ON FUZZY WEAKLY PREOPENFUNCTIONS

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ABSTRACT

In this paper,we introduce and characterize fuzzy weakly preopen functions between fuzzy topological spaces as natural dual to the fuzzy weakly precontinuous functions and also study these functions in relation to some other types of already known functions.

Keywords:*Fuzzy preopen sets,fuzzy weakly open functions, fuzzy extermally disconnected spaces, fuzzy almost compct spaces.*

1.Introduction

The concept of fuzzy sets was introduced by Prof.L.A.Zadeh in his classical paper [8].After the discovery of the fuzzy subsets, much attention has been paid to generalize the basic concepts of classical topology in fuzzy setting and thus a modern theory of topology is developed. The notion of fuzzy subsets naturally plays a significant role in the study of fuzzytopology which was introduced by C.L. Chang in 1968. In 1980,Ming and Ming , introduced the concepts of quasi-coincidence and *q*-neighbourhoodsby which the extensions of functions in fuzzy setting can very interestingly and effectivelybe carried out. In 1985, D.A. Rose defined weakly open functions in topological spaces.In 1997 J.H. Park, Y.B. Park and J.S. Park introduced the notion of weakly open functionsin between fuzzy topological spaces. In this paper we introduce and discuss the conceptof fuzzy weakly preopen functions which is weaker than fuzzy preopenfunctions, introduced by Miguel Caldas,Govindappa Navalagi, andRatnesh Saraf respectively and we obtained several characterizationsand properties of these functions. We also study these functions comparing withother types of already known functions. Here is seen that fuzzy preopenness implies fuzzyweakly preopenness but not conversely. But under a certain condition the converse is alsotrue. We also introduce and study the concept of fuzzy weakly preclosed functions.

Throughout this paper by (X, τ) or simply by X we mean a fuzzy topological space(fts, shorty) due to Chang . A fuzzy point in X with support $x \epsilon X$ and value p $(0 < p \leq 1)$is denoted by xp. Two fuzzy sets λ and α are said to be quasi-coincident (q-coincident,shorty) denoted by $\lambda q \alpha$, if there exists $x \epsilon X$ such that $\lambda(x) + \alpha(x) > 1$ and by$\bar{q}$wedenote "is not q-coincident". It is known that $\lambda \leq \alpha$ if and only if $\lambda \bar{q} (1 - \alpha)$. A fuzzy set$\lambda$ is said to be q-neighbourhood (q-nbd) of $x_p$if there is a fuzzy open set μ suchthat $x_p q \mu$ and $\mu \leq \lambda$ if $\mu(x) \leq \lambda(x)$ for all $x \epsilon X$.The interior, closure, and the complement of a fuzzy set λ in$X$ are denoted by $Int(\lambda)$,$Cl(\lambda)$ and $1 - \lambda$ respectively.

2.Preliminaries

Definition 2.1

A fuzzy set λ in a fts Xis called,

(i) Fuzzy semiopen if $\lambda \leq Cl(Int(\lambda))$.

(ii) Fuzzy semiclosedif $Int(Cl(\lambda)) \leq \lambda$.

(iii) Fuzzy preopen if $\lambda \leq Int(Cl(\lambda))$.

(iv) Fuzzy regular open if $\lambda = Int(Cl(\lambda))$.

(v) Fuzzy α-open if $\lambda \leq Int(Cl(Int(\lambda)))$.

(vi) Fuzzy β-openif $\lambda \leq Cl(Int(Cl(\lambda)))$.

Fuzzy semiclosed and Fuzzy semiopen sets are denoted by,

* $sCl(\lambda)=\wedge\{\lambda/\lambda\geq\mu$, μis fuzzy semiclosed set}

*$sInt(\lambda)=\vee\{\lambda/\lambda\geq \mu$, μ *is* fuzzy semiopen set}

Result:

A fuzzy set λ in a fts *X* is fuzzy semiclosed (resp . fuzzy semiopen) if and only if $\lambda=sCl(\lambda)$(resp.$\lambda=sInt(\lambda)$) .

