ON FUZZY PRE α-OPEN SETS AND FUZZY CONTRAPRE -α-CONTINUOUS FUNCTIONS IN FUZZY TOPOLOGICAL SPACES

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ABSTRACT *In this paper ,we introduce and study a new type of fuzzy generalized open sets in fuzzy topological spaces namely,Fuzzy pre-α-open sets,Fuzzy pre-α-continuous functions and fuzzy contra pre-α-continuous functions in Fuzzy topological spaces and we discuss the relation between these types of functions and each of fuzzy contra continuous functions and other weaker forms of fuzzy contra continuous functions.*

Keyword - *Fuzzy pre-α-open sets,Fuzzy pre-α-continuous functions, fuzzy contra pre-α-continuous functions.*

INTRODUCTION: The concept of fuzzy set was introduced by Zadeh .The notation of a fuzzy subsets naturally plays a significant role in the study of fuzzy topology was introduced by Chang. Bin Shahna have introduced the concept of fuzzy α-open sets. Sabiha I.Mahmoodintroduced and studied new type of generalized open sets in topological spaces namely,pre α-open sets, pre α-continuous functions ,contra pre α-continuous functions . In this paper, we introduce and study a new class of fuzzy open sets, namely, Fuzzy pre-α-open sets and we show that the family of all fuzzy pre-α-open sets in a fuzzy topological space (X,τ) form a fuzzy topology on X which is finer than τ . This class of fuzzy open sets is placed properly between the class of fuzzy open sets and each of fuzzy β-open sets, fuzzy b-open sets, fuzzy semi-open sets,fuzzy pre-open sets, fuzzy α-open sets, fuzzy α-generalized open sets, generalized fuzzy α-open sets and generalized semi open sets respectively. The characterizations and basic properties offuzzy pre-α-open sets and fuzzy pre-α-closed sets have been studied. Moreover, We use this fuzzy open sets to define and study a new type of fuzzy generalized open sets in fuzzy topological spaces namely , Fuzzy pre-α-open sets ,Fuzzy pre-α-continuous functions and fuzzy contra pre-α-continuous functions in Fuzzy topological spaces and we study the relation between these types of function and each of fuzzy contra continuous function and other weaker form of fuzzy contra continuous functions.

1.PRELIMINARIES:

First we recall the following definitions,theorems,proposition ,lemmas.

DEFINITION 1.1: A subset A of a fuzzy topological spaces$(X,\boldsymbol{\tau})$is called

i) A fuzzy β-open set[13] if $A \leq cl\left(int\left(cl(A)\right)\right)$.
ii) A fuzzy $b$open set[13] if $A \leq int(cl(A)) \cup cl(int(A))$.
iii) A fuzzy semi- open set [1]if $A \leq cl\left(int(A)\right)$.
iv) A fuzzy pre -open set[2]if $A \leq int\left(cl(A)\right)$.
v) An fuzzy α-open set[2]if $A \leq int(cl\left(int(A)\right))$.

The fuzzy closure(respectively semi-closure) of a subset A of a fuzzy topological space is the intersection of all closed sets (respectively semi-closed) which contains A and is denoted by (respectively).

DEFINITION 1.2:

A subset A of a fuzzy topological spaces$(X,\boldsymbol{\tau})$is said to be

i) A fuzzy generalized closed set [7] if $cl(A) \leq V$ whenever $A \leq V$ and Visfuzzyopen in X.
ii) A fuzzy α-generalized closed set [12]if $\alpha cl(A) \leq V$ whenever $A \leq V$and $V$is fuzzy open in X.
iii) A fuzzy generalized α-closed set [12]if $\alpha cl(A) \leq V$ whenever $A \leq V$and $V$is fuzzy α −open in X.
iv) A fuzzy generalized semi-closed set [7]if $scl(A) \leq V$ whenever $A \leq V$and $V$is fuzzy open in X.
v) A fuzzy $s * g$-closed set [6]if $cl(A) \leq V$ whenever $A \leq V$and $V$is fuzzy semi-open in X.

