ON THE FEEDBACK-FEEDFORWARD CONTROL FOR ROBOT MANIPULATOR USED DISTURBANCE ESTIMATOR

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ABSTRACT

In this paper, the feedback-feedforward controllers combining with the uncertainty and disturbance identifiers is designed for the robot manipulator. That is an iterative learning controller(ILC) proposed to achieve precise tracking control of robotic manipulators over a finite time interval. The learning is done in a feedback configuration and the learning law updated the feedback-feedforward input from the plant input of the previous trials. The designed PD tracking controller for robotic manipulators with the proposed disturbance estimator will be also compared with the learning control process which is applied to the reference tracking control of a two link robot manipulator and good tracking performance is obtained in the simulation.

Keyword: *Iterative methods;* ***robotic manipulators****; tracking systems*

1. INTRODUCTION

Robotic manipulators are widely used in industries to perform repetitive tasks. The application of robotic manipulators increases the productivity and quality of products to a greater extent. Therefore, many control methods have been established and applied to them to ensure that they will achieve the required accuracy. The performance of a control system can become worse as input disturbances and model uncertainties exist. Thus, there have been various control methods proposed for disturbance rejection such as adaptive control, robust control and disturbance observer based control. In this work, there have been several methods for designing disturbance observers as in [7, 8, 9]. Most of these disturbance observers were designed in combination with controllers in continuous time domain such as nonlinear controller, adaptive controller and robust controller. In practice, these Perturbation estimators can significantly improve the performance of system such as [12, 13, 14].

The main strategy of the iterative learning control is to improve the quality of control iteratively by using information obtained from previous trials to obtain the control input that does the desired output trajectory [15,17]. P-type ILC controller designing method for nonlinear system were proposed in [4, 5]. Comparisons between P-type ILC controllers with other ILC ones have been made in [3]. The stability and convergence analysis of this method were only performed for discrete-time systems.

It is shown that the convergence condition has no terms reflecting the controller dynamics and thus the feedback controllers have no effect on the convergence condition [2]. But the performance of learning can be improved greatly. By applying learning control, the performance of repetitive tasks is improved by utilizing data gathered in the previous cycles.

2. PRELIMINARIES

2.1 Dynamical Model

For motion control, consider dynamics of an n link robot manipulator given by a set of uncertainties and unknown input disturbances model as (1)

$$\left[M(q)+\Delta M\right]\ddot{q}+\left[C(q,\dot{q})+\Delta C\right]\dot{q}+\left[g(q)+\Delta g\right]=\tau+d(t) \qquad (1)$$

where M(q) is the nxn inertia matrix and C, g, are, respectively, the nx1 vectors of the Coriolis and centrifugal forces, the gravity loading. And τ is the nx1 torque vector of joint control inputs to be designed. $q, \dot{q}$ and $\ddot{q}$ are the nx1 vectors representing angular position, velocity and acceleration, respectively

And

$$d(t) - \Delta M\ddot{q} - \Delta C\dot{q} - \Delta g = \tau^d \qquad (2)$$

is an summaried unknown function vector, which consists of matched input disturbances and model uncertainties

$$M(q)\ddot{q} + C(q,\dot{q})\dot{q} = \tau + \tau^d$$

The summaried unknown function vector τ^d will be compensated by its estimated value $\hat{\tau}^d$. If this is already done in such a way, the system (3) in Fig. 1 is nearly not dependent on the input disturbance τ^d, and it is considered as an approximate model of the system without input disturbance.

$$\tau = u - \hat{\tau}^d, M(q)\ddot{q} + C(q,\dot{q})\dot{q} = \tau + \tau^d = u - \hat{\tau}^d + \tau^d$$

$$\ddot{q} = -M(q)^{-1}C(q,\dot{q})\dot{q} + M(q)^{-1}\left(u - \hat{\tau}^d + \tau^d\right) \qquad (3)$$

$$\ddot{q} \approx -M(q)^{-1}C(q,\dot{q})\dot{q} + M(q)^{-1}u \qquad (4)$$

2.2 Disturbance estimator design

In this work an aforementioned disturbance estimator will be builded for the system (3) based on state feedback which was presented in [16,17,18].