Definition 2.2

Let $f:(X,\tau)\rightarrow(Y,\sigma)$be a function from a fts (X,τ) into a fts (Y,σ).The function *f* is called :

(i) Fuzzy semiopen if$f(\lambda)$ is a fuzzy semiopen set of *Y* foreach fuzzy open set λ in *X*.
(ii) Fuzzy weakly open if $f(\lambda) \leq Int(f(Cl(\lambda)))$ for each fuzzyopen set λ in *X*.
(iii) Fuzzy almost open (written as f.a.o.N) if $f(\lambda)$ is a fuzzyopen set of *Y* for each fuzzy regular open set λ in *X*.
(iv) Fuzzy β-open if $f(\lambda)$ is a fuzzy β-open set of *Y* for each fuzzy open set λ in *X*.

3.Fuzzy Weakly Semiopen Functions

Definition 3.1

A function $f:(X,\tau_1)\rightarrow(Y,\tau_2)$ is said to be fuzzy weaklypreopen if$f(\lambda)\leq pInt(f(Cl(\lambda)))$ for each fuzzy open subset λ of *X*.

Clearly, every fuzzy weakly open function is fuzzy preopen and everyfuzzy preopenfunction is alsofuzzy weakly preopen.

Theorem 3.2

For a function $f:(X,\tau_1)\rightarrow(Y,\tau_2)$, the following conditions are equivalent:

(i)*f* is fuzzy weakly preopen,

(ii) $f(Int_\theta(\lambda)) \leq pInt(f(\lambda))$ for every fuzzy subset λ of *X*,

(iii) $Int_\theta(f^{-1}(\alpha)) \leq f^{-1}(pInt(\alpha))$ for every fuzzy subset α of *Y*,

(iv)$f^{-1}(pCl(\alpha)) \leq Cl_\theta(f^{-1}(\alpha))$ for every fuzzy subset α of *Y*.

(v) For each fuzzy θ-open set λ in *X*, $f(\lambda)$ is fuzzy preopen in *Y*,

(vi) For any fuzzy set α of *Y* and any fuzzy θ-closed set λ in *X* containing $f^{-1}(\alpha)$, where *X* is fuzzy regular space ,there existsa fuzzy preclosed set δ in *Y* containing α such that $f^{-1}(\delta)\leq\lambda$.

Proof:

$(i)\rightarrow(ii)$:Let λ be any fuzzy subset of *X* and x_pa fuzzy point in $Int_\theta(\lambda)$.

Then,there exists a fuzzy open *q*-nbd γ of $x_p$such that $\gamma \leq Cl(\gamma)\leq \lambda$.

Then, $f(\gamma) \leq f(Cl(\gamma)) \leq f(\lambda)$.

Since*f* is fuzzy weakly preopen, $f(\gamma) \leq pInt(f(Cl(\gamma))) \leq pInt(f(\lambda))$.

It implies that $f(x_p)$ is a point in $pInt(f(\lambda))$. This shows that $x_p \in f^{-1}(pInt(f(\lambda)))$.

Thus $Int_\theta(\lambda) \leq f^{-1}(pInt(f(\lambda)))$, and so $f(Int_\theta(\lambda)) \leq pInt(f(\lambda))$.

$(ii) \to (i)$:Let μ be a fuzzy open set in X. As $\mu \leq Int_\theta(Cl(\mu))$ implies,

$f(\mu) \leq f(Int_\theta(Cl(\mu))) \leq pInt(f(Cl(\mu)))$.

Hence f is fuzzy weakly preopen.

$(ii) \to (iii)$:Let α be any fuzzy subset of Y.

Then by (ii), $f(Int_\theta(f^{-1}(\alpha))) \leq pInt(\alpha)$.

Therefore $Int_\theta(f^{-1}(\alpha)) \leq f^{-1}(pInt(\alpha))$.

$(iii) \to (ii)$:This is obvious.

$(iii) \to (iv)$:Let α be any fuzzy subset of Y. Using (iii), we have

$1 - Cl_\theta(f^{-1}(\alpha)) = Int_\theta(1 - f^{-1}(\alpha))$

$= Int_\theta(f^{-1}(1 - \alpha))$

$\leq f^{-1}(pInt(1 - \alpha))$

$= f^{-1}(1 - pInt(\alpha))$

$= 1 - (f^{-1}(pCl(\alpha))$.

