OSCILLATION CRITERIA OF SECOND ORDER NONLINEAR INTERVAL FORCED DIFFERENTIAL EQUATIONS

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ABSTRACT

We present oscillation criteria of second order nonlinear interval forced differential equations. These criteria involve the use of averaging functions. Our theorems are stated in general form. We are interested in obtaining results on the oscillatory behavior of solutions of the second-order nonlinear forced class differential equations. Our results are based on the information on a sequence of subintervals of $[t_0, \infty)$ *only rather than on the whole half line.*

Keywords *Oscillation, second order, nonlinear, forced differential equations.*

1.1 INTRODUCTION

Consider the oscillation behavior for the interval forced second-order non-linear differential equation. $[m(t)\psi(y(t))\phi(y'(t))]' + r(t)f(y(t)) = e(t), t \geq t_0,$ (1.1.1)

Where the functions

$m, r, e \in C([t_0, \infty), R)$,and$\phi, \psi, f \in C(R, R)$.

Throughout this paper we shall assume that

i).$m(t) > 0, t \geq t_0, and\ \psi(y) > 0, yf(y) > 0\ for\ all\ y \neq 0;$

ii). ϕ be continuously differentiable and satisfying

$|\phi(y)|^{\frac{\alpha+1}{\alpha}} \leq \gamma_1 y\phi(y)$

for some constants$\alpha > 0, \quad \gamma_1 > 0$ and for all $y \in R$.

We will find that equation (1.1.1) can be considered as a natural generalization of the following differential equation. $y''(t)+r(t)f(y(t))=e(t), t\geq t_0$ (1.1.2)

$$\left(m(t)y'(t)\right)'+r(t)y(t)=e(t), t\geq t_0 \qquad (1.1.3)$$

$$(m(t)|y'(t)|^{\sigma-1}y'(t))'\, r(t)|y(t)|^{\tau-1}y(t)=e(t),$$
$$\sigma>0, \tau\geq\sigma, t\geq t_0. \qquad (1.1.4)$$

The more general forced second-order nonlinear differential equations of the form

$$(m(t)\psi(y(t))|y'(t)|^{\sigma-1}y'(t))'+r(t)f(y(t))=e(t), t\geq t_0 \qquad (1.1.5)$$

In this paper we will give some interval oscillation criteria for Eq.(1.1).Equation (1.1.1) through some new averaging functions

$H(t,s)\in C(D,R)$,which satisfy

i).$H(t,t)=0, H(t,s)>0$ for $t>s$;

ii). H has partial derivatives $\frac{\partial H}{\partial t}$ and $\frac{\partial H}{\partial s}$on D.

Such that

$$\frac{\partial H}{\partial t}=h_1(t,s)\sqrt{H(t,s)}$$

$$\frac{\partial H}{\partial s}=-h_2(t,s)\sqrt{H(t,s)}$$

Where $D=\{(t,s): t_0\leq s\leq t<\infty\}, h_1, h_2\in L(D,R^+)$.

1.2 $f(x)$BE MONOTONE INCREASING

The oscillation for equation (1.1.1) is

$$[m(t)\psi(y(t))\phi(y'(t))]'+r(t)f(y(t))=e(t), t\geq t_0$$

under the assumptions.

i).$m(t)>0, t\geq t_0$ and $\psi(y)>0, yf(y)>0$ for all $y\neq 0$.

ii) ϕ be continuously differentiable and satisfying

$$|\phi(y)|^{\frac{\alpha+1}{\alpha}}\leq\gamma_1 y\phi(y)$$

for some constants$\alpha>0, \gamma_1>0$ and for all $y\in R$,and the following assumption:

iii) $f'(y)$ $exists$, $yf(y)>0$ for $y\neq 0$ and $\frac{f'(y)}{(\psi(y)|f(y)|^{\alpha-1})^{\frac{1}{\alpha}}}\geq\gamma_2>0$for some nonnegative constant γ_2 and for all $y\in R/\{0\}$.

