NUMERICAL RESOLUTION OF MODIFIED VERSION OF FORCED REAL FRACTIONAL ORDER VAN DER POL OSCILLATOR EQUATION

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ABSTRACT

In this paper we will present one method of numerical resolution of modified version of forced real fractional order Van Der Pol Oscillator equation. . Numerical method for calculation of fractional derivative are the main tools for this resolution.

***Keys Words** : Fractional derivative, Van Der Pol*

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1 Introduction

The concept of the fractional derivative is a subject almost old as the classical calculus that we know today . However this theory can be considered as a new subject also, since a little more than thirty years . In recent years, there has been considerable development in the resolution of fractional differential equations (see examples in [1], [2], [3]) . Drawing on [9],[12], we decided to give a numerical resolution of modified version of forced real fractional order the Van Der Pol Oscillator.

The Van der Pol oscillator (VPO) represents a nonlinear system with an interesting behavior that exhibits naturally in several applications. It has been used for study and design of many models including biological phenomena, such as the heartbeat,neurons, acoustic models, radiation of mobile phones, and as a model of electrical oscillators (implemented with a tunnel diode, memristor or operating amplifier).The VPO model was used by Van der Pol in 1920 to study oscillations in vacuum tube circuits. In the standard form, it is given by a nonlinear differential equation of type see [9] [12],:

$$y''(t)+\varepsilon(y^2(t)-1)y'(t)+y(t)=A\cos(\omega t) \quad (1)$$

where ε is the control parameter, A the amplitude, ω the frequency.
Equation (1) can be rewritten into its state-space representation as follows:

$$\begin{cases} \dfrac{dy_1}{dt}=y_2(t) \\ \dfrac{dy_2}{dt}=-y_1(t)-\varepsilon(y_1^2(t)-1)y_1(t)+A\cos(\omega t) \end{cases} \quad (2)$$

with an equilibrium point in origin

The system to solve is the modified version of the fractional order the Van Der Pol Oscillator equation in the following form: : .

$$\begin{cases} {}_0D_t^{q_1} y_1(t) = y_2(t) \\ {}_0D_t^{q_2} y_2(t) = -y_1(t) - \varepsilon(y_1^2(t)-1)y_1(t) + A\cos(\omega t) \end{cases} \quad (3)$$

where

- q_1 and q_2 are orders ($0 < q_1, q_2 < 2$)
- $\varepsilon > 0, \omega > 0, A > 0$
- $y_1(t); y_2(t)$ are the unknown functions
- ${}_0D_t^q$ is fractional derivative

Its resolution requires the fractional derivative theory see[10]

2 Numerical methods for calculation of fractional derivatives
For numerical calculation of fractional-order derivatives we can use the relation
(1) derived from the Grunwald-Letnikov definition. This approach is based on the fact that for a wide class of functions, three definitions , Grunwald-Letnikov , Riemann Liouville and Caputo are equivalent. The relation to the explicit numerical approximation of q-th derivative at the points kh , (k=1,2,….) has the following form :

$$_{(k-\frac{L_m}{h})}D_{t_k}^q f(t) \approx h^{-q} \sum_{j=0}^{k} (-1)^j \binom{q}{j} f(t_{k-j}) \quad (4)$$

where L_m is the "memory length", $t_k = kh$, h is the time step of calculation and $(-1)^j \binom{q}{j}$ are binomial coefficients $c_j^{(q)}$ $(j = 0,1,.......)$

For their calculation we can use the following expression [10]

$$c_0^{(q)} = 1 \qquad c_j^{(q)} = (1 - \frac{1+q}{j}) c_{j-1}^{(q)} \quad (5)$$

Then, general numerical solution of the fractional differential equation

$$_aD_t^q y(t) = f(y(t), t) \quad (6)$$

can be expressed as

$$y(t_k) = f(y(t_k), t_k)h^q - \sum_{j=v}^{k} c_j^{(q)} y(t_{k-j}) \quad [9] \quad (7)$$

For the *memory term* expressed by the sum, a "short memory" principle can be used.
Then the lower index of the sums in relations (4) will be $v = 1$ for $k < (Lm/h)$
and $v = k-(Lm/h)$ for $k > (Lm/h)$, or without using the "short memory" principle,
we put $v = 1$ for all k.
Obviously, for this simplification we pay a penalty in the form of some inaccuracy.
see [11].