Now the state-space formulations of the arm dynamics may be obtained by defining the position/velocity state

$$\begin{cases} \dot{x} = A(x)x + B\left(u - \hat{\tau}^d + \tau^d\right) \\ y = (I,0)x \end{cases} \qquad (5)$$

$$x = \begin{pmatrix} q \\ \dot{q} \end{pmatrix}, A = \begin{pmatrix} 0 & I \\ 0 & -M(q)^{-1}C(q,\dot{q}) \end{pmatrix}, B\begin{pmatrix} 0 \\ -M(q)^{-1} \end{pmatrix}$$

And, 0, I are the zeros and the identity matrix of dimension nxn, respectively.

The compensated system (3) will be controlled so that its output *q(t)* converges to a desired reference *r(t)*

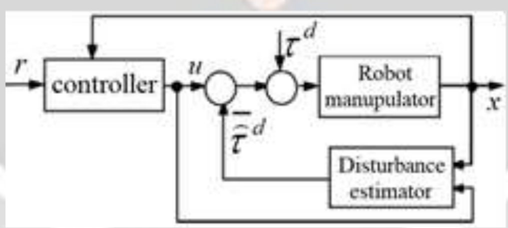

Fig -1: Suggested control scheme

The notation of *k* as the current working period, i.e. with $t = kT+\tau$, $0 \le \tau < T$ and $u(t)=u_k(\tau)$, $x(t)=x_k(\tau)$, then (5) is as:

$$\begin{cases} \dot{x}_k(\tau) = Ax_k(\tau) + B\left(u - \hat{\tau}_d(\tau) + \tau_d(\tau)\right) \\ y_k(\tau) = (I,0)x_k(\tau) \end{cases} \qquad (6)$$

The purpose of the design of disturbance estimator (see Fig.1) is now to estimate $\hat{\tau}_d$ from measured values of $x_k(\tau)$ for compensating τ_d.

Denote the last two measured values of $x_k(\tau)$, one at current time instant *iTs* and the other at previous time instant τ-$Ts=(i-1)Ts$, with $x_k(iTs)$, $x_k((i-1)T_S)$, respectively, where T_S is an arbitrarily small chosen constant

$$\dot{x}_k(iT_S) \approx \frac{x_k(iT_S) - x_k((i-1)T_S)}{T_S}$$

$$\frac{x_k(iT_S)-x_k((i-1)T_S)}{T_S}\approx Ax_k(iT_S)+B\left(u(iT_S)-\hat{\tau}_d((i-1)T_S)+\tau_d(iT_S)\right) \tag{7}$$

The obtained equation (6) will be used for calculating the estimated value $\hat{\tau}_d(iT_S)$ at the current time instant in a straightforward manner as follows. First, both the sign $\approx$ and $\tau_d(iT_S)$ in (7) are replaced with = and $\hat{\tau}_d(iT_S)$,respectively.

$$\frac{x_k(iT_S)-x_k((i-1)T_S)}{T_S}=Ax_k(iT_S)+B\left(u(iT_S)-\hat{\tau}_d((i-1)T_S)+\hat{\tau}_d(iT_S)\right)$$

Then,calculate

$$\hat{\tau}_k^d(iT_S)=\left(B^TB\right)^{-1}B^T\left(\frac{x_k(iT_S)-x_k((i-1)T_S)}{T_s}-Ax_k(iT_S)\right)+\left(u_k(iT_S)-\hat{\tau}_k^d((i-1)T_S)\right) \tag{8}$$

Thus, we have an estimation algorithm (8) for the unknown input disturbance $\tau_d(t)$ [16,17,18]

3. DESIGN CONTROLLERS FOR STABILIZING ROBOT MANIPULATOR

3.1 PD Controller Design

Through the years there have been proposed many sorts of robot control schemes. As it happens, most of them can be considered as special cases of the class of computed-torque controllers. Computed torque, at the same time, is a special application of feedback linearization of nonlinear systems. One way to select the auxiliary control signal *u(t)* is as the proportional-plus-derivative (*PD*) feedback**.**