Therefore, we obtain $f^{-1}(pCl(\alpha)) \leq Cl_\theta(f^{-1}(\alpha))$.

(iv)$\to(iii)$:Similary we obtain, $1 - f^{-1}(pInt(\alpha)) \leq 1 - Int_\theta(f^{-1}(\alpha))$, for

every fuzzy subset α of Y,

i.e., $Int_\theta(f^{-1}(\alpha)) \leq f^{-1}(pInt(\alpha))$.

(iv)$\to$(v):Let λ be a fuzzy θ-open set in X.

Then $1 - f(\lambda)$ is a fuzzy set in Y and by (iv) , $f^{-1}(pCl(1 - f(\lambda))) \leq Cl_\theta(f^{-1}(1 - f(\lambda)))$.

Therefore, $1 - f^{-1}(pInt(f(\lambda))) \leq Cl_\theta(1 - \lambda) = 1 - \lambda$.

Then, we have $\lambda \leq f^{-1}(pInt(f(\lambda)))$ which implies $f(\lambda) \leq pInt(f(\lambda))$.

Hence $f(\lambda)$ is fuzzy preopen in Y.

$(v) \to (iv)$:Let α be any fuzzy set in Y and λ be a fuzzy θ-closed set in X

such that $f^{-1}(\alpha) \leq \lambda$. Since $1 - \lambda$ is fuzzy θ-open in X, by (v), $f(1 - \lambda)$ is fuzzy preopen in Y.

Let $\delta = 1 - f(1 - \lambda)$. Then δ is fuzzy preclosed and $\alpha \leq \delta$.

Now, $f^{-1}(\delta) = f^{-1}(1 - f(1 - \lambda)) = 1 - f^{-1}(f(1 - \lambda)) \leq \lambda$.

(vi)$\to (iv)$: Let α be any fuzzy set in Y. Then by Corollary 3.6 of [10]

$\lambda = Cl_\theta(f^{-1}(\alpha))$ is fuzzy θ-closed set in X and $f^{-1}(\alpha) \leq \lambda$.

Then there exists a fuzzy preclosed set δ in Y containing α such that $f^{-1}(\delta) \leq \lambda$. Since δ is fuzzy preclosed $f^{-1}(pCl(\alpha)) \leq f^{-1}(\delta) \leq Cl_\theta(f^{-1}(\alpha))$.

Hence the proof.

Theorem 3.3

Let $f : (X,\tau_1) \rightarrow (Y,\tau_2)$ be a function. Then the following statements are equivalent .

(i) f is fuzzy weakly preopen:

(ii) For each fuzzy point x_p in X and each fuzzy open set μ of Xcontaining $x_p$there exists a

preopen set δ containing $f(x_p)$ such that δ ≤ $f(Cl(\mu))$.

Proof

(i)→(ii) : Let $x_p \epsilon X$ and μ be a fuzzy open set in X containing.Since f is fuzzy weakly preopen.$f(\mu) \leq pInt(f(Cl(\mu)))$.

Let δ= $pCl (f (Cl(\mu))$.Hence δ≤$f(Cl(\mu))$, with δcontaining $f(x_p)$.

(ii)→(i):Let μ be a fuzzy open set in X and let $y_p \epsilon f(\mu)$. Itfollowing from (ii) δ ≤ $f(Cl(\mu))$ for some δ preopen in Y containing y_p.

Hence we have, $y_p \epsilon \delta \leq pInt(f(Cl(\mu)))$.This shows that$f(\mu) \leq pCl (f(Int(\mu)))$,

i.e., f is a fuzzy weakly preopen function.

Hence the proof.

Theorem 3.4

Let $f(X,\tau_1) \rightarrow (Y,\tau_2)$be a bijective function.Then the following statements are equivalent:

(i)f is a fuzzy weakly preopen

(ii) $pCl(f(\lambda)) \leq f (Cl(\lambda))$ for each fuzzy open set λ in X.

(iii)$pCl(f(Int(\alpha)) \leq f(\alpha)$ for each fuzzy closed set α in X.

