NEURAL NETWORKS USING CONVERGENCE OF DISCRETE-TIME NEURAL NETWORKS WITH DELAYS AND DYNAMIC RECURRENT NEURAL NETWORKS

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ABSTRACT

Neural networks are being used to solve all kinds of problems from a wide range of disciplines. The topic is highly interdisciplinary in nature, and so it is extremely difficult to develop an introductory and comprehensive treatise on the subject in a short manuscript. An LMI(linear matrix inequality) approach and an embedding technique are employed to derive some sufficient conditions for the global exponential stability of discrete-time neural networks with time-dependent delays and constant parameters. An discrete recurrent neural network described by a set of difference equations may be used to approximate uniformly a state-space trajectory producd by either na discrete time nonlinear system or a continuous function on a closed discrete-time interval.

Key words: neural network, LMI(linear matrix inequality), discrete-time neural networks, time-dependent, discrete recurrent neural network

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INTRODUCTION

When a neural network updated discretely, the model describing the network is in the form of system of differential equations. Also, in numerical simulations and practical implementation of a continuous-time neural network, discretization is needed, which leads again to a system of difference equations. Therefore it is of both theoretical and practical importance to study the dynamics of discrete-time neural networks. In this paper, we consider the discrete-time neural network model with constant parameters and variable delays,

$$x_i(n+1) = a_i x_i(n) + \sum_{j=1}^{m} w_{ij} g_j\left(x_j(n-k(n))\right) + I_i, \quad n = 0,1,2\ldots \tag{1}$$

And a model with time-dependent parameters and constant delay,

$$x_i(n+1) = a_i(n) x_i(n) + \sum_{j=1}^{m} w_{ij}(n) g_j\left(x_j(n-k)\right) + I_i(n), \quad n = 0,1,2\ldots \tag{2}$$

In, $x_i(n+1) = a_i x_i(n) + \sum_{j=1}^{m} w_{ij} g_j\left(x_j(n-k(n))\right) + I_i, \quad n = 0,1,2\ldots k(n)$ are positive integers with $0 \le k(n) \le k$ (nor necessarily increasing $a \in (0,1), \ \ i \in \{1,2,\ldots,m\} := N(1,m)$.

In, $x_i(n+1) = a_i(n) x_i(n) + \sum_{j=1}^{m} w_{ij}(n) g_j\left(x_j(n-k)\right) + I_i(n), \quad n = 0,1,2\ldots$

$$a_i(n) \to a_i, \qquad w_{ij}(n) \to w_{ij}, \qquad I_i(n) \to \ I_i \ \ as\, n \to \infty, \ \ i,j \in N(1,m).$$

We will apply an LMI approach and an embedding technique to derive some delay-dependent and delay-independent conditionds under the system $x_i(n+1) = a_i x_i(n) + \sum_{j=1}^{m} w_{ij} g_j\left(x_j(n-k(n))\right) + I_i, n = 0,1,2\ldots$ Admits a unique equilibrium and which is globally exponentially stable.

We point out the LMI approach has been used by Liao, Chen and Snachez to establish some stability criteria for delayed continuous-time neural networks and for discrete-time periodic systems. In this paper, we attempt to establish some LMI based stability criteria for the discrete time neural network model $x_i(n+1) = a_i x_i(n) + \sum_{j=1}^{m} w_{ij} g_j\left(x_j(n-k(n))\right) + I_i, \quad n = 0,1,2\ldots.$ Which can be easily tested by efficient and reliable algorithms.Where an embedding techniques was used for a simple discrete-time neural network model having specific performance, and for the attractivity of some Hopfield type continuous-time neural networks with delays, we apply the embedding technique to system ,

$$x_i(n+1) = a_i x_i(n) + \sum_{j=1}^{m} w_{ij} g_j\left(x_j(n-k(n))\right) + I_i, \quad n = 0,1,2\ldots$$

To derive some sufficient conditions for its exponential stability. That system

$$x_i(n+1) = a_i(n) x_i(n) + \sum_{j=1}^{m} w_{ij}(n) g_j\left(x_j(n-k)\right) + I_i(n), \quad n = 0,1,2\ldots$$

Has the anatomous system

$$x_i(n+1) = a_i x_i(n) + \sum_{j=1}^{m} w_{ij} g_j\left(x_j(n-k(n))\right) + I_i, \quad n = 0,1,2\ldots$$