The complement of a fuzzy g-closed (respectively αg-closed ,$g\alpha$-closed , $s * g$-closed) set is called a fuzzy$g$-open

DEFINITION 1.3:

A function $f:(X,\tau) \rightarrow (Y,\sigma)$ is said to be:

i) Fuzzy contracontinuous[6] if $f^{-1}(V)$ is closed set in X for every fuzzy open set Vin Y.
ii) Fuzzy contrasemi-continuous[6] if $f^{-1}(V)$ is s-closed set in X for every fuzzy open set$V$in Y.
iii) Fuzzy contraα-continuous [8]if $f^{-1}(V)$ is α-closed set in X for every fuzzy open set Vin Y.
iv) Fuzzy contrapre-continuous [8]if $f^{-1}(V)$ is pre-closed set in X for every fuzzy open set Vin Y.
v) Fuzzy contra$b$-continuous [8]if $f^{-1}(V)$ is b-closed set in X for every fuzzy open set Vin Y.
vi) Fuzzy contraβ-continuous [8]if $f^{-1}(V)$ is β-closed set in X for every fuzzy open set Vin Y.
vii) Fuzzy Contra generalized continuous[11]if $f^{-1}(V)$ is g-closed set in X for every fuzzy open set V in Y.
viii) Fuzzy Contra generalized semi-continuous [11]if $f^{-1}(V)$ is gs-closed set in X for every fuzzy open set Vin Y.

ix) Fuzzy Contra generalized α-continuous [11]if $f^{-1}(V)$ is $g\alpha$-closed set in $X$for every open set V in Y.
x) Fuzzy contraα-generalized continuous [11]if $f^{-1}(V)$ is αg-closed set in X for every fuzzy open set Vin Y.
xi) Fuzzy contra$s*g$-continuous [7]if $f^{-1}(V)$ is $s*g$-closed set in X for every fuzzy open set Vin Y.

DEFINITION 1.4:[**11**]

Let A be a subset of a fuzzy topological spaces $(\boldsymbol{X},\boldsymbol{\tau})$. Then

$\cap\{U\in\tau: A\leq U\}$is called the kernel of A and is denoted by ker(A).

LEMMA 1.5:
Let A and B be a subset of a fuzzy topological space (X,τ).Then

i) $X\in\ker(A)$if and only if $A\cap F\neq\phi$ for any closed subset F of X containing x.
ii) $A\leq\ker(A)$and if A is fuzzy open in X , then $A=\ker(A)$.
iii) If$A\leq B$then $\ker(A)\leq\ker(B)$.

DEFINITION1.6: Let (X,τ) be a fuzzy toplogical space and $A\leq X$.Then

i) The fuzzy pre-closure of A ,denoted by $pcl(A)$ is the intersection of all fuzzy pre closed subset of$X$ which contains A.
ii) The fuzzy pre-interior of A ,denoted by $pint(A)$ is the union of all fuzzy pre-open subsets of X which are contained in A.

PROPOSITION1.7: Let (X,τ) be a fuzzy toplogical space and $A,B\leq X$.Then

i) $int(A)\leq pint(A)\leq A$.
ii) $A\leq pcl(A)\leq cl(A)$.
iii) $A$is fuzzy pre-closed iff $pcl(A)$.
iv) $pcl(pcl(A))=pcl(A)$.
v) $X-pint(A)=pcl(X-A)$.
vi) If $A\leq B$,Then$pcl(A)\leq pcl(B)$.
vii) $\cup_{\alpha\in\wedge}pcl(U_\alpha)\leq pcl(\cup_{\alpha\in\wedge}U_\alpha)$
viii) $X\in pcl(A)$iff for every fuzzy pre-open set U containing x,$U\cap A\neq\phi$.
ix) If V is open set in X and A is a fuzzy pre-open set in X ,then$V\cap A$ is a fuzzy pre-open set in X.

THEOREM1.8: Let $X\times Y$ be the product space of fuzzy topological spaces (X,τ) and (Y,τ) .If $A_1\leq X$and $A_2\leq Y$Then $pcl(A_1)\times pcl(A_2)=pcl(A_1\times A_2)$.

2. FUZZY PRE-$\boldsymbol{\alpha}$-OPEN SET In this section we introduce a new type of sets ,namely Fuzzy pre-α-open sets and we show the family of all fuzzy pre-α-open subsets of a fuzzy topological spaces(X,τ)form a topology on X whish is finer than τ.