An alternative linear state equation of the form may be written as

$$\dot{x}=\begin{bmatrix}0 & I\\ 0 & 0\end{bmatrix}x=\begin{bmatrix}0\\ I\end{bmatrix}\left[M(x_1)^{-1}C(x_1,x_1)+M(x_1)^{-1}\left(u+\delta\right)\right] \tag{9}$$

$$\delta=-\hat{\tau}_d(\tau)+\tau_d(\tau)$$

Then the overall robot arm input becomes

$$u=M(q)v=M(q)\left(\ddot{q}_d+K_v\dot{e}+K_pe\right)+N$$

Where

$$v=v_{ff}+v_{fb},v_{ff}=\ddot{q}_d,v_{fb}=K_v\dot{e}+K_pe$$

are the control input, the feedforward input, and the feedback control input, respectively.

The closed-loop error dynamics are

$$\ddot{e}+K_v\dot{e}+K_pe-\hat{\tau}^d+M^{-1}(q)\tau^d=0$$

or in state-space form,

$$\frac{d}{dt}\begin{bmatrix}e\\ \dot{e}\end{bmatrix}=\begin{bmatrix}0 & I\\ -K_p & -K_v\end{bmatrix}\begin{bmatrix}e\\ \dot{e}\end{bmatrix}+\begin{bmatrix}0\\ I\end{bmatrix}\left[\hat{\tau}^d-M^{-1}(q)\tau^d\right]=\frac{d}{dt}\begin{bmatrix}e\\ \dot{e}\end{bmatrix}=\phi p+Bv,v=\hat{\tau}^d-M^{-1}(q)\tau^d \tag{10}$$

Use a lyapunov function:

$$V(p)=\frac{1}{2}p^TPp$$

together with (9), we have:

$$\frac{dV}{dt}=\frac{1}{2}\left(\phi p+Bv\right)^TPp+p^TP\left(\phi p+Bv\right)=$$

$$\frac{dV}{dt}=-p^T\begin{pmatrix}K_1^2 & 0\\ 0 & K_2^2-K_1\end{pmatrix}p+v^T(0,I)\begin{pmatrix}2K_1K_2 & K_1\\ K_1 & K_2\end{pmatrix}p$$

$$\frac{dV}{dt}\leq a\left(\mu-a\left|p\right|\right)\left|p\right|$$

Where ϕ is the Hurwitz matrix. It is usual to take the nxn gain matrices diagonal so that

$$K_v = diag\left(k_{vi}\right) = diag\left(a\right), K_p = diag\left(k_{pi}\right) = diag\left(\sqrt{ab}\right)$$

have *b-1>a>0* is optional

$$\left|\hat{\tau}^d - M^{-1}\left(q\right)\tau^d\right| \le \mu$$

Thus, when the tracking error *p* is far from the origin, then there is $\dot{V} < 0$, and so, *p* decreases or the tracking error is going to the origin.

It is important to note that although selecting the *PD* gain matrices diagonal results in decoupled control at the outer-loop level, it does not result in a decoupled joint-control strategy. This is because multiplication by *M(q)* and addition of the nonlinear feedforward terms N in the inner loop scrambles the signal u(t) among all the joints. Thus, information on all joint positions and velocities is generally needed to compute the control for any one given joint.

3.2 Iterative learning controller design

Since many robotic applications involve repetitive motions, it is natural to consider the use of data gathered in previous cycles to improve the performance of the manipulator in subsequent cycles. This is the basic idea of repetitive control. Consider the robot model given in picture 2 and suppose that one is given a desired joint trajectory *r(t)* on finite time intervals $t=kT+\tau$, $0\le\tau<T$, (here, *k* as the current working period). The reference trajectory *r(t)* is used in repeated trails of the manipulator, assuming either that the trajectory is periodic.

