# OPTIMIZATION METHODS FOR INTERFERENCES REDUCTION BETWEEN MIMO RADAR AND SYSTEM OF CELLULAR COMMUNICATION

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**ABSTRACT**

*For the Radar MIMO, the Uniform Layer Antenna permit to beam the propagation on any specific angular. The interference with the cellular network could not convenient for that indeed about the envelope of the Radar. In this fact, the spectrum sharing is a new approach to solve this problem. The projection on Null Space could reduce the interference by inserting signal to transmit with the value of this projection. The solver of this problem could translate by a nonlinear problem without constraints. The choice of the most optimization algorithm could be problematic without having any algorithm to evaluate and select one of them because the random behavior of the interference's noise could be more advantages or more disadvantage than the other methods. Le author's contribution consists to create new criteria for the performance of the envelope by basing on the surface occupied of the transversal schema of the propagation and creation any selection algorithm for the beamforming of best resolution about FACE or Finite Alphabet Constant Envelope BPSK/QPSK on condition to null space. The BADS, NM, PSO are the candidate choose for the solver optimization. The output of the algorithm contains the algorithm of resolution and the binary signal to transmit for having a good deviation of the beamforming and capabilities to resist with the interference.*

**Keyword:** *NSP, MIMO, BPSK, QPSK, FACE*

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## 1. INTRODUCTION

The MIMO Radar and cellular network use the same band. To avoid interference, the MIMO Radar should send the signal BPSK and QPSK with the projection to null space. Our Works consists to make modelization noise and to determine the signal to send more robust against noise by using optimization equation. Most of the resolution of problem will be study like the algorithm NM, BADS and Soothe goal consists to select the best solution based on the surface occupied by the envelope. [1] [2] [3] [4]

## 2. MIMO RADAR

The MIMO Radar is a system with multi-antenna formed by $n_T$ transmission of antenna $n_R$ reception of antenna. The frequency of signal to be sent verify the equation [5]:

$$f_c = \frac{c}{\lambda_c} \tag{1}$$

The received signal will be localized on the angular $\theta_k$ defined by:

$$r_k(n) = \sum_{m=1}^{n_T} e^{-j(m-1)\pi sin(\theta_k)} x_m(n), \tag{2}$$

$$n = 1,2 \dots N\_s$$

The signal to be sent follow the equation:

$$x(n) = \begin{bmatrix} x_1(n) & x_2(n) & \dots & x_{n_T}(n) \end{bmatrix} \tag{3}$$

The ULA is defined by $(\theta_k)$ :

$$a(\theta_k) = \begin{bmatrix} 1 & e^{-j\pi \sin(\theta_k)} & e^{-j2\pi \sin(\theta_k)} & \dots & e^{-j\pi(M-1)\sin(\theta_k)} \end{bmatrix} \tag{4}$$

We could deduct that:

$$r_k(n) = a^T(\theta_k)x(n) \tag{5}$$

The received power by the destiny will be localized on the angular:

$$P(\theta_k) = E\{a^T(\theta_k).x(n).x^T(n).a(\theta_k)\} \tag{6}$$

$$P(\theta_k) = a^T(\theta_k).R.a(\theta_k) \tag{7}$$

## 2. THE DESIRED PROPAGATION

R is the correlation matrix with the transmitted signal. The MIMO Radar uses analogue automatism based on the desired signal $\emptyset(\theta_k)$ [6].

$$\emptyset(\theta_k) = \begin{cases} 0 \; si \; \theta_k \notin \; \theta_{BW} \\ P_{max} \; si \; \theta_k \in \; \theta_{BW} \end{cases} \tag{8}$$

$\theta_{BW}$ is the angular of the beam width.

## 3. OPTIMIZATION OF PROPAGATION

To follow the function $\emptyset(\theta_k)$, The MSE between the signal's power and the desired propagation should be minimal as possible :

$$J(R) = MSE\big(P(\theta_k) - \emptyset(\theta_k)\big) = MSE\big(\, a^H(\theta_k).R.a(\theta_k) - \emptyset(\theta_k)\big)$$

$$J(R) = \frac{1}{K}\sum_{k=1}^{K}(a^H(\theta_k).R.a(\theta_k) - \emptyset(\theta_k)\,)^2 \tag{9}$$

But, for having FACE or Finite Alphabet Constant Envelope, the covariance matrix R could not be choosing freely. It should verify 2 conditions:

- Like R is a covariance matrix, it should be positive semi-definitive by the constraints C1.

- Like the envelope CE of the propagation should be constant, all antennas should transmit the same power, the diagonal of R follows the constraints 2.

$$\text{C1}: V^H R V \geq 0, \qquad \forall V$$

$$\text{C2}: R(m,m) = c\,, \qquad m = 1,2,\ldots.n_T$$

The equation to form the envelope is defined by the optimization with constraints nonlinear:

$$\min_R \frac{1}{K}\sum_{k=1}^{K}(a^H(\theta_k).R.a(\theta_k) - \emptyset(\theta_k))^2$$
$$subject\ to \ \begin{matrix} V^H RV \geq 0, & \forall V \\ R(m,m)=c\,, & m=1,2,\ldots.n_T \end{matrix}$$

The resolution of this problem could be very difficult; because the constraints could have many solutions. For the MIMO Radar, this problem could be transformed to optimization nonlinear no constraints by inserting the auxiliary matrix W($\psi$). The two constraints C1 and C2 signifies that the MIMO RADAR guaranties the transmission with the same power each antenna, the received signal will be defined by:

$$r_k(n) = \sum_{q=1}^{n_T}\sum_{p=1}^{n_T} w_{p,q} x_p(n) e^{-j(q-1)\pi sin(\theta_k)} \tag{10}$$