1.3THEOREM

Suppose

i). $m(t) > 0, t \geq t_0$, and $\psi(y) > 0, yf(y) > 0$ $for\ all\ y \neq 0$.

ii). ϕ be continuously differentiable and satisfying

$|\phi(y)|^{(\alpha+1)/\alpha} \leq \gamma_1 y\phi(y)$.

for some constants $\alpha > 0, \gamma_1 > 0$ and for all $y \in R$.

iii). $f'(y)\ exists, yf(y) > 0\ for\ y \neq 0\ and\ \frac{f'(y)}{(\psi(y)|f(y)|^{\alpha-1})^{\frac{1}{\alpha}}} \geq \gamma_2 > 0$ for some nonnegative constant γ_2 and for all $y \in R/\{0\}$ be fulfilled and for any $T \geq t_0$, there exist $T \leq a_1 < b_1 \leq a_2 < b_2$ such that

$$e(t) \begin{cases} \leq 0, & t \in [a_1, b_1], \\ \geq 0, & t \in [a_2, b_2], \end{cases}$$

(1.3.1)

If there exist some $c_i \in (a_i, b_i), i = 1,2, H(t,s)$ satisfying

a) $H(t,t) = 0, H(t,s) > 0\ for\ t > s$;

b) H has derivatives $\frac{\partial H}{\partial t}\ and\ \frac{\partial H}{\partial s}\ on\ D$

Such that

$$\frac{\partial H}{\partial t} = h_1(t,s)\sqrt{H(t,s)}, \frac{\partial H}{\partial s} = -h_2(t,s)\sqrt{H(t,s)}$$

And a positive function $\rho \in c'([T_0, \infty), R)$.

Such that

$$\int_{a_i}^{c_i} \left[H^{\alpha+1}(s,a_i)r(s)\rho(s) - \frac{(\alpha\Upsilon)^\alpha}{(\alpha+1)^{\alpha+1}} m(s)\rho(s){H_1}^{\alpha+1}(s,a_i)\right] ds > 0,$$

$$\int_{c_i}^{b_i} \left[H^{\alpha+1}(b_i,s)r(s)\rho(s) - \frac{(\alpha\Upsilon)^\alpha}{(\alpha+1)^{\alpha+1}} m(s)\rho(s){H_2}^{\alpha+1}(b_i,s)\right] ds > 0,$$

(1.3.2)

For $i = 1,2$, where $\gamma = \frac{\gamma_1}{\gamma_2}$

$$H_1(t,s) = \left|(\alpha+1)h_1(t,s)\sqrt{H(t,s)} + H(t,s)\frac{\rho'(t)}{\rho(t)}\right|,$$

$$H_2(t,s) = \left|(\alpha+1)h_2(t,s)\sqrt{H(t,s)} + H(t,s)\frac{\rho'(s)}{\rho(s)}\right|$$

Then equation (1.1.1) is oscillatory.

PROOF

Given,

$[m(t)\psi(y(t))\phi(y'(t))]' + r(t)f(y(t)) = e(t), t \geq t_0$

To prove,

The equation (3.1.1) is oscillatory.

Now we take a contrary assume that. The equation (1.1.1) is no oscillatory. Suppose $y(t)$ be a non oscillatory solution of equation(1.1.1).Let $y(t) \neq 0$ on $[T_0, \infty)$ for some sufficiently large $T_0 \geq t_0$.

Define

$w(t) = \rho(t)\frac{m(t)\psi(y(t))\phi(y'(t))}{f(y(t))}, t \geq T_0$

(1.3.3)

The differentiating (1.3.3) and we have using the assumption condition.

$$w'(t) = -r(t)\rho(t) + \frac{e(t)}{f(y(t))}\rho(t) - \frac{\rho(t)m(t)\psi(y(t))}{f^2(y(t))}$$

$$[y'(t)\phi(y'(t))]f'(y(t)) + \frac{\rho'(t)}{\rho(t)}w(t)$$

$$\leq -r(t)\rho(t) + \frac{e(t)}{f(y(t))}\rho(t) - \frac{1}{\Upsilon}\frac{|w(t)|^{\frac{(\alpha+1)}{\alpha}}}{(m(t)\rho(t))^{\frac{1}{\alpha}}} + \frac{\rho'(t)}{\rho(t)}w(t)$$

(1.3.4)

By the assumptions we can choose $a_i, b_i \geq T_0$ for $i = 1,2$. Such that $e(t) \leq 0$ on the interval $I_1 = [a_1, b_1]$ with a_1, b_1 and $y(t) > 0$, or $e(t) \geq 0$ on the interval $I_2 = [a_2, b_2]$ and $y(t) < 0$ on the Intervals I_1 and I_2, (1.3.4) imply that $w(t)$ satisfies.

$$w'(t) \leq -r(t)\rho(t) - \frac{1}{\Upsilon}\frac{|w(t)|^{\frac{(\alpha+1)}{\alpha}}}{(m(t)\rho(t))^{\frac{1}{\alpha}}} + \frac{\rho'(t)}{\rho(t)}w(t) \quad (1.3.5)$$