Proof

(i)→(iii)**:** Let α be a fuzzy closed set in X. Then we have $f(1\text{-}\alpha)=1\text{-}f(\alpha) \leq pInt(f(Cl(1\text{-}\alpha)))$

and so $1\text{-}f(\alpha) \leq 1\text{-} pCl(f(Int(\alpha)))$.Hence $pCl(f(Int(\alpha))) \leq f(\alpha)$

(iii)→(ii): Let λ be a fuzzy open set in X.Since $Cl(\lambda)$ is a fuzzy closed set and $\lambda \leq Int(Cl(\lambda))$

by (iii) we have $pCl(f(\lambda)) \leq pCl(f(Int(Cl(\lambda))) \leq f (Cl(\lambda))$.

(ii)→(iii): Similar to (iii)→(ii).

(iii)→(i):Clear.

Hence the proof.

Definition 3.5

A function f: $(X,\tau_1) \rightarrow (Y,\tau_2)$ is said to satisfy the fuzzy weakly pre open interiority condition if $pInt\big(f\big(Cl(\lambda)\big)\big) \leq f(\lambda)$ for every fuzzy open subset λ of X.

Recall that, a function $f: (X,\tau_1) \rightarrow Y,(\tau_2)$ is said to be fuzzy strongly continuous [2], if for every fuzzy subset λ of X, $f\big(Cl(\lambda)\big) \leq f(\lambda)$.

Obviously, every fuzzy strongly continuous function satisfies the fuzzy weakly pre open interiority condition but the converse does not hold as is shown by the following example.

Theorem 3.6

If f : (X,τ_1)→(Y,τ_2) is fuzzy weakly preopen and satisfies the fuzzy weakly preopen interiority condition, then f is fuzzy pre open.

Proof:

Let λbe a fuzzy open subset of X.

Since f is fuzzy weakly pre open if $(f\big(Cl(\lambda)\big)) \geq f(\lambda)$.

However, because f satisfies the fuzzy weakly preopen interiority condition, $pInt\left(f\big(Cl(\lambda)\big)\right) = f(\lambda)$ and therefore $f(\lambda)$is fuzzy preopen.

Hence the proof.

Theorem 3.7

(i) If f : (X,τ_1)→(Y,τ_2) is fuzzy preopen and fuzzy contra –open ,

then f is a fuzzy weakly preopen function.

(ii) If f : (X,τ_1)→(Y,τ_2) is fuzzycontra –closed , then f is a fuzzy

weakly preopen function

Proof:

(i) Let λ fuzzy open subset of X.

Since f is fuzzy pre-open $Int\left(Cl\big(f(\lambda)\big)\right) \geq f(\lambda)$ and since f is fuzzy contra – open ,$f(\lambda)$is fuzzy closed .

Therefore

$$Int\left(Cl\big(f(\lambda)\big)\right) \geq f(\lambda) = Int\big(f(\lambda)\big)$$

$$\leq Int\left(f\big(Cl(\lambda)\big)\right)$$

$$\leq pInt(f(Cl(\lambda))) .$$

(ii)Let be an fuzzy open subset of X.

Then , we have $f(\lambda) \leq f(Cl(\lambda))) \leq pInt(f(Cl(\lambda)))$.
The converse of the theorem does not hold.

Hence the proof.

Theorem 3.8

Let X be a fuzzy regular space. Then $f: (X, \tau_1) \to (Y, \tau_2)$ is fuzzy weakly preopen if

and only if f is fuzzy preopen.

Proof

The sufficiency is clear. Necessity.Let $\boldsymbol{\lambda}$ be a non-null fuzzy open subset of X.

For each x_s fuzzy point in λ

Let μ_{xp} be a fuzzy open set such that $x_p \in \mu_{xp} \leq Cl(\mu_{xp}) \leq \lambda$Hence , we obtain that

$\lambda = \vee\{\mu_{xp} : x_p \in \lambda\} = \vee\{Cl(\mu_{xp}) : x_p \in \lambda\}$ and

$f(\lambda) = \vee\{f(\mu_{xp}) : x_p \in \lambda\} \leq \vee\{Int(f(Cl(\mu_{xp})) : x_p \in \lambda\}$

$\leq \vee\{pInt(f(Cl(\mu_{xp})) : x_p \in \lambda\}$

$\leq pInt(f(\vee\{Cl(\mu_{xp})) : x_p \in \lambda\}$

$= pInt(f(\lambda))$.