3 Discretization
The discretiozation is obtained by (7), which leads to the equations in form :

$$\begin{cases} y_1(t_k) = y_2(t_{k-1})h^{q_1} - \sum_{j=\nu}^{k} c_j^{(q_1)} x(t_{k-j}) \\ y_2(t_k) = (-y_1(t_k) - \varepsilon(y^2(t_k)-1)y_2(t_{k-1}))h^{q_2} - \sum_{j=\nu}^{k} c_j^{(q_2)} y_2(t_{k-j}) + A\cos(\omega t_k) \end{cases} \quad (8)$$

Where $t_k = hk$, k=1,2,……,N. for $N = \frac{T}{h}$, T is the simulation time and $(y_1(0), y_2(0))$ is a start point (initial conditions). The binomial coefficients $c_j^{(q_i)}, i = 1, 2, 3$ are calculated according to relation (5)

4 Simulation

4.1 Programming

The programming under MATLAB of results of the dicretizations above is :

```
T=input('simulation time T=')
h=input('time step h=')
N=T/h
q1=input('real order of frac der eq1 q1=')
q2=input('real order of frac der eq2  q2=')
eps=input('control parameter eps=')
om=input('frequency omega=')
A=input('amplitude A=')
cp1=1;
cp2=1;
 X0=input('start point of x=')
 Y0=input ('start point of y=')
 x(1)=X0; y(1)=Y0;
for j=1:N
     c1(j)=(1-(1+q1)/j)*cp1;
     c2(j)=(1-(1+q2)/j)*cp2;
     cp1=c1(j); cp2=c2(j);
end
for i=2:N
     U(i) = 0;
for j=1:i-1
    U(i) = U(i) + c1(j)*x(i-j);
end
V(i)=0;
for j=1:i-1

    V(i)=V(i) + c2(j)*y(i-j);
end
     x(i)=(y(i-1))*h^q1 -U(i);
     y(i)=(-x(i)-eps*(x(i)^2-1)*y(i-1))*h^q2 -V(i)+A*cos(om*h*i);
end

 X=x(1:N)
 Y=y(1:N)
 plot(X,Y)
```

In this program we take $x(t_k) = x_k \quad y(t_k) = y_k \quad y_1 = x \quad y_2 = y$

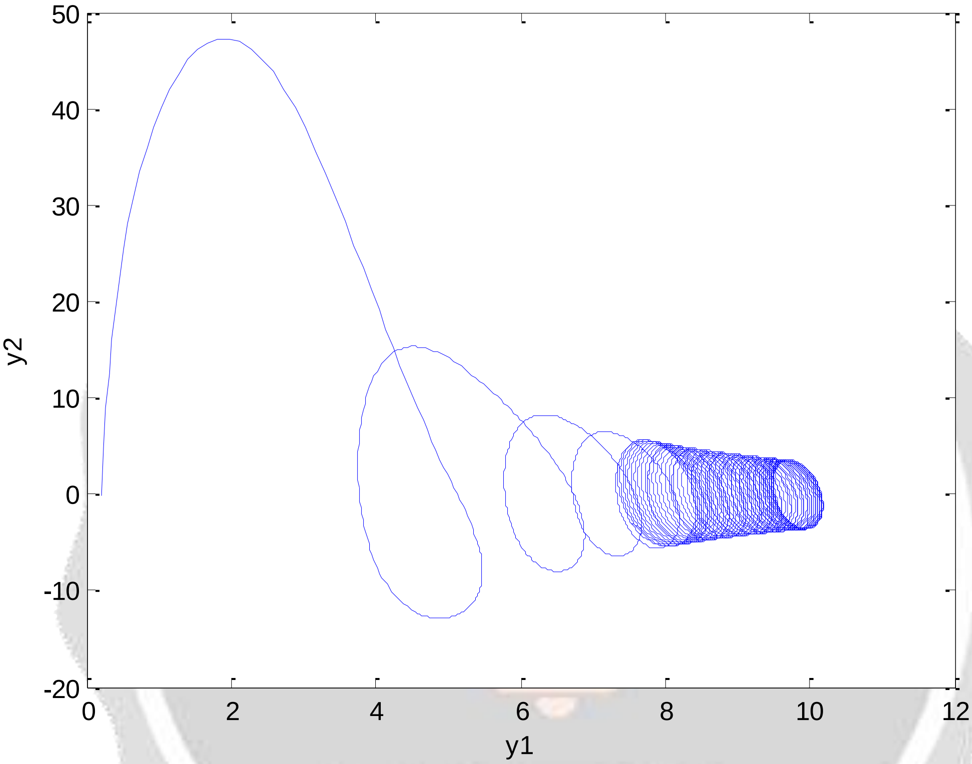
4.2 Graphic representation

For this simulation we take the value from [9] and [12] , hence :

$eps = 1 \quad A = 5 \quad q_1 = 1.2 \quad q_2 = 0.8$ x0=0.2 $y_0 = -0.2$

In this simulation we don't use the short memory then we take $\nu = 1$ see [11]

For the time step h=0.005, T=60



Graphical representation of the approximate solution of fractional order modified version of real fractional order the Van Der Pol Oscillator.

Conclusion

The above results are some of what we got.Our perseptive is to apply the fractional derivative method for the equations in physical sciences :

Stationary and temporal quantum, electromagnetism and thermal

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