DEFINITION2.1:A subset A of a fuzzy topological space (X,τ) is called a fuzzy pre-α-open set if$A\leq int(pcl(int(A)))$**.** The complement of a fuzzy pre-α-open set is defined to be fuzzy pre-α-closed. The family of all fuzzy pre-α-open subsets of X is denoted by $\tau^{pre-\alpha}$.

Clearly,every fuzzy open set is a fuzzy pre-α-open set , but the converse is not true .

THEOREM2.2: Every fuzzy pre-α-open set is a fuzzy α-open (respectively fuzzy $g\alpha$-open, αg-open ,fuzzy β -open,fuzzypre-open,fuzzy b -open)set.**PROOF:**Let A be any fuzzy pre- α -open set in X ,then $A \leq int(pcl(int(A)))$.Since $(pcl(int(A))) \leq int(cl(int(A)))$, thus $A \leq int(cl(int(A)))$, therefore A is an fuzzy α-open set in X.Since every α-open set is $g\alpha$-open (respectively fuzzy αg-open ,fuzzy β-open,fuzzy pre-open ,fuzzy b-open)set .Thus every fuzzy pre-α-open set isα-open (respectively fuzzy $g\alpha$-open, αg-open ,fuzzy β-open ,fuzzy pre-open ,fuzzy b-open)set.

THEOREM2.3: Every fuzzy pre- α -open set is fuzzy semi-open and fuzzy gs -open set. **PROOF:**Let A be any fuzzy pre- α -open set in X ,then $A \leq int(pcl(int(A)))$ .Since $(pcl(int(A))) \leq (pcl(int(A))) \leq cl(int(A))$, thus $A \leq (cl(int(A)))$, therefore A is an fuzzy semi-open set in X.Since every fuzzy semi-open set is fuzzy gs-open.Thus every fuzzy pre-α-open set is fuzzy semi -open and fuzzy gs-open set.

THEOREM2.4:A subset A of a fuzzy topological space (X, τ) is a fuzzy pre-α-open set if and only if there is an fuzzy open set V such that $V \leq A \leq int(pcl(V))$.**PROOF :**Assume that A is a fuzzy pre- α -open set in X , then $A \leq int(pcl(int(A)))$ .Since $int(A) \leq A$, then $int(A) \leq A \leq int(pcl(int(A)))$ .Put $V = int(A)$, There is an fuzzy open set V of X such that $V \leq A \leq int(pcl(V))$.Conversely , Assume that there is an fuzzy open set V of X such that $V \leq A \leq int(pcl(V))$ .Since $\leq A \Rightarrow V \leq int(A) \Rightarrow pcl(V) \leq pcl(int(A))$ $\Rightarrow int(pcl(V)) \leq int(pcl(int(A))$.But$A \leq int(fpcl(V))$,Thus$A \leq int(pcl(int(A))$.Therefore A is a fuzzy pre-α-open set in X.

THEOREM2.5: If V is a fuzzy open set in (X, τ) , Then $V \cap pcl(A) \leq pcl(V \cap A)$ for any subset A of X.

THEOREM2.6: The family of all fuzzy pre-α-open set in a fuzzy topological space (X, τ) form a fuzzy topology on X .**PROOF:**i)Since $\phi \leq int(pcl(int(\phi)))$ and $X \leq int(pcl(int(X)))$ .Then $\phi, X \in \tau^{pre-\alpha}$.

ii)Let$A, B \in \tau^{pre-\alpha}$. To prove that $A \cap B \in \tau^{pre-\alpha}$.By theorem (2.4) there are $U, V \in \tau$. Show that $U \leq A \leq int(pcl(U))$ and $\leq B \leq int(pcl(V))$ .Consider $U \cap V \in \tau$ and $U \cap V \leq A \cap B$ .Hence $A \cap B \leq int(pcl(U)) \cap int(pcl(V)) = int(int(pcl(U)) \cap pcl \leq int(pcl\left(int(pcl(U)) \cap (V)\right)$ (bytheorem2.5)$\leq int(pcl(pcl(U)) \cap (V) \leq int(pcl(pcl(U \cap V)$ (bytheorem2.5)$= int(pcl(U \cap V))$ (by proposition 1.7(iv)) .Therefore $U \cap V \leq A \cap B \leq int\ (pcl(U \cap V))$.Thus by the theorem 2.4 ,$A \cap B \in \tau^{pre-\alpha}$ .iii)Let$\{V_\alpha : \alpha \in A\}$ be any family of fuzzy pre-α-open sets in , Then $V_\alpha \leq int(pcl(int(V_\alpha))$ for each $\alpha \in A$.Therefore by the proposition (1.7(vii)).We get

