Mathematical Modeling for Life Cycle Forecasting of Zirconia Based Furnace Wall

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ABSTRACT

Furnaces are most almost continuously used for melting of materials. Induction furnaces are more invaluable as no fuel is required. It is a difficulty to search out life cycle of Induction Melting Furnace Wall beneath load variant. The induction melting furnace wall is made-up of zirconia which is one variety of refractory material. The failure happens because of cyclic thermal stresses due to heating and cooling cycles. Temperature distribution and thermal stress distribution fields of the induction melting furnace refractory wall have been calculated by utilizing explicit finite difference analysis centred on the bodily description of its failure under low cycle thermal fatigue stipulations. The life span of the refractory wall is required to be found out by means of essential thermal stresses created throughout the refractory wall of induction melting furnace wall from modified S – log N Curve.

Key words: *Advanced mathematical modeling, Temperature distribution, Stress distribution, Explicit finite difference method, Zirconia*

I. INTRODUCTION

Furnace is a device used to identify a closed place right here heat is applied to a body in order to elevate its temperature. The supply of heat could also be by fuel or electricity. Most commonly, metals and alloys and frequently non-metals are heated in furnaces. The purpose of heating defines the temperature of heating and heating rate. Increase in temperature softens the metals. They grow to be amenable to deformation. This softening happens with or and not using an alternate within the steel composition. Heating to critical temperatures (under the critical temperature) of the metallic softens it via relieving the internal stresses. Alternatively, metals heated to temperatures above the crucial temperatures results in changes in crystal structures and recrystallization like annealing. Additional some metals and alloys are melted, ceramic products vitrified, coals coked, metals like zinc are vaporized and plenty of different methods are performed in Furnaces.

Induction furnaces are widely used in the iron industry for the casting of the different grades of cast iron products. Refractory wall of induction melting furnace is a key component which is used as insulation layer. It is made of ramming mass like silica, alumina, magnesia etc. The refractory wall is directly influenced by the thermal cycling of the high temperature molten iron in the furnace. Thermal fatigue failure is easy to happen for it because of the larger phase transformation thermal stresses and it has a shorter life. This can cause serious production accidents. Therefore, the service life problem of the refractory wall has always been a focus of attention in the application of this to the industry. (A V K Suryanarayana, Fuels Furnaces Refractory and Pyrometry).

Here, Explicit Finite Difference Method is used to find out temperature and thermal stress variation with respect to time.

II. DEVELOPMENT OF ADVANCED EXPLICIT FINITE DIFFERENCE MODEL

We have divided Induction Furnace Wall into a Nodal Network as shown in Fig. 1. It is divided into 24 nodes. We have derived Explicit Finite Difference Equations for all nodes as per the boundary conditions applied to it. The furnace wall is having thermal conduction heat transfer between different nodes. It is having atmospheric heat convection ha applied from top side of the furnace wall which is open to atmosphere. It is having heat convection from molten metal from inside which is hi. It is having heat convection ho from cooling water which is circulating outside the furnace wall. (Yunus A Cengel, Heat and Mass Transfer).

To solve this advanced heat transfer problem of induction melting furnace wall which is made from Zirconia, the following initial and boundary conditions, material properties and basic assumptions are made:

Refractory Materials for induction furnace wall meets the basic assumptions in the science of mechanics.

- Environmental Temperature is homogeneous at 27° C.
- Ignore the influence of heat radiation.
- Ignore the effect of gravity field.
- The surface of induction melting furnace wall is clean.
- The initial temperature of the induction melting furnace is set 27° C and it is agreement with the ambient temperature during solving the problem.
- Heat convections are considered constant for this analysis.
- Scarp material input inside furnace is considered uniform for our analysis.

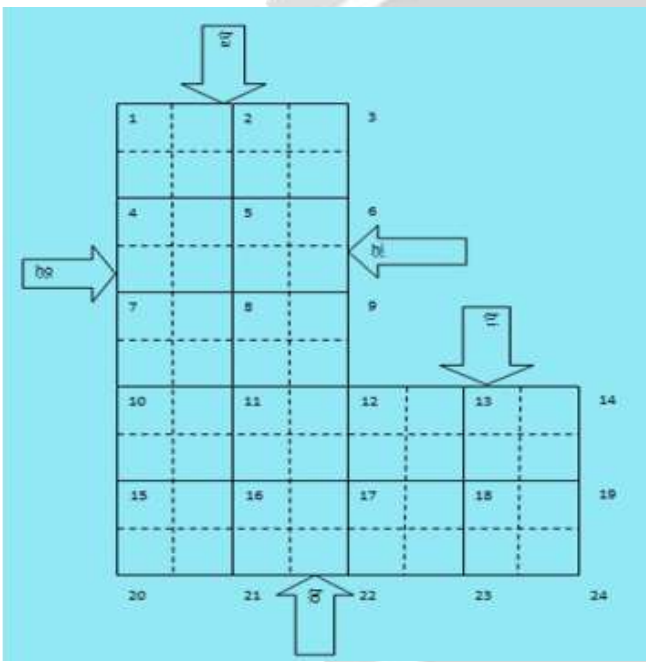

Fig.1 Nodal network for finite difference method

Node 1:

$$ha\frac{\Delta x}{2}(T_\infty - T_1^i)+ ho\frac{\Delta y}{2}(T_\infty - T_1^i) + k\frac{\Delta y}{2}\frac{T_2^i - T_1^i}{\Delta x} + k\frac{\Delta x}{2}\frac{T_4^i - T_1^i}{\Delta y} = \rho\frac{\Delta x}{2}\frac{\Delta y}{2}C\frac{T_1^{i+1}-T_1^i}{\Delta t}$$

$$T_1^{i+1} = ((ha\frac{\Delta x}{2}(T_\infty - T_1^i)+ ho\frac{\Delta y}{2}(T_\infty - T_1^i) + k\frac{\Delta y}{2}\frac{T_2^i - T_1^i}{\Delta x} + k\frac{\Delta x}{2}\frac{T_4^i - T_1^i}{\Delta y})\frac{4\Delta t}{\rho(\Delta x\Delta y}) + T_1^i$$

T[1][i+1] = (((0.5*ha*x*(To-T[1][i])) +(ho*y*(Ta-T[1][i])/2)+(0.5*k*y*(T[2][i]-T[1][i])/x)+(0.5*k*x*(T[4][i]-T[1][i])/y))*((4*t)/(r*c*x*y))) +T[1][i];

Node 2:

$$ha\Delta x(T_\infty - T_2^i)+ k\frac{\Delta y}{2}\frac{T_1^i - T_2^i}{\Delta x} + k\frac{\Delta y}{2}\frac{T_3^i - T_2^i}{\Delta x} + k\Delta x\frac{T_5^i - T_2^i}{\Delta y} = \rho\Delta x\frac{\Delta y}{2}C\frac{T_2^{i+1}-T_2^i}{\Delta t}$$

$$T_2^{i+1} = ((ha\Delta x(T_\infty - T_2^i)+ k\frac{\Delta y}{2}\frac{T_1^i - T_2^i}{\Delta x} + k\frac{\Delta y}{2}\frac{T_3^i - T_2^i}{\Delta x} + k\Delta x\frac{T_5^i - T_2^i}{\Delta y})\frac{2\Delta t}{\rho(\Delta x\Delta y}) + T_2^i$$

T[2][i+1] = (((ha*x*(To-T[2][i]))+(0.5*k*y*(T[1][i]-T[2][i])/x)+(0.5*k*y*(T[3][i]-T[2][i])/x) +(0.5*k*x*(T[5][i]-T[2][i])/y))*((2*t)/(r*c*x*y))) +T[2][i];

Node 3:

$$ha\frac{\Delta x}{2}(T_\infty - T_3^i) + hi\frac{\Delta y}{2}(T_h - T_3^i) + k\frac{\Delta y}{2}\frac{T_2^i - T_3^i}{\Delta x} + k\frac{\Delta x}{2}\frac{T_6^i - T_3^i}{\Delta y} = \rho\frac{\Delta x}{2}\frac{\Delta y}{2}C\frac{T_3^{i+1} - T_3^i}{\Delta t}$$

$$T_3^{i+1} = \left(\left(ha\frac{\Delta x}{2}(T_\infty - T_3^i) + hi\frac{\Delta y}{2}(T_h - T_3^i) + k\frac{\Delta y}{2}\frac{T_2^i - T_3^i}{\Delta x} + k\frac{\Delta x}{2}\frac{T_6^i - T_3^i}{\Delta y}\right)\frac{4\Delta t}{\rho(\Delta x\Delta y}\right) + T_3^i$$