As its limiting system. In this paper, we will obtain a convergence result for the asymptotically autonomous system $x_i(n+1) = a_i(n) x_i(n) + \sum_{j=1}^{m} w_{ij}(n) g_j\left(x_j(n-k)\right) + I_i(n), \quad n = 0,1,2\ldots$

By relating (2) to (1).

i.e.,$x_i(n+1) = a_i(n) x_i(n) + \sum_{j=1}^{m} w_{ij}(n) g_j\left(x_j(n-k)\right) + I_i(n), n = 0,1,2\ldots$

to $x_i(n+1) = a_i x_i(n) + \sum_{j=1}^{m} w_{ij} g_j\left(x_j(n-k(n))\right) + I_i, \quad n = 0,1,2\ldots$

With relatively chain transitive sets. In section 2 we establish some criteria for exponential stability of,

$$x_i(n+1) = a_i x_i(n) + \sum_{j=1}^{m} w_{ij} g_j\left(x_j(n-k(n))\right) + I_i, \quad n = 0,1,2\ldots$$

By combining the LMI approach, Liapunov functional method, embedding technique and the comparison method for discrete monotone system. In section 3 is devoted to the convergence of the asymptotic discrete-time

neural networks, $x_i(n+1) = a_i(n) x_i(n) + \sum_{j=1}^{m} w_{ij}(n) g_j\left(x_j(n-k)\right) + I_i(n), \quad n = 0,1,2\ldots$

In which the chain transitive set and the strong attractivity theorem play a crucial role.

EXPONENTIAL STABILITY OF

$$x_i(n+1) = a_i x_i(n) + \sum_{j=1}^{m} w_{ij} g_j\left(x_j(n-k(n))\right) + I_i, \quad n = 0,1,2\ldots$$

We use the following notations:$\lambda_M(W)$

The largest eigenvalue of the symmetric matrix W; $\lambda_M(W)$

The smallest eigenvalue of the symmetric matrix W; W^T

The transpose of the matrix W; W^{-1}

THe inverse of the matrix W; $\|x\| = (\sum_{i=1}^{m} x_i^2)$

The Euclidean norm of the vector $x = (x_1, x_2, \ldots\ldots, x_m)^T \in \mathbb{R}^m$ and $\|W\| = (max\lambda(W^T W))^{\frac{1}{2}}$

The matrix norm introduced by the Euclidean vector norm. The initial condition associated with

$$x_i(n+1) = a_i x_i(n) + \sum_{j=1}^{m} w_{ij} g_j\left(x_j(n-k(n))\right) + I_i, \quad n = 0,1,2\ldots$$

Are of the form ,

$$x_i(s) = \phi_i(s), \qquad i = N(1,m), \qquad s\epsilon N(-k,0). \tag{3}$$

Throughout this we assume that, For each $i = N(1,m)$, $g_i : \mathbb{R} \to \mathbb{R}$

Is globally Lipschitz continuous with, $sup_{u,v\in\mathbb{R},u\neq v} \frac{|g_i(u)-g_i(v)|}{|u-v|} = l_i$, and

$|g_i(u)| \leq M_i, \quad u \in \mathbb{R}, \quad M_i > 0.$

If we let, $x = (x_1, x_2, \ldots\ldots, x_m)^T$, $A = diag(a_1, a_2, \ldots a_m)$, $W = (w_{ij})_{n\times n}$, $I = (I_1, I_2, \ldots I_m)^T$,

$g(x(n)) = (g_1(x_1(n)), g_2(x_2(n)), \ldots\ldots\ldots g_m(x_m(n)))^T$,

Tnen, $x_i(n+1) = a_i x_i(n) + \sum_{j=1}^{m} w_{ij} g_j\left(x_j(n-k(n))\right) + I_i, \quad n = 0,1,2\ldots$

Can be written in form of matrices:

$$x(n+1) = Ax(n) + Wg[x(n-k(n))] + I, \quad n \in N(0) \tag{4}$$

As usual, a vector $x^* = (x_1^*, x_2^*, \ldots\ldots\ldots x_m^*)^T$. Is said to be the equilibrium of

$$x(n+1) = Ax(n) + Wg[x(n-k(n))] + I, \quad n \in N(0)$$

If it satisfies, $x^* = Ax^* + Wg(g^*) + I$. Based on our assumption on the activation functions, it is easily seems that $x(n+1) = Ax(n) + Wg[x(n-k(n))] + I, \quad n \in N(0)$. Admits atleast one equilibrium.