$$u = M(q)\left(v - K_p q - K_v \dot{q}\right) + N$$

$$v = v_{ff} + v_{fb}, v_{ff} = v_{i-1}, v_{fb} = Ke_i$$

$$\ddot{q} = v - K_p q - K_v \dot{q} + M^{-1}(q)\left(\tau^d - \hat{\tau}^d\right)$$

$$\dot{x} = \ddot{q}_d + \begin{bmatrix} K_p & 0 \\ 0 & K_v \end{bmatrix} x + M^{-1}(q)\tau^d - \hat{\tau}^d$$

After the disturbances τ^d defined in (2) are compensated with the compensator (3), the system (3) becomes LTI as described in (9), which is now rewritten in the ILC language for repetitive systems as follows:

$$\begin{cases} x_k(i+1) = \tilde{A}x_k(i) + \tilde{B}\left(u_k(i) - \hat{\tau}_d(i) + \tau_d(i)\right) \\ y_k(i) = q = \tilde{C}x_k(i) \end{cases} \Leftrightarrow \begin{cases} x_k(i+1) = \exp(AT_S)x_k(i) + \left(\int_0^{T_S} \exp(At)Bdt\right)\left(u_k(i) - \hat{\tau}_d(i) + \tau_d(i)\right) \\ y_k(i) = q = (I_n, 0_n)x_k(i) \end{cases}$$

Where i = 0,1, ... ,N = T/Ts, $x_k(N) = x_{k.+1}(0)$

Under the assumption that both matrices *Kp, Kv* are suitably chosen such that the matrix *K* given in (6) becomes Hurwitz, the next control task is now to determine an appropriate learning parameter *K* for a P-Type update law

$$v_{k+1}(i) = v_k(i) + Ke_k(i); e_k(i) = r_k(i) - q_k(i) \quad (11)$$

in order to satisfy the required convergence $\left\|e_k(i)\right\| \to 0$ for all i, or at least as close as possible to the origin.

In order to ensure convergence using the learning control scheme of the (11), one must choose the correct gains *K*, so that $\left\|I - \tilde{C}\tilde{B}K\right\| < 1$.

4. SIMULATION RESULTS AND DISCUSSION

In this example, a two link robot manipulator is used to illustrate the proposed controller and compare with the classical PD controller [2]. The two link robot manipulator is described as follows

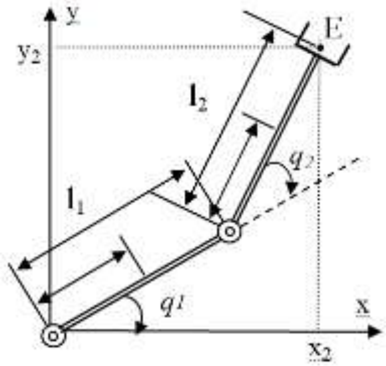

Fig.2 two link robot manipulator

$$\left[M(q)+\Delta M\right]\ddot{q}+\left[C(q,\dot{q})+\Delta C\right]\dot{q}+\left[g(q)+\Delta g\right]+d(t)=\tau$$

$$M(q)\ddot{q}+C(q,\dot{q})\dot{q}=\tau+\tau_d$$

Where

$$\begin{bmatrix} M_{11} & M_{12} \\ M_{21} & M_{22} \end{bmatrix}\begin{bmatrix} \ddot{q}_1 \\ \ddot{q}_2 \end{bmatrix}+\begin{bmatrix} -C_{12}\dot{q}_2 & -C_{12}(\dot{q}_1+\dot{q}_2) \\ C_{12}\dot{q}_1 & 0 \end{bmatrix}\begin{bmatrix} \dot{q}_1 \\ \dot{q}_2 \end{bmatrix}+\begin{bmatrix} g_1(q) \\ g_2(q) \end{bmatrix}=\begin{bmatrix} \tau_1 \\ \tau_2 \end{bmatrix}$$