T[3][i+1] = (((ha*x*(To-T[3][i])*0.5) +(hi*y*(Th-T[3][i])*0.5) +(0.5*k*y*(T[2][i]-T[3][i])/x)+(0.5*k*x*(T[6][i]-T[3][i])/y)) *((4*t)/(r*c*x*y))) +T[3][i];

Node 4:

$$ho\Delta y(T_\infty - T_4^i) + k\Delta y\frac{T_5^i - T_4^i}{\Delta x} + k\frac{\Delta x}{2}\frac{T_1^i - T_4^i}{\Delta y} + k\frac{\Delta x}{2}\frac{T_7^i - T_4^i}{\Delta y} = \rho\frac{\Delta x}{2}\Delta y C\frac{T_4^{i+4} - T_4^i}{\Delta t}$$

$$T_4^{i+4} = \left(\left(ho\Delta y(T_\infty - T_4^i) + k\Delta y\frac{T_5^i - T_4^i}{\Delta x} + k\frac{\Delta x}{2}\frac{T_1^i - T_4^i}{\Delta y} + k\frac{\Delta x}{2}\frac{T_7^i - T_4^i}{\Delta y}\right)\frac{2\Delta t}{\rho(\Delta x\Delta y}\right) + T_4^i$$

T[4][i+1] = (((ho*y*(Ta-T[4][i])) +(k*y*(T[5][i]-T[4][i])/x)+(0.5*k*x*(T[1][i]-T[4][i])/y) + (0.5*k*x*(T[7][i]-T[4][i])/y)) *((2*t)/(r*c*x*y))) +T[4][i];

Node 5:

$$k\Delta y\frac{T_4^i - T_5^i}{\Delta x} + k\Delta y\frac{T_6^i - T_5^i}{\Delta x} + k\Delta x\frac{T_2^i - T_5^i}{\Delta y} + k\Delta x\frac{T_8^i - T_5^i}{\Delta y} = \rho\Delta x\Delta y C\frac{T_5^{i+1} - T_5^i}{\Delta t}$$

$$T_5^{i+1} = \left(\left(k\Delta y\frac{T_4^i - T_5^i}{\Delta x} + k\Delta y\frac{T_6^i - T_5^i}{\Delta x} + k\Delta x\frac{T_2^i - T_5^i}{\Delta y} + k\Delta x\frac{T_8^i - T_5^i}{\Delta y}\right)\frac{\Delta t}{\rho(\Delta x\Delta y}\right) + T_5^i$$

T[5][i+1] = (((k*y*(T[4][i]-T[5][i])/x)+(k*y*(T[6][i]-T[5][i])/x) + (k*x*(T[8][i]-T[5][i])/y)) *((t)/(r*c*x*y))) +T[5][i];

Node 6:

$$hi\Delta y(T_h - T_6^i) + k\Delta y\frac{T_2^i - T_6^i}{\Delta x} + k\frac{\Delta x}{2}\frac{T_3^i - T_6^i}{\Delta y} + k\frac{\Delta x}{2}\frac{T_9^i - T_6^i}{\Delta y} = \rho\frac{\Delta x}{2}\Delta y C\frac{T_6^{i+1} - T_6^i}{\Delta t}$$

$$T_6^{i+1} = \left(\left(hi\Delta y(T_h - T_6^i) + k\Delta y\frac{T_2^i - T_6^i}{\Delta x} + k\frac{\Delta x}{2}\frac{T_3^i - T_6^i}{\Delta y} + k\frac{\Delta x}{2}\frac{T_9^i - T_6^i}{\Delta y}\right)\frac{2\Delta t}{\rho(\Delta x\Delta y}\right) + T_6^i$$

T[6][i+1] = (((hi*y*(Th-T[6][i])) + (k*y*(T[2][i]-T[6][i])/x)+(0.5*k*x*(T[3][i]-T[6][i])/y) + (0.5*k*x*(T[9][i]-T[6][i])/y)) *((2*t)/(r*c*x*y))) +T[6][i];

Node 7:

$$ho\Delta y(T_\infty - T_7^i) + k\Delta y\frac{T_8^i - T_7^i}{\Delta x} + k\frac{\Delta x}{2}\frac{T_4^i - T_7^i}{\Delta y} + k\frac{\Delta x}{2}\frac{T_{10}^i - T_7^i}{\Delta y} = \rho\frac{\Delta x}{2}\Delta y C\frac{T_7^{i+7} - T_7^i}{\Delta t}$$

$$T_7^{i+7} = \left(\left(ho\Delta y(T_\infty - T_7^i) + k\Delta y\frac{T_8^i - T_7^i}{\Delta x} + k\frac{\Delta x}{2}\frac{T_4^i - T_7^i}{\Delta y} + k\frac{\Delta x}{2}\frac{T_{10}^i - T_7^i}{\Delta y}\right)\frac{2\Delta t}{\rho(\Delta x\Delta y}\right) + T_7^i$$

T[7][i+1] = (((ho*y*(Ta-T[7][i])) + (k*y*(T[8][i]-T[7][i])/x)+(0.5*k*x*(T[4][i]-T[7][i])/y) + (0.5*k*x*(T[10][i]-T[7][i])/y)) *((2*t)/(r*c*x*y))) +T[7][i];

Node 8:

$$k\Delta y\frac{T_7^i - T_8^i}{\Delta x} + k\Delta y\frac{T_9^i - T_8^i}{\Delta x} + k\Delta x\frac{T_5^i - T_8^i}{\Delta y} + k\Delta x\frac{T_{11}^i - T_8^i}{\Delta y} = \rho\Delta x\Delta y C\frac{T_8^{i+1} - T_8^i}{\Delta t}$$

$$T_8^{i+1} = \left(\left(k\Delta y\frac{T_7^i - T_8^i}{\Delta x} + k\Delta y\frac{T_9^i - T_8^i}{\Delta x} + k\Delta x\frac{T_5^i - T_8^i}{\Delta y} + k\Delta x\frac{T_{11}^i - T_8^i}{\Delta y}\right)\frac{\Delta t}{\rho(\Delta x\Delta y}\right) + T_8^i$$

T[8][i+1] = (((k*y*(T[7][i]-T[8][i])/x)+(k*y*(T[9][i]-T[8][i])/x) +(k*x*(T[5][i]-T[8][i])/y) + (k*x*(T[11][i]-T[8][i])/y)) *((t)/(r*c*x*y))) +T[8][i];

Node 9:

$$hi\Delta y(T_h - T_9^i) + k\Delta y\frac{T_8^i - T_9^i}{\Delta x} + k\frac{\Delta x}{2}\frac{T_6^i - T_9^i}{\Delta y} + k\frac{\Delta x}{2}\frac{T_{12}^i - T_9^i}{\Delta y} = \rho\frac{\Delta x}{2}\Delta y C\frac{T_9^{i+1} - T_9^i}{\Delta t}$$

$$T_9^{i+1} = ((hi\Delta y(T_h - T_9^i) + k\Delta y\frac{T_8^i - T_9^i}{\Delta x} + k\frac{\Delta x}{2}\frac{T_6^i - T_9^i}{\Delta y} + k\frac{\Delta x}{2}\frac{T_{12}^i - T_9^i}{\Delta y})\frac{2\Delta t}{\rho C\Delta x\Delta y}) + T_9^i$$

T[9][i+1] = (((hi*y*(Th-T[9][i]))+(k*y*(T[8][i]-T[9][i])/x)+ (0.5*k*x*(T[6][i]-T[9][i])/y) + (0.5*k*x*(T[12][i]-T[9][i])/y)) *((2*t)/(r*c*x*y))) +T[9][i];