LMI based criteria:

It follows $S > (\geq) 0$ means the matrix S is symmetric and positive definite (semi-positive definite).from the theory of matrices, we have the following theorem.

Theorem:

(1)$if\ A > 0, B > 0, \alpha > 0, then\ A + B > 0, \alpha A > 0;$

(2)$A = \begin{pmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{pmatrix} > 0\ if\ and\ only\ if\ A_{11} > 0\ and\ A_{22} - A_{21}A_{11}^{-1}A_{12} > 0;$

(3)$for\ any\ real\ matrix\ A, B, C\ and\ a\ scalar\ \in > 0\ with\ C > 0, the\ identity$

$A^T B + B^T A \leq \in A^T CA + \in^{-1} B^T C^{-1} B$ holds.

Proof:

Suppose x^* is an equilibrium system

of $x(n+1) = Ax(n) + Wg[x(n - k(n))] + I, \quad n \in N(0)$.

And

let $y(n) = x(n) - x^*$ and $f(y(n)) = g(x(n)) - g(x^*)$. Then the stability of equilibrium x^* of $x(n+1) = Ax(n) + Wg[x(n - k(n))] + I, \quad n \in N(0)$. Corresponds to that of the zero solution of the following system.

$$y(n+1) = Ay(n) + Wf[y(n - k(n))] \quad (5)$$

It follows that f has the property,

$$\|f(y)\| \leq \|L\|\|y\| \quad (6)$$

With $L = diag(l_1, l_2, \ldots\ldots l_m)$.

Theorem:

Assume that the time-dependent delay $k(n)$ is the bounded satisfying $0 \leq k(n) \leq k and\ \Delta k(n) = k(n+1) - k(n) < 1$. If there exist two squares $q > 1, \in > 0$ and two matrices $P > 0, R > 0$ such that,

$$\begin{pmatrix} R & W^T PA \\ \frac{q}{\in} APW & P - qAPA - LQL \end{pmatrix} > 0 \quad (7)$$

Then the equilibrium x^* of $x(n+1) = Ax(n) + Wg[x(n - k(n))] + I,$

Is exponentially stable. More precisely, for any solution $x(n)$ of

$x(n+1) = Ax(n) + Wg[x(n-k(n))] + I,$ The following inequality holds,

$$\|x(n) - x^*\|x^2 \le q^{-n}C_1 \quad sup_{s\in N(-k,0)}\|x(n) - x^*\|x^2 \qquad (8)$$

Where $C_1 = \frac{\lambda_M(P)+\delta\lambda_M(Q)\|L\|^2}{\lambda_M(P)}$, With $\delta = \frac{1-(\frac{1}{q})^k}{q-1}$

And $Q = q^{1+k}\epsilon R + q^{1+k}W^T PW > 0.$

Proof:

Define $V(n) = V(y(n))$ by

$$V(n) = q^n y^T(n)Py(n) + \sum_{s=n-k(n)}^{n-1} q^s f^T(y(s))Qf(y(s)) \qquad (9)$$

Then we have,

$\Delta V(n) =$
$V(n+1) - V(n) = q^{n+1}y^T(n+1)Py(n+1) - q^n y^T(n)Py(n) + \sum_{s=n+1-k(n+1)}^{n} q^s f^T(y(s))Qf(y(s)) - \sum_{s=n-k(n)}^{n-1} q^s f^T(y(s))Qf(y(s)) \le$
$q^{n+1}(Ay(n) + Wf(y(n-k(n)))^T P(Ay(n) + Wf(y(n-k(n)) - q^n y^T(y(n))Qf(y(n)) - q^{n-k(n)}f^T(y(n-k(n))Qf(y(n-k(n)),$

$\Delta V(n) \le$
$q^{n+1}y^T(n)APAy(n) + q^n y^T(N)Py(n) + q^n f^T(y(n))Qf(y(n)) + q^{n+1}[y^T(n)APWf(y(n-k(n)) + f^T(y(n-k(n))W^T PAy(n)] + f^T(y(n-k(n))(q^{n+1}W^T PW - q^{n-k(n)}Q)f(y(n-k(n)),$

