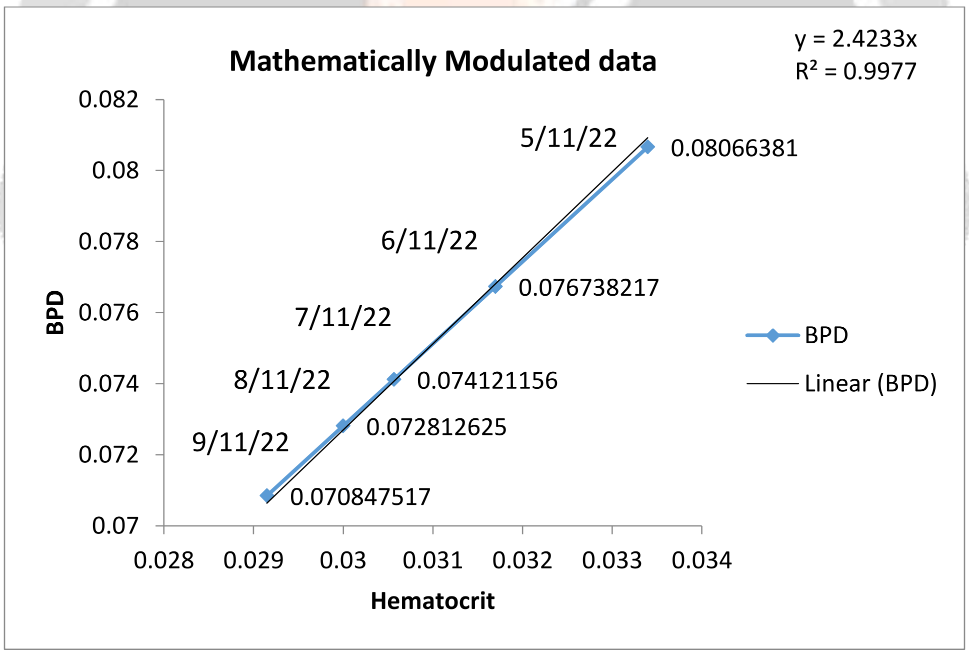
Table-2: Table of Hematocrit v/s Blood Pressure Drop in Pathological data

Graphically presentation

Graph-1: Table of Hematocrit v/s Blood Pressure Drop in Pathological data



Graph-2: Table of Hematocrit v/s Blood Pressure Drop in mathematically Modulated data



5. Conclusion

If 30% stenosed coronary artery presented figure, graph 2 (table 2) shows from 5/11/22 to 9/11/22 decreasing sense. According to trend line if the slope is less than 45 degree, fluctuations in blood pressure drop are decreased and we suggest for high dose. On the other way if the slope is greater than 45 degree, fluctuations in blood pressure drop are increased and we suggest for low dose during operation. According to this study work we observed that the slope is in decreasing sense then the fluctuation is also decreased and we suggest for high dose during operation.

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