Node 10:

$$ho\Delta y(T_\infty - T_{10}^i) + k\Delta y\frac{T_{11}^i - T_{10}^i}{\Delta x} + k\frac{\Delta x}{2}\frac{T_7^i - T_{10}^i}{\Delta y} + k\frac{\Delta x}{2}\frac{T_{15}^i - T_{10}^i}{\Delta y} = \rho\frac{\Delta x}{2}\Delta y C\frac{T_{10}^{i+10} - T_{10}^i}{\Delta t}$$

$$T_{10}^{i+10} = ((ho\Delta y(T_\infty - T_{10}^i) + k\Delta y\frac{T_{11}^i - T_{10}^i}{\Delta x} + k\frac{\Delta x}{2}\frac{T_7^i - T_{10}^i}{\Delta y} + k\frac{\Delta x}{2}\frac{T_{15}^i - T_{10}^i}{\Delta y})\frac{2\Delta t}{\rho C\Delta x\Delta y}) + T_{10}^i$$

T[10][i+1] = (((ho*y*(Ta-T[10][i]))+(k*y*(T[11][i]-T[10][i])/x)+ (0.5*k*x*(T[7][i]-T[10][i])/y) + (0.5*k*x*(T[15][i]-T[10][i])/y)) *((2*t)/(r*c*x*y))) +T[10][i];

Node 11:

$$k\Delta y\frac{T_{10}^i - T_{11}^i}{\Delta x} + k\Delta y\frac{T_{12}^i - T_{11}^i}{\Delta x} + k\Delta x\frac{T_8^i - T_{11}^i}{\Delta y} + k\Delta x\frac{T_{16}^i - T_{11}^i}{\Delta y} = \rho\Delta x\Delta y C\frac{T_{11}^{i+1} - T_{11}^i}{\Delta t}$$

$$T_{11}^{i+1} = ((k\Delta y\frac{T_{10}^i - T_{11}^i}{\Delta x} + k\Delta y\frac{T_{12}^i - T_{11}^i}{\Delta x} + k\Delta x\frac{T_8^i - T_{11}^i}{\Delta y} + k\Delta x\frac{T_{16}^i - T_{11}^i}{\Delta y})\frac{\Delta t}{\rho C\Delta x\Delta y}) + T_{11}^i$$

T[11][i+1] = (((k*y*(T[10][i]-T[11][i])/x)+ (k*y*(T[12][i]-T[11][i])/x)+ (k*x*(T[8][i]-T[11][i])/y) + (k*x*(T[16][i]-T[11][i])/y)) *((t)/(r*c*x*y))) +T[11][i];

Node 12:

$$hi\frac{\Delta x}{2}(T_h - T_{12}^i) + hi\frac{\Delta y}{2}(T_h - T_{12}^i) + k\Delta y\frac{T_{11}^i - T_{12}^i}{\Delta x} + k\frac{\Delta y}{2}\frac{T_{13}^i - T_{12}^i}{\Delta x} + k\frac{\Delta x}{2}\frac{T_9^i - T_{12}^i}{\Delta y} + k\Delta x\frac{T_{17}^i - T_{12}^i}{\Delta y} = \rho\frac{3\Delta x\Delta y}{4}C\frac{T_{12}^{i+1} - T_{12}^i}{\Delta t}$$

$$T_{12}^{i+1} = ((hi\frac{\Delta x}{2}(T_h - T_{12}^i) + hi\frac{\Delta y}{2}(T_h - T_{12}^i) + k\Delta y\frac{T_{11}^i - T_{12}^i}{\Delta x} + k\frac{\Delta y}{2}\frac{T_{13}^i - T_{12}^i}{\Delta x} + k\frac{\Delta x}{2}\frac{T_9^i - T_{12}^i}{\Delta y} + k\Delta x\frac{T_{17}^i - T_{12}^i}{\Delta y})\frac{4\Delta t}{3\rho C\Delta x\Delta y}) + T_{12}^i$$

T[12][i+1] = (((0.5*hi*x*(Th-T[12][i]))+(0.5*hi*y*(Th-T[12][i])) +(k*y*(T[11][i]-T[12][i])/x)+ (0.5*k*y*(T[13][i]-T[12][i])/x) + (0.5*k*x*(T[9][i]-T[12][i])/y) + (k*x*(T[17][i]-T[12][i])/y)) *((4*t)/(3*r*c*x*y))) +T[12][i];

Node 13:

$$hi\Delta x(T_h - T_{13}^i) + k\frac{\Delta y}{2}\frac{T_{12}^i - T_{13}^i}{\Delta x} + k\frac{\Delta y}{2}\frac{T_{14}^i - T_{13}^i}{\Delta x} + k\Delta x\frac{T_{18}^i - T_{13}^i}{\Delta y} = \rho\Delta x\frac{\Delta y}{2}C\frac{T_{13}^{i+1} - T_{13}^i}{\Delta t}$$

$$T_{13}^{i+1} = ((hi\Delta x(T_h - T_{13}^i) + k\frac{\Delta y}{2}\frac{T_{12}^i - T_{13}^i}{\Delta x} + k\frac{\Delta y}{2}\frac{T_{14}^i - T_{13}^i}{\Delta x} + k\Delta x\frac{T_{18}^i - T_{13}^i}{\Delta y})\frac{2\Delta t}{\rho C\Delta x\Delta y}) + T_{13}^i$$

T[13][i+1] = (((hi*x*(Th-T[13][i]))+(0.5*k*y*(T[12][i]-T[13][i])/x)+ (0.5*k*y*(T[14][i]-T[13][i])/x)+ (k*x*(T[18][i]-T[13][i])/y)) *((2*t)/(r*c*x*y))) +T[13][i];

Node 14:

$$hi\frac{\Delta x}{2}(T_h - T_{14}^i) + hi\frac{\Delta y}{2}(T_h - T_{14}^i) + k\frac{\Delta y}{2}\frac{T_{13}^i - T_{14}^i}{\Delta x} + k\frac{\Delta x}{2}\frac{T_{19}^i - T_{14}^i}{\Delta y} = \rho\frac{\Delta x}{2}\frac{\Delta y}{2}C\frac{T_{14}^{i+1} - T_{14}^i}{\Delta t}$$

$$T_{14}^{i+1} = ((hi\frac{\Delta x}{2}(T_h - T_{14}^i) + hi\frac{\Delta y}{2}(T_h - T_{14}^i) + k\frac{\Delta y}{2}\frac{T_{13}^i - T_{14}^i}{\Delta x} + k\frac{\Delta x}{2}\frac{T_{19}^i - T_{14}^i}{\Delta y})\frac{4\Delta t}{\rho C\Delta x\Delta y}) + T_{14}^i$$

T[14][i+1] = (((0.5*hi*x*(Th-T[14][i]))+(0.5*hi*y*(Th-T[14][i])) +(0.5*k*y*(T[13][i]-T[14][i])/x)+ (0.5*k*x*(T[19][i]-T[14][i])/y)) *((4*t)/(r*c*x*y))) +T[14][i];

Node 15:

$$ho\,\Delta y(T_{\infty} - T_{15}^{i}) + k\Delta y\,\frac{T_{16}^{i}-T_{15}^{i}}{\Delta x} + k\frac{\Delta x}{2}\frac{T_{10}^{i}-T_{15}^{i}}{\Delta y} + k\frac{\Delta x}{2}\frac{T_{20}^{i}-T_{15}^{i}}{\Delta y} = \rho\,\frac{\Delta x}{2}\Delta y\,C\,\frac{T_{15}^{i+15}-T_{15}^{i}}{\Delta t}$$

$$T_{15}^{i+15} = ((\,ho\,\Delta y(T_{\infty} - T_{15}^{i}) + k\Delta y\,\frac{T_{16}^{i}-T_{15}^{i}}{\Delta x} + k\frac{\Delta x}{2}\frac{T_{10}^{i}-T_{15}^{i}}{\Delta y} + k\frac{\Delta x}{2}\frac{T_{20}^{i}-T_{15}^{i}}{\Delta y})\,\frac{2\Delta t}{\rho C\Delta x\Delta y}\,) + T_{15}^{i}$$

T[15][i+1] = (((ho*y*(Ta-T[15][i]))+(k*y*(T[16][i]-T[15][i])/x)+ (0.5*k*x*(T[10][i]-T[15][i])/y)+(0.5*k*x*(T[20][i]-T[15][i])/y)) *((2*t)/(r*c*x*y))) +T[15][i];