$$\bigcup_{\alpha \in \wedge} V_\alpha \leq \bigcup_{\alpha \in \wedge} int(pcl(int(V_\alpha)))$$

$$\leq int(\bigcup_{\alpha \in \wedge} pcl(int(V_\alpha))) \leq int(pcl(\bigcup_{\alpha \in \wedge} int(V_\alpha))) \leq int(pcl(int(\bigcup_{\alpha \in \wedge} V_\alpha)))$$

Then$\bigcup_{\alpha \in \wedge} V_\alpha \in \tau^{pre-\alpha}$. Therefore τ^{pre_α} is a topology on X.

THEOREM2.7: Let B be a subset of a fuzzy topological space (X, τ). Then the following statement are equivalent:

i) B is fuzzy pre-α- closed.

ii) $cl(pint(cl(B)) \leq B$.

iii) There is a fuzzy closed subset F of X such that $cl(pint(cl(F)) \leq B$.

PROOF:

(i)⇒(ii).Since B is a fuzzy pre- α- closed set in $X \Rightarrow X-B$ is a fuzzy pre- α- open $\Rightarrow X-B \leq int(pcl(int(X-B))) \Rightarrow X-B \leq int(pcl(X-cl(b)))$. (By proposition(1.7) (v)).We get $X-pint(cl(B)) = pcl(X-c)$ Hence $X-B \leq int((X-pint(cl(B))) \Rightarrow X-B \leq X-cl(pint(cl(B))) \Rightarrow cl(pint(cl(B))) \leq B$.(ii)⇒(iii) Since $cl(pint(cl(B))) \leq B$ and $B \leq cl(B)$. Then $cl(pint(cl(B))) \leq B \leq cl(B)$ Put $F = cl(B)$,thus there is a closed subset F of X such that $cl(pint(F)) \leq B \leq F$.(iii)⇒(i) Assume that there is a closed subset F of X such that $cl(pint(F)) \leq B \leq F$ .Hence $X-F \leq X-B \leq X-cl(pint(F)) = int(X-pint(F))$ .Since $X-pint(F) = pcl(X-F)$,Then$X-F \leq X-B \leq X-int(pcl(F))$Hence $X-B$ is a fuzzy pre$-\alpha-$open set in .Thus B is a fuzzy pre$-\alpha-$closed set in X.

PROPOSITION2.8: If A is a fuzzy pre-α-open set in $(X,\tau)$and $A \leq B \leq int(pcl(A))$,then B is a fuzzy pre-α-open setin$X$.**PROOF:**Since A is a fuzzy pre-α-open set in X. Then by theorem (2.4), there is an open set V of X such that $V \leq A \leq int(pcl(V))$.Since$A \leq B \Rightarrow V \leq B$.But $int(pcl(A)) \leq int(pcl(V)) \Rightarrow V \leq B \leq int(pcl(V))$.Thus B is a fuzzy pre-$\alpha$-open set in X.

COROLLARY2.9: If A is a fuzzy pre-α-closed set in $(X,\tau)$and $cl(p\ int(A)) \leq B \leq A$,then$B$ is a fuzzy pre- α -closedsetin X .**PROOF:**Since $X-A \leq X-B \leq X-cl(pint\ (A)) = int(X-pint(A)) = in(pcl(X-A))$ then by proposition (2.8), $X-B$ is a fuzzy pre-α-open set in X.Thus B is a fuzzy pre-α-closed set in X.