Thus f is fuzzy preopen.

Hence the proof.

Note that, f: $(X,\tau_1) \to (Y,\tau_2)$ is said to be fuzzy contra-pre-semiclosed provide that $f(\lambda)$ is fuzzy pre-open for each fuzzy pre-closed λ of X.

Theorem 3.9

If f: $(X,\tau_1) \to (Y,\tau_2)$ is fuzzy weakly preopen and Y has the property that union of fuzzy pre-closed sets is fuzzy pre-closed and if for each fuzzy pre -closed subset α of X and each fiber $f^{-1}(y_p) \leq$ 1-α, there exist a fuzzy open subset μ of X for which $\alpha \leq \mu$ and $f^{-1}(y_q)\bar{q}Cl(\mu)$, then f is fuzzy contra-pre-semiclosed.

proof

Assume α is a fuzzy preclosed subset of X and let $y_p \in 1\text{-}f(\alpha)$.

Thus $f^{-1}(y_p) \leq$ 1-α and hence there exists a fuzzy open subset μ of X for which $\alpha \leq \mu$ and $f^{-1}(y_q)\bar{q}Cl(\mu)$.

Therefore $y_p \in 1\text{-}f(Cl(\mu)) \leq 1\text{-}f(\alpha)$. Since f is a fuzzy weakly preopen $f(\mu) \leq pInt(f(Cl(\mu)))$.

By complement , we obtain $y_p \in pCl(1\text{-}f(Cl(\mu))) \leq 1\text{-}f(\alpha)$.

Let $\delta_{\mathrm{yp}} = pCl(1\text{-}f(Cl(\mu)))$.

Then $\delta_{yp}$is a fuzzy preclosed subset of Y contining yp.

Hence $1\text{-}f(\beta) = \vee\{\delta_{yp} : yp \in 1\text{-}f(\beta)\}$ is a fuzzy preclosed and therefore $f(\beta)$ is fuzzy preopen.

Hence the proof.

Theorem 3.10

If $f : (X, \tau_1) \to (Y, \tau_2)$ is an f.a.o.N function, then it is a fuzzy

weakly preopen function.The converse is not generally true.

Proof

Let λ be a fuzzy open set in X.

Since f is f.a.o.N and $Int(Cl(\lambda))$ is fuzzy regular open, *f*(*Int*(*Cl*(λ))) is fuzzy open in *Y* and hence

f(λ) ≤ *f*(*Int*(*Cl*(λ)) ≤ *Int*(*f*(*Cl*(λ))) ≤ *pInt*(*f*(*Cl*(λ))).

This shows that *f* is fuzzy weakly preopen.

Hence the proof.

Theorem 3.11

If $f :(X,\tau_1)\rightarrow(Y,\tau_2)$ is a fuzzy weakly preopen and fuzzy continuous function, then *f* is a fuzzy α-open function.

Proof

Let λ be a fuzzy open set in *X*. Then by fuzzy weak preopenness of *f*, *f*(λ) ≤*pInt*(*f*(*Cl*(λ))).

Since *f* is fuzzy continuous *f*(*Cl*(λ)) ≤*Cl*(*f*(λ)) and since for any fuzzy subset α of *X*,

we obtain that,

f(λ) ≤ *pInt*(*f*(*Cl*(λ)))

≤*pCl*(*Int*(*f*(λ))) ≤*Int*(*Cl*(*Int*(*f*(λ)))).

Thus,*f*(λ) ≤*Cl*(*Int*(*Cl*(*f*(λ))) which showsthat*f*(λ) is a fuzzy α-open set in *Y* and hence by definition 1.3, *f* is a fuzzy α-open function.

Hence the proof.

Result

If $f : (X, \tau_1) \rightarrow (Y, \tau_2)$ is a fuzzy weakly preopen and fuzzy strongly continuous function. Then *f* is a fuzzy α-open function.

Recall,two non-null fuzzy sets λ and α in a fuzzy topological spaces *X*(i.e., neither λ nor α is o_x) are said to be fuzzy pre-separated if $\lambda\overline{q}pCl(\alpha)$ and $\alpha\overline{q}pCl(\lambda)$ orequivalently if there exist two fuzzy preopen sets μ and *v* such that λ≤ μ, α ≤*v*, $\lambda\overline{q}v$and $\alpha\overline{q}\mu$

A fuzzy topological space *X* which cannot be expressed as the union of two fuzzypre-separated sets is said to be a fuzzy pre-connected space.