Letting $B = W^T PAy, C = R,$

$y^T(n)APWf(y(n-k(n)) + f^T(y(n-k(n))W^T PAy(n) \le \epsilon f^T(y(n-k(n))Rf(y(n-k(n)) + \frac{1}{\epsilon}y^T(n)APWR^{-1}W^T PAy(n),$

Which shows,

$\Delta V(n) \le -q^n y^T(n)\left(P - qAPA - LQL - \frac{q}{\epsilon}APWR^{-1}W^T PA\right)y(n) - q^{n-k(n)}f^T(y(n-k(n))(Q - q^{1+K(N)}(\epsilon R + W^T PW))f^T(y(n-k(n)).$

Recalling that , $Q = q^{1+K(N)}(\epsilon R + W^T PW))f^T(y(n-k(n))$. We know from before theorem.

i.e., (1)$if\ A > 0, B > 0, \alpha > 0, then\ A + B > 0, \alpha A > 0;$

(2)$A = \begin{pmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{pmatrix} > 0$ *if and only if* $A_{11} > 0$ *and* $A_{22} - A_{21}A_{11}^{-1}A_{12} > 0$;

(3)*for any real matrix* A, B, C *and a scalar* $\in > 0$ *with* $C > 0$, *the identity*
$A^T B + B^T A \leq \in A^T CA + \in^{-1} B^T C^{-1} B$ holds.

Proof:

Suppose x^* is an equilibrium system of $x(n+1) = Ax(n) + Wg[x(n-k(n))] + I, \quad n \in N(0)$. And let $y(n) = x(n) - x^*$ and $f(y(n)) = g(x(n)) - g(x^*)$. Then the stability of equilibrium x^* of $x(n+1) = Ax(n) + Wg[x(n-k(n))] + I, \quad n \in N$. Corresponds to that of the zero solution of the following system. $y(n+1) = Ay(n) + Wf[y(n-k(n))]$. It follows that f has the property, $\|f(y)\| \leq \|L\|\|y\|$ With $= diag(l_1, l_2, \ldots\ldots l_m)$. That $Q > 0$ and $(Q - q^{1+K(N)}(\in R + W^T PW))f^T(y(n-k(n))$ this shows that,

$$\Delta V(n) \leq -q^n y^T(n)\Omega y(n),$$

Where, $\Omega = P - qAPA - LQL - \frac{q}{\in} APW R^{-1} W^T PA$.condition

$$\begin{pmatrix} R & W^T PA \\ \frac{q}{\in} APW & P - qAPA - LQL \end{pmatrix} > 0$$

And $\Omega > 0$, hence $\Delta V(n) \leq 0$. Therefore, we have

$$V(n) \leq V(0) = y^T(0)Py(0) + \sum_{s=-k(0)}^{-1} q^s f^T(y(s))Qf(y(s)) \leq \lambda_M(P)\|y(0)\|^2 + \sum_{s=-k}^{-1} \lambda_M(Q)\|L\|^2\|y(s)\|^2 q^s \leq \lambda_M(P) + \delta\lambda_M(Q)\|L\|^2 \quad sup_{s \in N(-K,0)}\|y(s)\|^2$$

On the other hand, from the definition of $V(n)$ it follows that,

$$V(y(n)) \geq q^n \lambda_m(P)\|y(n)\|^2$$

We then obtain $\|y(n)\|^2 \leq q^{-n} \frac{\lambda_M(P) + \delta\lambda_M(Q)\|L\|^2}{\lambda_M(P)} sup_{s \in N(-K,0)}\|y(s)\|^2$

Which gives $\|x(n) - x^*\|x^2 \leq q^{-n} C_1 \ \ sup_{s \in N(-k,0)}\|x(n) - x^*\|x^2$

Hence the proof.

Conclusion:

It has been have proved in this note that a discrete-time DRNN may be used to uniformly approximate a discrete-time state-space trajectory which is produced by either a dynamic system or a continuous-time function to any degree of precision. The analytical results show that some of hidden units of such a DRNN may be selected as output units of the network and the states of these output units may be used to uniformly approximate a desired state-space trajectory. The proof is used in this note is constructive and may be extended to study the approximation issue for other types of DRNN's. also it has been indicated that this approximation process has to be carried out by an adaptive learning process.

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