THEOREM2.10:A subset$A$ of a fuzzy topological space (X,τ) is fuzzy pre-α-clopen (fuzzy pre-α-open and fuzzy pre-α-closed) if and only if A is clopen (open and closed).**PROOF:**Assume that A is a fuzzy pre-α-clopen set in X,then$A$ is fuzzy pre-α-open and fuzzy pre-α-closed set in X.Thus $A \leq int(pcl(int(A)))$and $(pint(cl(A)))$ .By proposition ((1.7)(i)(ii)),we get ,$pcl(A) \leq cl(A)$ and $int(A) \leq pint(A)$,then $A \leq int(pcl(int(A)))$ and $cl(pint(cl(A)))$ .Since $int(A) \leq A \Rightarrow cl(int(A)) \leq cl(A)$ ….….(1)Since$(cl(int(A))) \leq cl(int(A))$,then $A \leq int(cl(int(A))) \leq cl(int(A)) \Rightarrow cl(A) \leq cl(int(A))$ …………… (2)Therefore, from (1) and (2) ,we get $cl(int(A)) = cl(A)$ …………… (3)Similarly , since $A \leq cl(A) \Rightarrow int(A) \leq int(cl(A))$ …………… (4)Also, $(cl(A)) \leq cl(int(cl(A))) \leq A$, Then $int(cl(A)) \leq int(A)$…………… (5)Therefore from equation (4) and (5) ,we get $int(cl(A)) = int(A)$ .Since $int(cl(A)) = int(A) \Rightarrow cl(int(cl(A))) = cl(int(A)) = cl(A)$ by (3)But $cl(int(cl(A))) \leq A$,then $cl(A) \leq A$,Since $A \leq cl(A)$,therefore $A = cl(A)$.Hence A is closed in X. Similarly , since $cl(int(A)) = cl(A) \Rightarrow int\left(cl(int(A))\right) = int(cl(A)) = int(A)$ (by (6)).But $A \leq int(cl(int(A)))$, then $A \leq int(A)$,since $int(A) \leq A$,therefore$A = int(A)$.Hence A is open in X.Thus A is a clopen set in X.⇐ It is a obvious.

DEFINITION2.11:A subset N of a fuzzy topological space (X,τ) is called a fuzzy pre-α-neighbourhood of a point x in X if there is a fuzzy pre-α-open set O in X such that $x \in O \leq N$.

PROPOSITION2.12:A subset A of fuzzy topological space (X,τ) is fuzzy pre-α-open if and only if it is a fuzzy pre -α-neighbourhood of each of its points .**PROOF:**⇒If A is a fuzzy pre-α-open in $x \in A \leq A$ for each $\in A$ .Hence A is a fuzzy pre –α-neighbourhood of each of its points .Conversely ,Assume that A is fuzzy pre-α-neighbourhood of each of its points.Then for each $x \in A$ there is a fuzzy pre-α-open sets in X ,it is a union of fuzzy pre-α- open set.

DEFINITION2.13: Let A be a subset of fuzzy topological space (X,τ) then

i) The fuzzy pre $-\alpha$-closure of$A$, denoted by fp-α-$cl(A)$ is the intersection of all fuzzy pre $-\alpha$-closed subsets of X which contains A.
ii) The fuzzy pre $-\alpha$-interior of , denoted by fp-α-$int(A)$ is the union of all fuzzy pre-α-open sets of X which contains A.