Node 16:

$$k\Delta y\,\frac{T_{15}^{i}-T_{16}^{i}}{\Delta x} + k\Delta y\,\frac{T_{17}^{i}-T_{16}^{i}}{\Delta x} + k\Delta x\,\frac{T_{11}^{i}-T_{16}^{i}}{\Delta y} + k\Delta x\,\frac{T_{21}^{i}-T_{16}^{i}}{\Delta y} = \rho\Delta x\Delta y\,C\,\frac{T_{16}^{i+1}-T_{16}^{i}}{\Delta t}$$

$$T_{16}^{i+1} = ((\,k\Delta y\,\frac{T_{15}^{i}-T_{16}^{i}}{\Delta x} + k\Delta y\,\frac{T_{17}^{i}-T_{16}^{i}}{\Delta x} + k\Delta x\,\frac{T_{11}^{i}-T_{16}^{i}}{\Delta y} + k\Delta x\,\frac{T_{21}^{i}-T_{16}^{i}}{\Delta y})\,\frac{\Delta t}{\rho C\Delta x\Delta y}\,) + T_{16}^{i}$$

T[16][i+1] = (((k*y*(T[15][i]-T[16][i])/x)+ (k*y*(T[17][i]-T[16][i])/x)+(k*x*(T[11][i]-T[16][i])/y)+(k*x*(T[21][i]-T[16][i])/y)) *((t)/(r*c*x*y))) +T[16][i];

Node 17:

$$k\Delta y\,\frac{T_{16}^{i}-T_{17}^{i}}{\Delta x} + k\Delta y\,\frac{T_{18}^{i}-T_{17}^{i}}{\Delta x} + k\Delta x\,\frac{T_{12}^{i}-T_{17}^{i}}{\Delta y} + k\Delta x\,\frac{T_{22}^{i}-T_{17}^{i}}{\Delta y} = \rho\Delta x\Delta y\,C\,\frac{T_{17}^{i+1}-T_{17}^{i}}{\Delta t}$$

$$T_{17}^{i+1} = ((\,k\Delta y\,\frac{T_{16}^{i}-T_{17}^{i}}{\Delta x} + k\Delta y\,\frac{T_{18}^{i}-T_{17}^{i}}{\Delta x} + k\Delta x\,\frac{T_{12}^{i}-T_{17}^{i}}{\Delta y} + k\Delta x\,\frac{T_{22}^{i}-T_{17}^{i}}{\Delta y})\,\frac{\Delta t}{\rho C\Delta x\Delta y}\,) + T_{17}^{i}$$

T[17][i+1] = (((k*y*(T[16][i]-T[17][i])/x)+ (k*y*(T[18][i]-T[17][i])/x)+(k*x*(T[12][i]-T[17][i])/y)+(k*x*(T[22][i]-T[17][i])/y)) *((t)/(r*c*x*y))) +T[17][i];

Node 18:

$$k\Delta y\,\frac{T_{17}^{i}-T_{18}^{i}}{\Delta x} + k\Delta y\,\frac{T_{19}^{i}-T_{18}^{i}}{\Delta x} + k\Delta x\,\frac{T_{13}^{i}-T_{18}^{i}}{\Delta y} + k\Delta x\,\frac{T_{23}^{i}-T_{18}^{i}}{\Delta y} = \rho\Delta x\Delta y\,C\,\frac{T_{18}^{i+1}-T_{18}^{i}}{\Delta t}$$

$$T_{18}^{i+1} = ((k\Delta y\,\frac{T_{17}^{i}-T_{18}^{i}}{\Delta x} + k\Delta y\,\frac{T_{19}^{i}-T_{18}^{i}}{\Delta x} + k\Delta x\,\frac{T_{13}^{i}-T_{18}^{i}}{\Delta y} + k\Delta x\,\frac{T_{23}^{i}-T_{18}^{i}}{\Delta y}\,)\,\frac{\Delta t}{\rho C\Delta x\Delta y}\,) + T_{18}^{i}$$

T[18][i+1] = (((k*y*(T[17][i]-T[18][i])/x)+ (k*y*(T[19][i]-T[18][i])/x)+(k*x*(T[13][i]-T[18][i])/y)+(k*x*(T[23][i]-T[18][i])/y)) *((t)/(r*c*x*y))) +T[18][i];

Node 19:

$$hi\Delta y(T_{h} - T_{19}^{i}) + k\Delta y\,\frac{T_{18}^{i}-T_{19}^{i}}{\Delta x} + k\frac{\Delta x}{2}\frac{T_{14}^{i}-T_{19}^{i}}{\Delta y} + k\frac{\Delta x}{2}\frac{T_{24}^{i}-T_{19}^{i}}{\Delta y} = \rho\,\frac{\Delta x}{2}\Delta y\,C\,\frac{T_{19}^{i+1}-T_{19}^{i}}{\Delta t}$$

$$T_{19}^{i+1} = ((hi\Delta y(T_{h} - T_{19}^{i}) + k\Delta y\,\frac{T_{18}^{i}-T_{19}^{i}}{\Delta x} + k\frac{\Delta x}{2}\frac{T_{14}^{i}-T_{19}^{i}}{\Delta y} + k\frac{\Delta x}{2}\frac{T_{24}^{i}-T_{19}^{i}}{\Delta y})\,\frac{2\Delta t}{\rho C\Delta x\Delta y}\,) + T_{19}^{i}$$

T[19][i+1] = (((hi*y*(Th-T[19][i]))+(k*y*(T[18][i]-T[19][i])/x)+ (0.5*k*x*(T[14][i]-T[19][i])/y)+(0.5*k*x*(T[24][i]-T[19][i])/y)) *((2*t)/(r*c*x*y))) +T[19][i];

Node 20:

$$ho\,\frac{\Delta x}{2}(T_{\infty} - T_{20}^{i}) + ho\,\frac{\Delta y}{2}(T_{\infty} - T_{20}^{i}) + k\frac{\Delta y}{2}\frac{T_{21}^{i}-T_{20}^{i}}{\Delta x} + k\frac{\Delta x}{2}\frac{T_{15}^{i}-T_{20}^{i}}{\Delta y} = \rho\,\frac{\Delta x}{2}\frac{\Delta y}{2}\,C\,\frac{T_{20}^{i+20}-T_{20}^{i}}{\Delta t}$$

$$T_{20}^{i+20} = ((ho\,\frac{\Delta x}{2}(T_{\infty} - T_{20}^{i}) + ho\,\frac{\Delta y}{2}(T_{\infty} - T_{20}^{i}) + k\frac{\Delta y}{2}\frac{T_{21}^{i}-T_{20}^{i}}{\Delta x} + k\frac{\Delta x}{2}\frac{T_{15}^{i}-T_{20}^{i}}{\Delta y})\,\frac{4\Delta t}{\rho C\Delta x\Delta y}\,) + T_{20}^{i}$$

T[20][i+1] = (((0.5*ho*x*(Ta-T[20][i]))+(0.5*ho*y*(Ta-T[20][i]))+(0.5*k*y*(T[21][i]-T[20][i])/x)+ (0.5*k*x*(T[15][i]-T[20][i])/y))*((4*t)/(r*c*x*y))) +T[20][i];

Node 21:

$$ho\,\Delta x(T_{\infty} - T_{21}^{i}) + k\frac{\Delta y}{2}\frac{T_{20}^{i}-T_{21}^{i}}{\Delta x} + k\frac{\Delta y}{2}\frac{T_{22}^{i}-T_{21}^{i}}{\Delta x} + k\Delta x\,\frac{T_{16}^{i}-T_{21}^{i}}{\Delta y} = \rho\Delta x\,\frac{\Delta y}{2}\,C\,\frac{T_{21}^{i+1}-T_{21}^{i}}{\Delta t}$$

$$T_{21}^{i+1} = ((ho\,\Delta x(T_{\infty} - T_{21}^{i}) + k\frac{\Delta y}{2}\frac{T_{20}^{i}-T_{21}^{i}}{\Delta x} + k\frac{\Delta y}{2}\frac{T_{22}^{i}-T_{21}^{i}}{\Delta x} + k\Delta x\,\frac{T_{16}^{i}-T_{21}^{i}}{\Delta y})\,\frac{2\Delta t}{\rho C\Delta x\Delta y}\,) + T_{21}^{i}$$