On the one hand, multiplying $H^{\alpha+1}(t,s)$ through (1.3.5) and integrating it (with t replaced by S) over $[c_i, t)$ for $t \in [c_i, b_i), i = 1,2,$ by using assumption condition, we have for $s \in [c_i, t)$

$$\int_{c_i}^{t} H^{\alpha+1}(t,s)r(s)\rho(s)\,ds$$

$$\leq -\int_{c_i}^{t} H^{\alpha+1}(t,s)w'(s)ds$$

$$-\int_{c_i}^{t} H^{\alpha+1}(t,s)\left[\frac{\rho'(s)}{\rho(s)}w(s) - \frac{|w(s)|^{\frac{(\alpha+1)}{\alpha}}}{\Upsilon\big(m(s)\rho(s)\big)^{\frac{1}{\alpha}}}\right]ds$$

$$= H^{\alpha+1}(t,c_i)w(c_i) - \int_{c_i}^{t} (\alpha+1)H^{\alpha}(t,s)h_2(t,s)\sqrt{H(t,s)}w(s)ds$$

$$+\int_{c_i}^{t} H^{\alpha+1}(t,s)\left[\frac{\rho'(s)}{\rho(s)}w(s) - \frac{|w(s)|^{\frac{(\alpha+1)}{\alpha}}}{\Upsilon\big(m(s)\rho(s)\big)^{\frac{1}{\alpha}}}\right]ds$$

$$\leq H^{\alpha+1}(t,c_i)w(c_i)$$

$$+\int_{c_i}^{t}\left[H^{\alpha}(t,s)H_2(t,s)|w(s)| - \frac{H^{\alpha+1}(t,s)}{\Upsilon\big(m(s)\rho(s)\big)^{\frac{1}{\alpha}}}|w(s)|^{\frac{\alpha+1}{\alpha}}\right]ds.$$

(1.3.6)

For a given t and s set

$$F(V) = H^{\alpha}H_2 - \frac{H^{\alpha+1}}{\Upsilon(m\rho)^{\frac{1}{\alpha}}}v^{\frac{\alpha+1}{\alpha}}, \quad v > 0.$$

Because of $F'(v) = H^{\alpha}H_2 - \frac{(\alpha+1)H^{\alpha+1}}{\alpha\Upsilon(m\rho)^{\frac{1}{\alpha}}}v^{\frac{1}{\alpha}}, and\ F(v)$ obtains its maximum at

$v = m\rho\left(\frac{\alpha\Upsilon H_2}{(\alpha+1)H}\right)^{\alpha}$,and

$$F(v) \leq F_{max} = \frac{(\alpha\Upsilon)^{\alpha}}{(\alpha+1)^{\alpha+1}}m\rho {H_2}^{\alpha+1}$$

(1.3.7)

Then we get, by using(1.3.7),

$$\int_{c_i}^{t} H^{\alpha+1}(t,s) r(s)\rho(s)\,ds \le H^{\alpha+1}(t,c_i)w(c_i) + \frac{(\alpha\Upsilon)^{\alpha}}{(\alpha+1)^{\alpha+1}}\int_{c_i}^{t} m(s)\rho(s){H_2}^{\alpha+1}(t,s)ds$$

(1.3.8)

Letting $t \to {b_i}^{-}$ in (1.3.6), we obtain

$$\int_{c_i}^{b_i} H^{\alpha+1}(b_i,s) r(s)\rho(s)\,ds \le H^{\alpha}(b_i,c_i)w(c_i) + \frac{(\alpha\Upsilon)^{\alpha}}{(\alpha+1)^{\alpha+1}}\int_{c_i}^{b_i} m(s)\rho(s){H_2}^{\alpha+1}(b_i,s)ds.$$

(1.3.9)

On the other hand, if we multiply $H^{\alpha+1}(s,t)$ through (1.3.5) and integrate it (with t replaced by s) over $(t,c_i]$ for $t \in (a_i,c_i], i = 1,2,$ instead by using assumption condition, we have for $s \in (t,c_i]$

$$\int_{t}^{s} H^{\alpha+1}(s,t) r(s)\rho(s)\,ds \le -\int_{t}^{c_i} H^{\alpha+1}(s,t)w'(s)ds + \int_{t}^{c_i} H^{\alpha+1}(s,t)\left[\frac{\rho'(s)}{\rho(s)}w(s) - \frac{|w(s)|^{\frac{(\alpha+1)}{\alpha}}}{\Upsilon\big(m(s)\rho(s)\big)^{\frac{1}{\alpha}}}\right]ds$$