$$M_{11}=m_2(l_1^2+l_1^2)/4+m_1l_1^2/4+m_2l_1l_2\cos(q_2)+J_2+J_2$$

$$M_{12}=M_{21}=m_2l_2\left(l_1+l_2\right)/4+m_2l_1l_2\cos(q_2)/2+J_2$$

$$M_{22}=m_2l_2/2+J_2$$

$$g_1(q)=gm_1l_1\cos(q_1)/2+gm_2l_1\cos(q_1)+l_2\cos(q_1+q_2)/2$$

$$g_2(q)=gm_2l_2\cos(q_1+q_2)/2$$

$$c_{11}=-g\dot{q}_2m_2l_1l_2\sin(q_2);c_{12}=c_{11}/2$$

$$c_{21}=\dot{q}_1m_2l_1l_2\sin(q_2)/2;c_{22}=o$$

m_1 and m_2 are mass of link1 and link2, l_1 and l_2 are length of link1 and link2, J_1 and J_2 are inertia moment of link1 and link2, g is gravity acceleration, $\tau_d(t)$ is unknown input disturbance, ΔM, ΔC, and Δg are model uncertainties.

Let $x=\begin{bmatrix} q \\ \dot{q} \end{bmatrix}$

and

$$-\Delta M\ddot{q}-\Delta C\dot{q}-\Delta g-d(t)=\tau_d$$

Then, the system can be rewritten as

$$\dot{x}=\begin{bmatrix} 0 & I \\ 0 & M(x_1)^{-1}C(x_1,x_1) \end{bmatrix}x=\begin{bmatrix} 0 \\ M(x_1)^{-1} \end{bmatrix}\left[\tau+\tau_d\right]=A(x)+B(x)\left[\tau+\tau_d\right]$$

For simulation, parameters for the planar robot are given as g = 9.81m/s^2, m_1 = 0.3kg, m_2 = 0.3kg, l_1 = 1m, l_2 = 0.8m, J_1 = J_2 = 0.6, x_0=[-1.5 1.5 -1.5080 0.4189]^T,

$$\tau_d=\begin{bmatrix} \text{0.1sin(0.3t)+0.2cos(0.1t)} \\ \text{0.3cos(0.2t)+0.2sin(0.5t)} \end{bmatrix}\text{-[g1;g2]}$$

is chosen as the unknown compound disturbance consisting of model error and unknown input disturbance.
The repetitive reference $r(t)$ is given as

$$r=\begin{bmatrix} \text{-1.5cos(0.4189t)-0.9sin(1.6755t)} \\ \text{sin(0.4189t)+1.5cos(1.6755t)]} \end{bmatrix}$$ and Ts=0.05s

A Proportional Derivative (PD) [1,2] is designed as

$$u=M(q)[\ddot{w}+K_pe+K_v\dot{e}]+C(q,\dot{q})\dot{q}$$

Where, $e = r - q$, $K_p = 2I_{2x2}$, $K_v = 5I_{2x2}$. This *PD* controller with the proposed disturbance estimator will be compared with the iterative learning controller.

Fig. 3 and Fig. 4 show the compound disturbances ($\tau_{d1}(t)$, $\tau_{d2}(t)$) and estimation errors (e_{d1}, e_{d2}) for the first disturbance and the second disturbance ($\tau_{d1}(t)$, $\tau_{d2}(t)$), respectively. The estimated disturbances converge to the origin disturbances with absolute approximation error of 5.10^{-2} after 1.5 (seconds)
For the simulation of iterative learning controller design, there are assigned

$$K_p = \begin{bmatrix} 30 & 0 \\ 0 & 8 \end{bmatrix}; K_v = \begin{bmatrix} 11 & 0 \\ 0 & 5 \end{bmatrix}; K = \begin{bmatrix} 0.86 & 0 \\ 0 & 0.35 \end{bmatrix}; T_S = 0.05s$$

After implementing the control algorithms presented in Subsection III. We obtain the simulation results exhibited in Figure 5 and Figure 6. Both joint variables converged on their desired references. Exactly after 100 trials the maximal value of the tracking error over the whole working period is approximately 0.01 for the first joint variable and 0.02 for the second joint variable, respectively.