THEOREM2.14 Let A and B be a subset of fuzzy topological space (X,τ) then

i) $int(A) \leq p\text{-}\alpha\text{-}int(A) \leq A$ and $A \leq p\text{-}\alpha\text{-}cl(A) \leq cl(A)$
ii) $p\text{-}\alpha\text{-}int(A)$ is a fuzzy pre-α- open set in X and $p\text{-}\alpha\text{-}cl(A)$ is a fuzzy pre-α- closed set in X
iii) If $A \leq B$, then $p\text{-}\alpha\text{-}int(A) \leq p\text{-}\alpha\text{-}int(B)$ and $p\text{-}\alpha\text{-}cl(A) \leq p\text{-}\alpha\text{-}cl(B)$
iv) A is a fuzzy pre-α-open iff $p\text{-}\alpha\text{-}int(A) = A$ and A is a fuzzy pre-α- closediff $p\text{-}\alpha\text{-}cl(A) = A$
v) $p\text{-}\alpha\text{-}int(A \cap B) = p\text{-}\alpha\text{-}int(A) \cap p\text{-}\alpha\text{-}int(B)$and p - α - $pcl(A \cup B) = p\text{-}\alpha\text{-}pcl(A) \cup p\text{-}\alpha\text{-}pcl(B)$.
vi) $p\text{-}\alpha\text{-}int(p\text{-}\alpha\text{-}(int(A)) = p\text{-}\alpha\text{-}int(A)$and $p\text{-}\alpha\text{-}cl(p\text{-}\alpha\text{-}cl(A)) = p\text{-}\alpha\text{-}cl(A)$
vii) $x \in p\text{-}\alpha\text{-}int(A)$ iff there is a fuzzy pre-α-open set U in X show that $x \in U \leq A$.
viii) $x \in p\text{-}\alpha\text{-}cl(A)$iff for every fuzzy pre-$\alpha$-open set U containing$X$,$U \cap A \neq \phi$.

PROOF: It is obvious.

PROPOSITION 2.15 Let (X,τ) and (Y,σ) be fuzzy topological space. If $A_1 \leq X$ and $A_2 \leq Y$ then $A_1 \times A_2$ is a fuzzy pre-α-open set in $X \times Y$iff $A_1$and A_2 are fuzzy pre-α-open sets in X and Y respectively.**PROOF:** ⇐since $A_1$and A_2 are fuzzy pre-α-open sets in X and Y respectively.then by definition 2.1 we get $A_1 \leq int(pcl(int(A_1))$ and $A_2 \leq int(pcl(int(A_1)$.Hence$A_1 \times A_2 \leq int(pcl(int(A_1))) \times int(pcl(int(A_2)) = int(pcl(int(A_1)) \times (pcl(int(A_2))$.By theorem $pcl(A_1) \times pcl(A_2) = pcl(A_1 \times A_2)$ therefore $A_1 \times A_2 \leq int(pcl(int(A_1 \times A_2)$Thus $A_1 \times A_2$is a fuzzy pre-α-open set in $X \times Y$.by the same way we can prove that $A_1$and A_2 are fuzzy pre-α-open set in $X \times Y$ respectively .if $A_1 \times A_2$ is a fuzzy pre-α-open set in $X \times Y$.

3.FUZZY CONTRA PRE$-\alpha-$CONTINUOUS FUNCTION IN FUZZY TOPOLOGICAL SPACES

In this section ,we introduce new types of functions , namely , fuzzy pre-α-continuous functions, fuzzy contra pre-α-continuous functions in fuzzy topological spaces.

DEFINITION3.1:A function $f:(X,\tau) \rightarrow (Y,\sigma)$ is called fuzzy pre-α-continuous if $f^{-1}(U)$ is a fuzzy pre-α-open sets in X for every fuzzy open set Uin Y .

DEFINITION3.2: A function $f:(X,\tau) \rightarrow (Y,\sigma)$ is called fuzzy contra pre-α-continuous if $f^{-1}(U)$ is a fuzzy pre-α-closed sets in X for every fuzzy open set Uin$Y$.

PROPOSITION3.3: Every fuzzy contra continuous function is fuzzy contra pre- α -continuous.**PROOF:** Follows from the definition (3.2) and the fact that every closed set is fuzzy pre-α-closed.

THEOREM3.4: Every fuzzy contra pre-α-continuous function is fuzzy contraα-continuous (fuzzy contra $g\alpha$-continuous ,fuzzy contra αg-continuous, fuzzy contra β-continuous, fuzzy contra pre-continuous,fuzzy contra$b$-continuous)function.**PROOF:**Follows from the theorem (2.2).