$$= -H^{\alpha+1}(c_i,t)w(c_i) + \int_t^{c_i} (\alpha+1)H^{\alpha}(s,t)h_1(s,t)\sqrt{H(s,t)}w(s)\,ds + \int_t^{c_i} H^{\alpha+1}(s,t)\left[\frac{\rho'(s)}{\rho(s)}w(s) - \frac{|w(s)|^{\frac{(\alpha+1)}{\alpha}}}{\Upsilon(m(s)\rho(s))^{\frac{1}{\alpha}}}\right]ds$$

$$\leq -H^{\alpha+1}(c_i,t)w(c_i) + \int_t^{c_i}\left[H^{\alpha}(s,t)H_1(s,t)|w(s)| - \frac{H^{\alpha+1}(s,t)}{\Upsilon(m(s)\rho(s))^{\frac{1}{\alpha}}}|w(s)|^{\frac{\alpha+1}{\alpha}}\right]ds$$

$$\leq -H^{\alpha+1}(c_i,t)w(c_i) + \frac{(\alpha\Upsilon)^{\alpha}}{(\alpha+1)^{\alpha+1}}\int_t^{c_i} m(s)\rho(s){H_1}^{\alpha+1}(s,t)\,ds$$

(1.3.10)

(We get the final"$\leq$ " in (1.3.10) by following the proof of (3.3.8).

Letting $t \to {a_i}^+$ in (1.3.10), it follows that

$$\int_{a_i}^{c_i} H^{\alpha+1}(s,a_i)r(s)\rho(s)\,ds \leq H^{\alpha+1}(c_i,a_i)w(c_i) + \frac{(\alpha\Upsilon)^{\alpha}}{(\alpha+1)^{\alpha+1}}\int_{a_i}^{c_i} m(s)\rho(s){H_1}^{\alpha+1}(s,a_i)\,ds.$$

(1.3.11)

Finally, dividing (1.3.9) and (1.3.11) by $H^{\alpha+1}(b_i,c_i)$ and $H^{\alpha+1}(c_i,a_i)$ respectively, and then adding them, we have the following inequality

$$\frac{1}{H^{\alpha+1}(c_i,a_i)}\int_{a_i}^{c_i} H^{\alpha+1}(s,a_i)r(s)\rho(s)\,ds$$
$$+\frac{1}{H^{\alpha+1}(b_i,c_i)}\int_{c_i}^{b_i} H^{\alpha+1}(b_i,s)m(s)\rho(s)\,ds$$
$$\leq \frac{1}{H^{\alpha+1}(c_i,a_i)}\frac{(\alpha\Upsilon)^\alpha}{(\alpha+1)^{\alpha+1}}\int_{a_i}^{c_i} m(s)\rho(s)H_1{}^{\alpha+1}(s,a_i)\,ds$$
$$+\frac{1}{H^{\alpha+1}(b_i,c_i)}\frac{(\alpha\Upsilon)^\alpha}{(\alpha+1)^{\alpha+1}}\int_{c_i}^{b_i} m(s)\rho(s)H_2{}^{\alpha+1}(b_i,s)\,ds,$$

(1.3.12)

Then

$$\int_{a_i}^{c_i}\left[H^{\alpha+1}(s,a_i)r(s)\rho(s) - \frac{(\alpha\Upsilon)^\alpha}{(\alpha+1)^{\alpha+1}}m(s)\rho(s)H_1{}^{\alpha+1}(s,a_i)\right]ds < 0,$$

$$\int_{c_i}^{b_i}\left[H^{\alpha+1}(b_i,s)r(s)\rho(s) - \frac{(\alpha\Upsilon)^\alpha}{(\alpha+1)^{\alpha+1}}m(s)\rho(s)H_2{}^{\alpha+1}(b_i,s)\right]ds < 0,$$

Which contradict to the condition(1.3.2).Hence the equation (1.1.1) is oscillatory.

$$\int_{a_i}^{c_i}\left[H^{\alpha+1}(s,a_i)r(s)\rho(s) - \frac{(\alpha\Upsilon)^\alpha}{(\alpha+1)^{\alpha+1}}m(s)\rho(s)H_1{}^{\alpha+1}(s,a_i)\right]ds > 0,$$

$$\int_{c_i}^{b_i}\left[H^{\alpha+1}(b_i,s)r(s)\rho(s) - \frac{(\alpha\Upsilon)^\alpha}{(\alpha+1)^{\alpha+1}}m(s)\rho(s)H_2{}^{\alpha+1}(b_i,s)\right]ds > 0,$$

Hence the proof.

CONCLUSION

Throughout this work, we discussed some definition and theorems on oscillation criteria of second order non linear interval forced differential equations and then we discussed monotone increase and monotone decrease estimate. Finally we establish that, when the second order non linear interval forced differential equations is oscillatory. We proved the equation of monotone increase and monotone decrease is oscillatory.

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