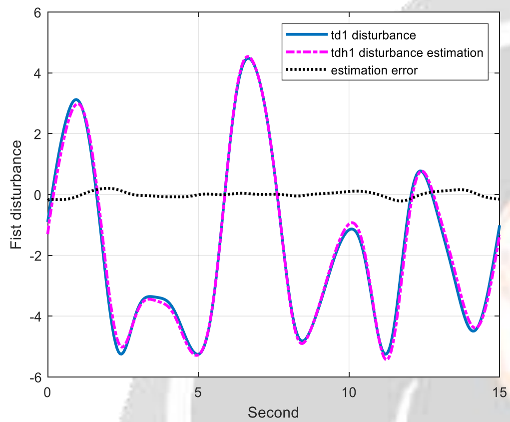

Fig.3 td1 disturbance and estimation error

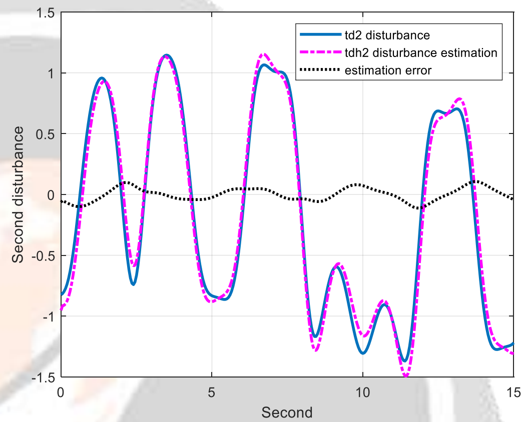

Fig.4 td2 disturbance and estimation error

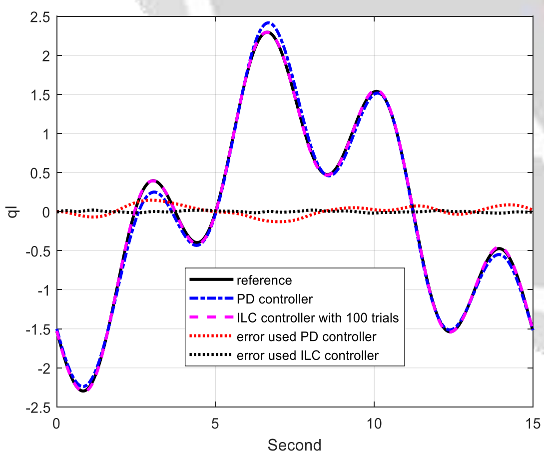

Fig.5 q1 and its tracking error.

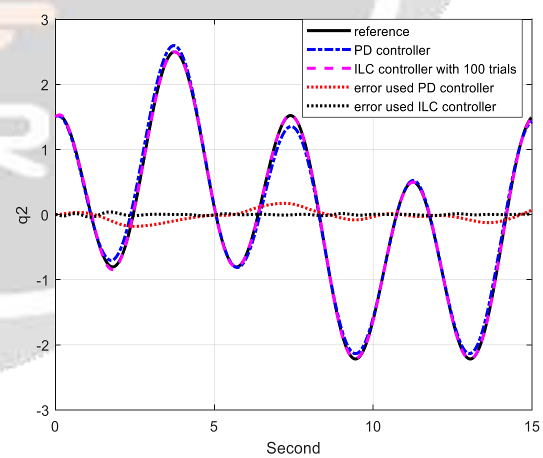

Fig.6 q2 and its tracking error.

The iterative learning control method provides better performance than the PD controller as shown in Fig. 5,6, and it takes less time to converge to the unknown disturbances for the estimator as in Fig. 3,4

5. CONCLUSIONS

A disturbance estimator and a path tracking algorithm were proposed for robot manipulator. This proposed method can be applied for robot manipulator with both unknown input disturbance and model's uncertainty. Numerical simulations for a two link robot manipulator were carried out to support the proposed input disturbance estimator and two controllers. All estimated disturbances converged to the unknown disturbances when using different controllers and the proposed iterative learning control produced stability and fast responses.

6. ACKNOWLEDGEMENT

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