THEOREM3.5: Every fuzzy contra pre-α-continuous function is fuzzy contra semi-continuous and fuzzy contrags-continuous.**PROOF:**Follows from the theorem (2.3)

PROPOSITION3.6: If $f:(X,\tau)\to(Y,\sigma)$is fuzzy contra pre-α-continuous, then $f^{-1}\big(int(B)\big)\leq p$-$\alpha$-$cl(f^{-1}(B))$.**PROOF:**Since $int(B)\leq B\Rightarrow f^{-1}(int(B))\leq f^{-1}(B)$.since $int(B)$ is an open set in Y and f is fuzzy contra pre-α-continuous , then by definition (3.2) $f^{-1}(int(B))$ is a fuzzy pre-α-closed set in X such that $f^{-1}(int(B))\leq f^{-1}(B)$ .Therefore by theorem (2.14(iv)) $f^{-1}(int(B))\leq p\alpha-cl(f^{-1}(B))$.

THEOREM3.7: Let $f:(X,\tau)\to(Y,\sigma)$ be a function .Then the following statement are equivalent:

i) f is a fuzzy contra pre-α-continuous
ii) For every closed subset $F$of ,$f^{-1}(F)$ is a fuzzy pre-α-open sets in X.
iii) For each $x\in X$ and each closed set F in Ycontaining $f(x)$,there exists a fuzzy pre-α-open set V in X such that $x\in V$ and $f(V)\leq F$.
iv) $f(p$-α-$cl(A))\leq \ker(f(A))$ for each subset $A$of X.
v) p-α-$cl(f^{-1}(B))\leq f^{-1}(\ker(B))$ for each subset $B$of Y.

PROOF: (i)⇔(ii)Itisobvious.(ii)⇒(iii) Let F be a closed set in Y show that $f(x)\in F$.To prove that , there is a fuzzy pre-α-open set V in X show that $x\in V$ and $f(V)\leq F$.Since $f(x)\in F$ and F is closed in$Y$, then by hypothesis $f^{-1}(F)$ is a fuzzy pre-α-open set in X show that$x\in f^{-1}(F)$ .Let $V=f^{-1}(F)\Rightarrow f(V)=f(f^{-1}(F))\leq F$.(iii)⇒(ii) Let F be any closed set in Y.To prove that $f^{-1}(F)$ is a fuzzy pre-α-open set in X. Let $x\in f^{-1}(F)\Rightarrow f(x)\in F$. By hypothesis there is a fuzzy pre-α-open set V in X show that $x\in V$ and $f(V)\leq F\Rightarrow x\in V\leq f^{-1}(F)$ .Thus by theorem (2.14(vii)) $f^{-1}(F)$ is a fuzzy pre-α-open set in X.(ii) ⇒(iv) Let A be any subset of X.assume that $y\notin\ker(f(A))$,Then by lemma (1.5(i)) there is a closed set F in Y .Show that $y\in F$ and $f(A)\cap F=\phi$.Thus ,we have $A\cap f^{-1}(F)=\phi$ and fuzzy$p$-α-$cl(A)\cap f^{-1}(F)=\phi$.Therefore we obtain $f(p$-α-$cl(A))\cap F=\phi$and$y\notin f(p$-α-$cl(A))$.Thus $f(p$-α-$cl(A))\leq\ker(f(A))$ for each subset A of X.

(iv)⇒(v) Let B be any subset of Y .by (iv) and lemma ((1.5).iii) we have$f\big(p$-α-$cl\big(f^{-1}(B)\big)\big)\leq\ker\big(f\big(f^{-1}(B)\big)\big)\leq\ker(B)$ .Therefore p-α-$cl\big(f^{-1}(B)\big)\leq f^{-1}(\ker(B))$ for each subset B of Y. (v)⇒(i) Let V be any open set of Y. Then by (v) and lemma ((1.5).ii), we have$p$-α-$cl\big(f^{-1}(V)\big)\leq f^{-1}(\ker(V))=f^{-1}(V)$ and p-α-$cl\big(f^{-1}(V)\big)=f^{-1}(V)$Therefore $f^{-1}(V)$ is a fuzzy pre-α-closed set in X.Thus f is fuzzy contra pre-α-continous.

CONCLUSION

we discussed and study a new type of fuzzy generalized open sets in fuzzy topological spaces namely,Fuzzy pre-α-open sets,Fuzzy pre-α-continuous functions and fuzzy contra pre-α-continuous functions in Fuzzy topological spaces and we discuss the relation between these types of functions and each of fuzzy contra continuous functions and other weaker forms of fuzzy contra continuous functions.Further we use for the future work.

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