T[21][i+1] = (((0.5*ho*x*(Ta-T[21][i]))+(0.5*k*y*(T[20][i]-T[21][i])/x)+(0.5*k*y*(T[22][i]-T[21][i])/x)+ (k*x*(T[16][i]-T[21][i])/y))*((2*t)/(r*c*x*y))) +T[21][i];

Node 22:

$$ho\,\Delta x(T_{\infty} - T_{22}^{i}) + k\frac{\Delta y}{2}\frac{T_{21}^{i} - T_{22}^{i}}{\Delta x} + k\frac{\Delta y}{2}\frac{T_{23}^{i} - T_{22}^{i}}{\Delta x} + k\Delta x\,\frac{T_{17}^{i} - T_{22}^{i}}{\Delta y} = \rho\,\Delta x\,\frac{\Delta y}{2}\,C\,\frac{T_{22}^{i+1} - T_{22}^{i}}{\Delta t}$$

$$T_{22}^{i+1} = ((ho\,\Delta x(T_{\infty} - T_{22}^{i}) + k\frac{\Delta y}{2}\frac{T_{21}^{i} - T_{22}^{i}}{\Delta x} + k\frac{\Delta y}{2}\frac{T_{23}^{i} - T_{22}^{i}}{\Delta x} + k\Delta x\,\frac{T_{17}^{i} - T_{22}^{i}}{\Delta y})\,\frac{2\Delta t}{\rho C\Delta x\Delta y}) + T_{22}^{i}$$

T[22][i+1] = (((ho*x*(Ta-T[22][i]))+(0.5*k*y*(T[21][i]-T[22][i])/x)+(0.5*k*y*(T[23][i]-T[22][i])/x)+
(k*x*(T[17][i]-T[22][i])/y))*((2*t)/(r*c*x*y)))
+T[22][i];

Node 23:

$$ho\,\Delta x(T_{\infty} - T_{23}^{i}) + k\frac{\Delta y}{2}\frac{T_{22}^{i} - T_{23}^{i}}{\Delta x} + k\frac{\Delta y}{2}\frac{T_{24}^{i} - T_{23}^{i}}{\Delta x} + k\Delta x\,\frac{T_{18}^{i} - T_{23}^{i}}{\Delta y} = \rho\,\Delta x\,\frac{\Delta y}{2}\,C\,\frac{T_{23}^{i+1} - T_{23}^{i}}{\Delta t}$$

$$T_{23}^{i+1} = ((ho\,\Delta x(T_{\infty} - T_{23}^{i}) + k\frac{\Delta y}{2}\frac{T_{22}^{i} - T_{23}^{i}}{\Delta x} + k\frac{\Delta y}{2}\frac{T_{24}^{i} - T_{23}^{i}}{\Delta x} + k\Delta x\,\frac{T_{18}^{i} - T_{23}^{i}}{\Delta y})\,\frac{2\Delta t}{\rho C\Delta x\Delta y}) + T_{23}^{i}$$

T[23][i+1] = (((ho*x*(Ta-T[23][i]))+(0.5*k*y*(T[22][i]-T[23][i])/x)+(0.5*k*y*(T[24][i]-T[23][i])/x)+
(k*x*(T[18][i]-T[23][i])/y))*((2*t)/(r*c*x*y))) +T[23][i];

Node 24:

$$ho\,\frac{\Delta x}{2}(T_{\infty} - T_{24}^{i}) + hi\,\frac{\Delta y}{2}(T_{h} - T_{24}^{i}) + k\frac{\Delta y}{2}\frac{T_{23}^{i} - T_{24}^{i}}{\Delta x} + k\frac{\Delta x}{2}\frac{T_{19}^{i} - T_{24}^{i}}{\Delta y} = \rho\,\frac{\Delta x}{2}\frac{\Delta y}{2}\,C\,\frac{T_{24}^{i+1} - T_{24}^{i}}{\Delta t}$$

$$T_{24}^{i+1} = ((ho\,\frac{\Delta x}{2}(T_{\infty} - T_{24}^{i}) + hi\,\frac{\Delta y}{2}(T_{h} - T_{24}^{i}) + k\frac{\Delta y}{2}\frac{T_{23}^{i} - T_{24}^{i}}{\Delta x} + k\frac{\Delta x}{2}\frac{T_{19}^{i} - T_{24}^{i}}{\Delta y})\,\frac{4\Delta t}{\rho C\Delta x\Delta y}) + T_{24}^{i}$$

T[24][i+1] = (((0.5*ho*x*(Ta-T[24][i]))+(0.5*hi*y*(Th-T[24][i])) +(0.5*k*y*(T[23][i]-T[24][i])/x)+
(0.5*k*x*(T[19][i]-T[24][i])/y))*((4*t)/(r*c*x*y)))
+T[24][i];

III. PROGRAMMING & SOLUTION

With the help of a computer program we can solve the matrix created by finite difference equations for 24 nodes. We can calculate temperature distribution and stress distribution with respect to time.

Table 1 Material Property and Boundary Conditions

Material Properties and Boundary Conditions for Zirconia			Unit
1	Internal Film Co-efficient hi	200	W/m^2 K
2	External Film Co-efficient ho	40	W/m^2 K
3	Atmosphere Film Co-efficient ha	10	W/m^2 K
4	Density	5000	Kg/m^3
5	Time Interval Δt	10	Seconds
6	Thermal Conductivity k	1.2	W/m K
7	Temperature outside Furnace Wall	303	Kelvin
8	Temperature inside Furnace Wall	1873	Kelvin
9	Temperature of Air	303	Kelvin
10	Specific Heat	780	J/kg K
11	Elasticity Constant	240000	N/ m^2
12	Thermal Expansion Co-efficient	0.00000086	m/ K
13	Ultimate Stress	600	MPa