Theorem 3.12

If $f : (X,\tau_1) \rightarrow (Y, \tau_2)$ is an injective fuzzy weakly preopen function of aspace *X* onto a fuzzy pre-connected space *Y*, then *X* is fuzzy connected.

Proof

If possible, let *X* be not connected. Then there exist fuzzy separated sets α and γ in *X* such that *X* = α∪γ.

Since α and γ are fuzzy separated, there exist two fuzzy open sets μ and *v* such that α≤γ , γ≤*v*, $\alpha\overline{q}v$ and $\gamma\overline{q}\mu$.

Hence we have *f*(α) ≤*f*(μ), *f*(γ) ≤*f*(*v*), *f*(α) $\overline{q}$ *f*(ν)and *f*(γ) $\overline{q}$ *f*(μ).

Since *f* is fuzzy weakly preopen, we have *f*(μ) ≤*pInt*(*f*(*Cl*(μ))) and *f*(*v*) ≤ *pInt*(*f*(*Cl*(*v*))) and since μ and *v* are fuzzy open and also fuzzy closed, we have *f*(*Cl*(μ)) =*f*(μ), *f*(*Cl*(*v*)) = *f*(*v*).

Hence *f*(μ) and *f*(*v*) are fuzzy preopen in *Y*. Therefore, *f*(α) and *f*(γ) arefuzzy pre-separated sets in *Y* and

Y = *f*(*X*) = *f*(α∪γ) = *f*(α) ∪*f*(γ).

Hence this contrary to thefact that *Y* is fuzzy pre-connected.

Thus X is fuzzy connected.

Hence the proof.

Definition 3.13

A space X is said to be fuzzy hyper-connected if every non-null fuzzyopen subset of X is fuzzy dense in X

Theorem 3.14

If X is a fuzzy hyper-connected space, then a function $f : (X, \tau_1) \rightarrow (Y, \tau_2)$is fuzzy weakly preopen if and only if $f(X)$ is fuzzy preopen in Y.

Proof

The necessity is clear.

For the sufficiency observe that for any fuzzy open subsetλ of X,

$f(\lambda) \leq f(X)$

$= pInt(f(X))$

Hence the proof.

4. Conclusion

We have discussed about the fuzzy weakly preopen functions between fuzzy topological spaces as natural dual to the fuzzy weakly precontinuous functions .

5. References

[1] K.K. Azad, "On Fuzzy Semicontinuity, Fuzzy AlmostContinuity and Fuzzy WeaklyContinuity", *J. Math. Anal. Appl*., 82, (1981), 14-32.

[2] C.L. Chang, "Fuzzy Topological Spaces", *J. Math. Anal. Appl*., 24, (1968), 182-192.

[3] Miguel Caldas, Govindappa Navalagi and Ratnesh Saraf, "Weakly Preopen and Weakly Preclosed Functions in Fuzzy Topology"Vol.4,Nos.1-2,Jan-Dec 2011,pp.37-48.

[4] P.P. Ming, and L.Y. Ming, "Fuzzy Topology I. NeighborhoodStructure of Fuzzy Point andMoore-Smith Convergence",*J. Math. Anal. Appl*., 76, (1980), 571-594.

[5] Govindappa Navalagi, Ratnesh Saraf, Miguel Caldas " On Weak Forms of Fuzzy semi-preopen and Fuzzysemi-preclosed Functions" Vol.2 , No.1, Jan-June 2011, pp. 1-10.

[6] J.H. Park, Y.B. Park, and J.S Park, "Fuzzy Weakly OpenMappings and Fuzzy RS-compactSets", *Far East J. Math. Sci. Special*, (1997), Part II, 201-212.

[7] D.A. Rose , On Weak Openness and Almost Openness, Internet. J. Math. I& Math. Sci., 7, pp.35-40, (19840.

[8] L.A. Zadeh, "Fuzzy Sets", *Inform. and Control*, 8, (1965), 338-353.