COMPUTER PROGRAM

```
#include<iostream.h>
#include<conio.h>
#include<math.h>
#include<complex.h>
#include<fstream.h>
void main()
{       clrscr;
ofstreammyfile;
myfile.open ("outputfdm.txt");
myfile<<"Writing this to a file.\n";
float T[25][500];
```

```
float S[25][500];
float hi,ho,ha,To,r,t,k,Ta,c,Th,E,Af,Sut,Se,m,N,L;
cout<<"Enter in W/m2 K value of hi";
cin>>hi;
cout<<"Enter in W/m2 K value of ho";
cin>>ho;
cout<<"Enter in W/m2 K value of ha";
cin>>ha;
cout<<"Enter in Kg/m3 value of density";
cin>>r;
cout<<"Enter in seconds time interval delta t";
cin>>t;
cout<<"Enter in W/m K value of thermal conductivity";
cin>>k;
cout<<"Enter in Kelvin Temperature outside Furnace Wall";
cin>>Ta;
cout<<"Enter in Kelvin Temperature inside Furnace Wall";
cin>>Th;
cout<<"Enter in Kelvin Temperature of Air";
cin>>To;
cout<<"Enter in J/Kg K value of Specific Heat";
cin>>c;
cout<<"Enter in N/m2 value of Elasticity Constant";
cin>>E;
cout<<"Enter in m/K Thermal Expansion Co-efficient";
cin>>Af;
cout<<"Enter in MPA value of Ultimate Stress";
cin>>Sut;
T[1][0]=300;
T[2][0]=300;
T[3][0]=300;
T[4][0]=300;
T[5][0]=300;
T[6][0]=300;
T[7][0]=300;
T[8][0]=300;
T[9][0]=300;
T[10][0]=300;
T[11][0]=300;
T[12][0]=300;
T[13][0]=300;
T[14][0]=300;
T[15][0]=300;
T[16][0]=300;
T[17][0]=300;
T[18][0]=300;
T[19][0]=300;
T[20][0]=300;
T[21][0]=300;
T[22][0]=300;
T[23][0]=300;
T[24][0]=300;
for (inti=0; i<=270;i++)
{
T[1][i+1] = (((0.5*ha*0.06*(To-T[1][i])) +(ho*0.3*(Ta-T[1][i])/2)+(0.5*k*0.3*(T[2][i]-T[1][i])/0.06)+(0.5*k*0.06*(T[4][i]-T[1][i])/0.3) )*((4*t)/(r*c*0.06*0.3)))+T[1][i];
T[2][i+1] = (((ha*0.06*(To-T[2][i]))+(0.5*k*0.3*(T[1][i]-T[2][i])/0.06)+(0.5*k*0.3*(T[3][i]-T[2][i])/0.06) +(0.5*k*0.06*(T[5][i]-T[2][i])/0.3))*((2*t)/(r*c*0.06*0.3)))+T[2][i];
T[3][i+1] = (((ha*0.06*(To-T[3][i])*0.5) +( hi*0.3*(Th-T[3][i])*0.5 ) +(0.5*k*0.3*(T[2][i]-T[3][i])/0.06)+(0.5*k*0.06*(T[6][i]-T[3][i])/0.3)) *((4*t)/(r*c*0.06*0.3)))+T[3][i];
T[4][i+1] = ((( ho*0.3*(Ta-T[4][i])) +(k*0.3*(T[5][i]-T[4][i])/0.06)+(0.5*k*0.06*(T[1][i]-T[4][i])/0.3) + (0.5*k*0.06*(T[7][i]-T[4][i])/0.3)) *((2*t)/(r*c*0.06*0.3)))+T[4][i];
T[5][i+1] = (((k*0.3*(T[4][i]-T[5][i])/0.06)+(k*0.3*(T[6][i]-T[5][i])/0.06) + (k*0.06*(T[8][i]-T[5][i])/0.3)) *((t)/(r*c*0.06*0.3)))+T[5][i];
T[6][i+1] = (((hi*0.3*(Th-T[6][i])) + (k*0.3*(T[2][i]-T[6][i])/0.06)+(0.5*k*0.06*(T[3][i]-T[6][i])/0.3) + (0.5*k*0.06*(T[9][i]-T[6][i])/0.3)) *((2*t)/(r*c*0.06*0.3)))+T[6][i];
T[7][i+1] = (((ho*0.3*(Ta-T[7][i])) + (k*0.3*(T[8][i]-T[7][i])/0.06)+(0.5*k*0.06*(T[4][i]-T[7][i])/0.3) + (0.5*k*0.06*(T[10][i]-T[7][i])/0.3)) *((2*t)/(r*c*0.06*0.3)))+T[7][i];
T[8][i+1] = (((k*0.3*(T[7][i]-T[8][i])/0.06)+( k*0.3*(T[9][i]-T[8][i])/0.06 ) +(k*0.06*(T[5][i]-T[8][i])/0.3) + (k*0.06*(T[11][i]-T[8][i])/0.3)) *((t)/(r*c*0.06*0.3)))+T[8][i];
```

```
T[9][i+1] = (( hi*0.3*(Th-T[9][i]))+(k*0.3*(T[8][i]-T[9][i])/0.06)+ (0.5*k*0.06*(T[6][i]-T[9][i])/0.3) + (0.5*k*0.06*(T[12][i]-T[9][i])/0.3))
*((2*t)/(r*c*0.06*0.3)))+T[9][i];
T[10][i+1] = (( ho*0.18*(Ta-T[10][i]))+(k*0.18*(T[11][i]-T[10][i])/0.06)+ (0.5*k*0.06*(T[7][i]-T[10][i])/0.18) + (0.5*k*0.06*(T[15][i]-
T[10][i])/0.18)) *((2*t)/(r*c*0.06*0.18)))+T[10][i];
T[11][i+1] = (( (k*0.18*(T[10][i]-T[11][i])/0.06)+ (k*0.18*(T[12][i]-T[11][i])/0.06)+ (k*0.06*(T[8][i]-T[11][i])/0.18) + (k*0.06*(T[16][i]-
T[11][i])/0.18)) *((t)/(r*c*0.06*0.18)))+T[11][i];
T[12][i+1] = (( (0.5*hi*0.12*(Th-T[12][i]))+( 0.5*hi*0.18*(Th-T[12][i])) +(k*0.18*(T[11][i]-T[12][i])/0.12)+ (0.5*k*0.18*(T[13][i]-
T[12][i])/0.12)+(0.5*k*0.12*(T[9][i]-T[12][i])/0.18) + (k*0.12*(T[17][i]-T[12][i])/0.18)) *((4*t)/(3*r*c*0.12*0.18))) +T[12][i];
T[13][i+1] = (( hi*0.18*(Th-T[13][i]))+(0.5*k*0.18*(T[12][i]-T[13][i])/0.18)+ (0.5*k*0.18*(T[14][i]-T[13][i])/0.18)+ (k*0.18*(T[18][i]-
T[13][i])/0.18)) *((2*t)/(r*c*0.18*0.18)))+T[13][i];
T[14][i+1] = (( (0.5*hi*0.18*(Th-T[14][i]))+(0.5*hi*0.18*(Th-T[14][i])) +(0.5*k*0.18*(T[13][i]-T[14][i])/0.18)+ (0.5*k*0.18*(T[19][i]-
T[14][i])/0.18)) *((4*t)/(r*c*0.18*0.18)))+T[14][i];
T[15][i+1] = (( ho*0.06*(Ta-T[15][i]))+(k*0.06*(T[16][i]-T[15][i])/0.06)+ (0.5*k*0.06*(T[10][i]-T[15][i])/0.06)+( 0.5*k*0.06*(T[20][i]-
T[15][i])/0.06)) *((2*t)/(r*c*0.06*0.06)))+T[15][i];
T[16][i+1] = (( (k*0.06*(T[15][i]-T[16][i])/0.06)+ (k*0.06*(T[17][i]-T[16][i])/0.06 )+(k*0.06*(T[11][i]-T[16][i])/0.06)+( k*0.06*(T[21][i]-
T[16][i])/0.06)) *((t)/(r*c*0.06*0.06)))+T[16][i];
T[17][i+1] = (( (k*0.06*(T[16][i]-T[17][i])/0.12)+ (k*0.06*(T[18][i]-T[17][i])/0.12 )+(k*0.12*(T[12][i]-T[17][i])/0.06)+( k*0.12*(T[22][i]-
T[17][i])/0.06)) *((t)/(r*c*0.12*0.06)))+T[17][i];
T[18][i+1] = (( (k*0.06*(T[17][i]-T[18][i])/0.18)+ (k*0.06*(T[19][i]-T[18][i])/0.18 )+(k*0.18*(T[13][i]-T[18][i])/0.06)+( k*0.18*(T[23][i]-
T[18][i])/0.06)) *((t)/(r*c*0.18*0.06)))+T[18][i];
T[19][i+1] = (( hi*0.06*(Th-T[19][i]))+(k*0.06*(T[18][i]-T[19][i])/0.18)+ (0.5*k*0.18*(T[14][i]-T[19][i])/0.06)+( 0.5*k*0.18*(T[24][i]-
T[19][i])/0.06)) *((2*t)/(r*c*0.18*0.06)))+T[19][i];
T[20][i+1] = (( (0.5*ho*0.06*(Ta-T[20][i]))+( 0.5*ho*0.06*(Ta-T[20][i]))+(0.5*k*0.06*(T[21][i]-T[20][i])/0.06)+ (0.5*k*0.06*(T[15][i]-
T[20][i])/0.06))*((4*t)/(r*c*0.06*0.06)))+T[20][i];
T[21][i+1] = (( (0.5*ho*0.06*(Ta-T[21][i]))+(0.5*k*0.06*(T[20][i]-T[21][i])/0.06)+( 0.5*k*0.06*(T[22][i]-T[21][i])/0.06)+ (k*0.06*(T[16][i]-
T[21][i])/0.06))*((2*t)/(r*c*0.06*0.06)))+T[21][i];
T[22][i+1] = (( (ho*0.12*(Ta-T[22][i]))+(0.5*k*0.06*(T[21][i]-T[22][i])/0.12)+( 0.5*k*0.06*(T[23][i]-T[22][i])/0.12)+ (k*0.12*(T[17][i]-
T[22][i])/0.06))*((2*t)/(r*c*0.12*0.06)))+T[22][i];
T[23][i+1] = (( (ho*0.18*(Ta-T[23][i]))+(0.5*k*0.06*(T[22][i]-T[23][i])/0.18)+( 0.5*k*0.06*(T[24][i]-T[23][i])/0.18)+ (k*0.18*(T[18][i]-
T[23][i])/0.06))*((2*t)/(r*c*0.18*0.06)))+T[23][i];
T[24][i+1] = (( (0.5*ho*0.18*(Ta-T[24][i]))+( 0.5*hi*0.06*(Th-T[24][i])) +(0.5*k*0.06*(T[23][i]-T[24][i])/0.18)+ (0.5*k*0.18*(T[19][i]-
T[24][i])/0.06))*((4*t)/(r*c*0.18*0.06)))+T[24][i];
S[14][i] = E*Af*T[14][i];
}
for (i=271; i<=360;i++)
{
hi=ha;
Th=Ta;
T[1][i+1] = (((0.5*ha*0.06*(To-T[1][i])) +(ho*0.3*(Ta-T[1][i])/2)+(0.5*k*0.3*(T[2][i]-T[1][i])/0.06)+(0.5*k*0.06*(T[4][i]-T[1][i])/0.3)
)*((4*t)/(r*c*0.06*0.3)))+T[1][i];
T[2][i+1] = (((ha*0.06*(To-T[2][i]))+(0.5*k*0.3*(T[1][i]-T[2][i])/0.06)+(0.5*k*0.3*(T[3][i]-T[2][i])/0.06) +(0.5*k*0.06*(T[5][i]-
T[2][i])/0.3))*((2*t)/(r*c*0.06*0.3)))+T[2][i];
T[3][i+1] = (((ha*0.06*(To-T[3][i])*0.5) +( hi*0.3*(Th-T[3][i])*0.5 ) +(0.5*k*0.3*(T[2][i]-T[3][i])/0.06)+(0.5*k*0.06*(T[6][i]-T[3][i])/0.3))
*((4*t)/(r*c*0.06*0.3)))+T[3][i];
T[4][i+1] = ((( ho*0.3*(Ta-T[4][i])) +(k*0.3*(T[5][i]-T[4][i])/0.06)+(0.5*k*0.06*(T[1][i]-T[4][i])/0.3) + (0.5*k*0.06*(T[7][i]-T[4][i])/0.3))
*((2*t)/(r*c*0.06*0.3)))+T[4][i];
T[5][i+1] = (((k*0.3*(T[4][i]-T[5][i])/0.06)+(k*0.3*(T[6][i]-T[5][i])/0.06) + (k*0.06*(T[8][i]-T[5][i])/0.3)) *((t)/(r*c*0.06*0.3)))+T[5][i];
T[6][i+1] = (((hi*0.3*(Th-T[6][i])) + (k*0.3*(T[2][i]-T[6][i])/0.06)+(0.5*k*0.06*(T[3][i]-T[6][i])/0.3) + (0.5*k*0.06*(T[9][i]-T[6][i])/0.3))
*((2*t)/(r*c*0.06*0.3)))+T[6][i];
T[7][i+1] = (((ho*0.3*(Ta-T[7][i])) + (k*0.3*(T[8][i]-T[7][i])/0.06)+(0.5*k*0.06*(T[4][i]-T[7][i])/0.3) + (0.5*k*0.06*(T[10][i]-T[7][i])/0.3))
*((2*t)/(r*c*0.06*0.3)))+T[7][i];
T[8][i+1] = (((k*0.3*(T[7][i]-T[8][i])/0.06)+( k*0.3*(T[9][i]-T[8][i])/0.06 ) +(k*0.06*(T[5][i]-T[8][i])/0.3) + (k*0.06*(T[11][i]-T[8][i])/0.3))
*((t)/(r*c*0.06*0.3)))+T[8][i];
T[9][i+1] = (( hi*0.3*(Th-T[9][i]))+(k*0.3*(T[8][i]-T[9][i])/0.06)+ (0.5*k*0.06*(T[6][i]-T[9][i])/0.3) + (0.5*k*0.06*(T[12][i]-T[9][i])/0.3))
*((2*t)/(r*c*0.06*0.3)))+T[9][i];
T[10][i+1] = (( ho*0.18*(Ta-T[10][i]))+(k*0.18*(T[11][i]-T[10][i])/0.06)+ (0.5*k*0.06*(T[7][i]-T[10][i])/0.18) + (0.5*k*0.06*(T[15][i]-
T[10][i])/0.18)) *((2*t)/(r*c*0.06*0.18)))+T[10][i];
T[11][i+1] = (( (k*0.18*(T[10][i]-T[11][i])/0.06)+ (k*0.18*(T[12][i]-T[11][i])/0.06)+ (k*0.06*(T[8][i]-T[11][i])/0.18) + (k*0.06*(T[16][i]-
T[11][i])/0.18)) *((t)/(r*c*0.06*0.18)))+T[11][i];
T[12][i+1] = (( (0.5*hi*0.12*(Th-T[12][i]))+( 0.5*hi*0.18*(Th-T[12][i])) +(k*0.18*(T[11][i]-T[12][i])/0.12)+ (0.5*k*0.18*(T[13][i]-
T[12][i])/0.12)+(0.5*k*0.12*(T[9][i]-T[12][i])/0.18) + (k*0.12*(T[17][i]-T[12][i])/0.18)) *((4*t)/(3*r*c*0.12*0.18))) +T[12][i];
T[13][i+1] = (( hi*0.18*(Th-T[13][i]))+(0.5*k*0.18*(T[12][i]-T[13][i])/0.18)+ (0.5*k*0.18*(T[14][i]-T[13][i])/0.18)+ (k*0.18*(T[18][i]-
T[13][i])/0.18)) *((2*t)/(r*c*0.18*0.18)))+T[13][i];
T[14][i+1] = (( (0.5*hi*0.18*(Th-T[14][i]))+(0.5*hi*0.18*(Th-T[14][i])) +(0.5*k*0.18*(T[13][i]-T[14][i])/0.18)+ (0.5*k*0.18*(T[19][i]-
T[14][i])/0.18)) *((4*t)/(r*c*0.18*0.18)))+T[14][i];
T[15][i+1] = (( ho*0.06*(Ta-T[15][i]))+(k*0.06*(T[16][i]-T[15][i])/0.06)+ (0.5*k*0.06*(T[10][i]-T[15][i])/0.06)+( 0.5*k*0.06*(T[20][i]-
T[15][i])/0.06)) *((2*t)/(r*c*0.06*0.06)))+T[15][i];
T[16][i+1] = (( (k*0.06*(T[15][i]-T[16][i])/0.06)+ (k*0.06*(T[17][i]-T[16][i])/0.06 )+(k*0.06*(T[11][i]-T[16][i])/0.06)+( k*0.06*(T[21][i]-
T[16][i])/0.06)) *((t)/(r*c*0.06*0.06)))+T[16][i];
```

```
T[17][i+1] = (( (k*0.06*(T[16][i]-T[17][i])/0.12)+ (k*0.06*(T[18][i]-T[17][i])/0.12 )+(k*0.12*(T[12][i]-T[17][i])/0.06)+( k*0.12*(T[22][i]-T[17][i])/0.06)) *((t)/(r*c*0.12*0.06)))+T[17][i];
T[18][i+1] = (( (k*0.06*(T[17][i]-T[18][i])/0.18)+ (k*0.06*(T[19][i]-T[18][i])/0.18 )+(k*0.18*(T[13][i]-T[18][i])/0.06)+( k*0.18*(T[23][i]-T[18][i])/0.06)) *((t)/(r*c*0.18*0.06)))+T[18][i];
T[19][i+1] = (( (hi*0.06*(Th-T[19][i]))+(k*0.06*(T[18][i]-T[19][i])/0.18)+ (0.5*k*0.18*(T[14][i]-T[19][i])/0.06)+( 0.5*k*0.18*(T[24][i]-T[19][i])/0.06)) *((2*t)/(r*c*0.18*0.06)))+T[19][i];
T[20][i+1] = (( (0.5*ho*0.06*(Ta-T[20][i]))+( 0.5*ho*0.06*(Ta-T[20][i]))+(0.5*k*0.06*(T[21][i]-T[20][i])/0.06)+ (0.5*k*0.06*(T[15][i]-T[20][i])/0.06))*((4*t)/(r*c*0.06*0.06)))+T[20][i];
T[21][i+1] = (( (0.5*ho*0.06*(Ta-T[21][i]))+(0.5*k*0.06*(T[20][i]-T[21][i])/0.06)+( 0.5*k*0.06*(T[22][i]-T[21][i])/0.06)+ (k*0.06*(T[16][i]-T[21][i])/0.06))*((2*t)/(r*c*0.06*0.06)))+T[21][i];
T[22][i+1] = (( (ho*0.12*(Ta-T[22][i]))+(0.5*k*0.06*(T[21][i]-T[22][i])/0.12)+( 0.5*k*0.06*(T[23][i]-T[22][i])/0.12)+ (k*0.12*(T[17][i]-T[22][i])/0.06))*((2*t)/(r*c*0.12*0.06)))+T[22][i];
T[23][i+1] = (( (ho*0.18*(Ta-T[23][i]))+(0.5*k*0.06*(T[22][i]-T[23][i])/0.18)+( 0.5*k*0.06*(T[24][i]-T[23][i])/0.18)+ (k*0.18*(T[18][i]-T[23][i])/0.06))*((2*t)/(r*c*0.18*0.06)))+T[23][i];
T[24][i+1] = (( (0.5*ho*0.18*(Ta-T[24][i]))+( 0.5*hi*0.06*(Th-T[24][i])) +(0.5*k*0.06*(T[23][i]-T[24][i])/0.18)+ (0.5*k*0.18*(T[19][i]-T[24][i])/0.06))*((4*t)/(r*c*0.18*0.06)))+T[24][i];
S[14][i] = E*Af*T[14][i];
 }
for(i=0; i<=360;i=i+6)
{
myfile<<"MAXIMUM TEMPERATURE AFTER  "<<i/6<<" MINUTES"<<T[14][i]<<" KELVIN"<<endl;
myfile<<"MAXIMUM STRESS AFTER  "<<i/6<<" MINUTES "<<S[14][i]<<" MPa"<<endl;            }
(i=0; i<=300; i++)
{
if (i==270)
{
myfile<<"S"<<S[14][i];
Se=0.15*Sut;
m = (Sut-Se)/7;
N = 7 -((S[14][i]-Se)/m);
L = pow(10,N);
myfile<<"N"<<N;
myfile<<"LIFE CYCLE"<<L;
myfile.close();
}
}
getch();   }
```

IV. RESULTS AND DISCUSSION

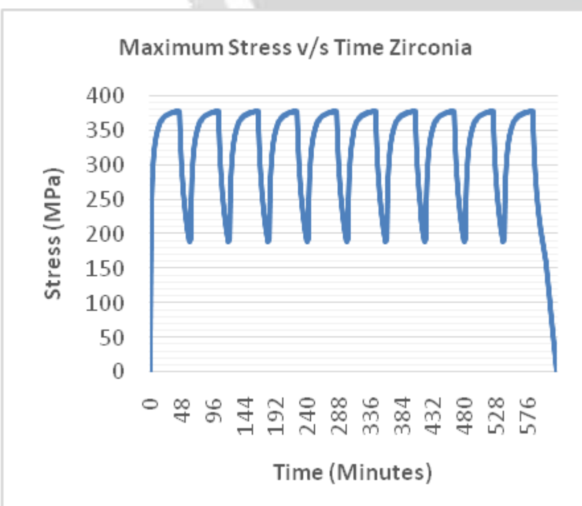

Fig.2 Maximum thermal stress v/s time graph for zirconia

We can see from the Fig. 3 that maximum thermal stress is increasing from initial condition 0 MPa and reaches to maximum stress 377 MPa in 45 minutes and then starts reducing and reaches to 188 MPa in next 15 minutes. It again starts increasing and reaches to maximum stress 377 MPa after 105 minutes and again it starts reducing. There are 10 similar thermal stress cycles in one day.

Stress-Life Method

To determine life of any component by Stress-Life Method, we need to find out ultimate strength and endurance limit of the component for the required material.

We know the values of ultimate stress for these all materials. Zirconia is having ultimate strength of 600 MPa. We can find out Se' from the equation given below or we can say dividing value of ultimate strength.

We know value of ultimate stress of zirconia is 600 Mpa.

We know the relation between Sut and Se' so that we can find out Se'.

Se' = 0.5 * Sut= 300 MPa

Endurance Limit Modifying Factors

We have seen that the rotating-beam specimen used in the laboratory to determine endurance limits is prepared very carefully and tested under closely controlled conditions.

It is unrealistic to expect the endurance limit of a mechanical or structural member to match the values obtained in the laboratory.

Some differences include

Material: composition, basis of failure, variability
Manufacturing: method, heat treatment, fretting corrosion, surface condition, stress concentration
Environment: corrosion, temperature, stress state, relaxation times
Design: size, shape, life, stress state, stress concentration, speed, fretting, galling

Marin identified factors that quantified the effects of surface condition, size, loading, temperature, and miscellaneous items. The question of whether to adjust the endurance limit by subtractive corrections or multiplicative corrections was resolved by an extensive statistical analysis of a 4340 (electric furnace, aircraft quality) steel, in which a correlation coefficient of 0.85 was found for the multiplicative form and 0.40 for the additive form.

A Marin equation is therefore written as

Se = ka*kb*kc*kd*ke*kf* Se'
Where,
ka= surface condition modification factor = 0.84
kb= size modification factor = 0.98
kc= load modification factor = 0.96
kd= temperature modification factor = 0.45
ke= reliability factor = 0.9
kf= miscellaneous-effects modification factor = 0.95
*S*e'= specimen endurance limit
Se = endurance limit at the critical location of a machine part in the geometry and condition of use.
We can find out different factor like surface finish factor, size factor, loading factor, temperature factor, reliability factor, miscellaneous effects factor as per the guideline. (Joseph E. Shigley*et al*, Machine Engineering Design)
Now, we can find out endurance limit for all different materials.
Se = ka*kb*kc*kd*ke*kf*Se' = 100 MPa

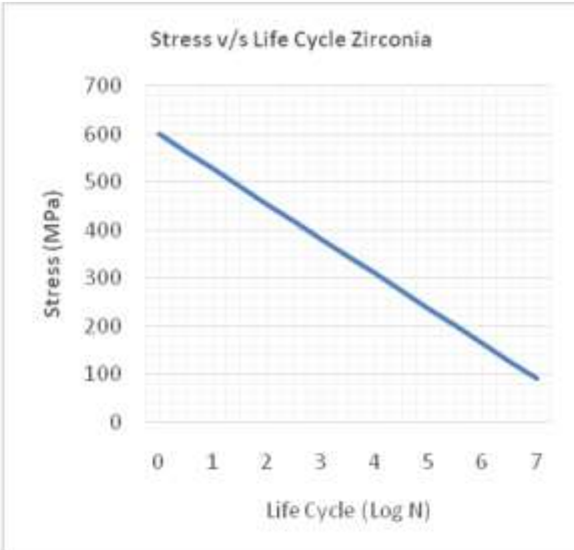

Fig. 2 Stress v/s Life Cycle for Zirconia

CONCLUSION

Induction Melting Furnaces are highly used now- a-days for melting of different kinds of materials. The problem comes from the zirconia of losing its material properties and failure occurs within 1100-1200 hours of lifetime. It will disturb production schedule as it requires time to replace the induction melting furnace wall of zirconia. Explicit finite difference analysis is done for induction melting furnace refractory wall and validation is done with respect to experimental Results. Explicit finite difference analysis is done with respect to actual working conditions of induction furnace and material properties of zirconia. Then S – log N Curves are plotted for Life Span Prediction.

We had found life span of different materials like zirconia is 1100-1200 cycles. From the results of experimental study and explicit finite difference analysis of thermal fatigue failure of induction melting furnace wall, it can be seen that finite difference model exactly predicts the failure of the induction furnace refractory wall and the definite solution conditions in the finite difference numerical calculation are accurate. The fatigue life of the induction melting furnace refractory wall under thermal fatigue working conditions was predicted using stress-life method by plotting S – log N curves for zirconia on the basis of explicit finite difference calculations and maximum thermal stress in the induction melting furnace refractory wall. We can use it as a linear to increase lifespan or we can use premixed zirconia for economical and better working lifespan of induction melting furnace wall.

The accuracy of the fatigue life prediction for the induction melting furnace refractory wall depends upon temperature and thermal stress spectrum calculated at the critical point by explicit finite difference method and S-log N curves prepared from the material properties and boundary condition for zirconia.

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