This vector could be written like:

$$r_k(n) = a^H(\theta_k) W(\psi) x(n) \tag{11}$$

with:

$$W(\psi) = \begin{matrix} w_{11} & w_{12} & \cdots & w_{1n_T} \\ w_{21} & w_{22} & \cdots & w_{2n_T} \\ \vdots & \vdots & \vdots & \vdots \\ w_{n_T 1} & w_{n_T 2} & \cdots & w_{n_T n_T} \end{matrix} \tag{12}$$

The received localized on the angular $\theta_k$ is defined by:

$$P(\theta_k) = E\{a^H(\theta_k).W.x(n).x^H(n).W^H.a(\theta_k)\}$$

$$P(\theta_k) = a^H(\theta_k).W.W^H.a(\theta_k) \tag{13}$$

The function to be minimized for having the angular $\emptyset(\theta_k)$ is:

$$J(R) = MSE\big(P(\theta_k) - \emptyset(\theta_k)\big) = MSE\big(\ a^T(\theta_k).R.a(\theta_k) - \emptyset(\theta_k)\big)$$

$$J(R) = \frac{1}{K}\sum_{k=1}^{K}(a^T(\theta_k).W.W^T.a(\theta_k) - \emptyset(\theta_k))^2 \tag{14}$$

For having a FACE-Envelope, The average power transmitted on the antenna q will be:

$$P_{av}(q) = E\left\{\left|\sum_{p=1}^{n_T} w_{pq} x_p(n)\right|^2\right\}$$

$$P_{av}(q) = \ E\left\{\left|W_q^T.x(n).x^T(n).W_q\right|^2\right\}$$

$$P_{av}(q) = W_q^T W_q \tag{15}$$

This equation signify that the power transmitted of the antenna q is equal to the norm of the q-th vector column of W.

$$W(\psi)=\begin{bmatrix} 1 & \sin(\psi_{21}) & \sin(\psi_{31})\sin(\psi_{32}) & \cdots & \prod_{m=1}^{n_T-1}\sin(\psi_{n_T m}) \\ 0 & \cos(\psi_{21}) & \sin(\psi_{31})\cos(\psi_{32}) & \cdots & \prod_{m=1}^{n_T-2}\sin(\psi_{n_T m})\cos(\psi_{n_T,n_T-1}) \\ 0 & 0 & \cos(\psi_{31}) & \ddots & \vdots \\ \vdots & \vdots & \ddots & \ddots & \sin(\psi_{n_T 1})\cos(\psi_{n_T 2}) \\ 0 & 0 & \cdots & 0 & \cos(\psi_{n_T 1}) \end{bmatrix} \quad (16)$$

In this fact, the angular $\psi$ wich is the variable to determine for minimized the function J is:

$$\psi = \left[\psi_{21}, \psi_{31}, \psi_{32}, \dots, \psi_{n_T,1}, \psi_{n_T,2}, \dots, \psi_{n_T,n_T-1}\right] \quad (17)$$

$$card\{\psi\} = \frac{(n_T - 1) * n_T}{2} \quad (18)$$

$$P(\psi, \theta_k) = a^T(\theta_k).W(\psi).W^T(\psi).a(\theta_k) \quad (19)$$

The optimization without constraints will be written by:

$$min\, J(\psi) = \frac{1}{K}\sum_{k=1}^{K}(P(\psi) - \emptyset(\theta_k))^2 \quad (20)$$

By using FACE-BPSK, the number of variable of the matrix W does not change. Therefore, the variable of number to optimize is:

$$n_{var} = (n_T - 1) * \frac{n_T}{2} = \frac{(n_T - 1) * n_T}{2} \quad (21)$$

The generation of the waveform uses those 2 operators: the GRV or Gaussian Random Value and the SVD or Singular Value Decomposition

## 4. GENERATOR GRV OR GAUSSIAN RANDOM VALUE

### 4.1 Definitions

The random vector $X = (X_1, X_2, \dots, X_n)$ is a GRV if his repartition function is [9]:

$$f_X(x) = \frac{1}{(2\pi)^{\frac{n}{2}}|\sum|^{\frac{1}{2}}} e^{-\frac{1}{2}(x-\mu)^H \Sigma^{-1}(x-\mu)} \quad (22)$$

Like $\mu$ is the mean value and $\sum$ is the matrix covariance.

### 4.2 Decomposition SVD or Singular Value Decomposition

The GRV generate the sampled signal FACE-BPSK or FACE-QPSK using the SVD.

$$X = USV^H \quad (23)$$

$U$ is the Eigen matrix vector of $XX^H$ verifying : $U^H U = I$

$V$ is the Eigen matrix vector of $X^H X$ verifying : $VV^H = I$

S is a diagonal matrix with the positive or null terms.

## 5. NONLINEAR SYSTEM RESOLUTION

### 5.1 Definition

The optimization function with constraint is defined by [10] [11]:

$$\begin{array}{ll} \min & f(x) \\ s.t. & \\ & \begin{cases} h(x) \le 0 \\ g(x) = 0 \\ LB \le x \le UB \end{cases} \end{array} \quad (24)$$

The dimension of x is n: $x = \begin{bmatrix} x_1 & x_2 & \cdots & x_n \end{bmatrix}$. For the no constraints, h(x) and g(x) do not exist.

*5.2. Function gradient and hessian*

$$gradf = \nabla f = \begin{bmatrix} \frac{\partial f}{\partial x_1} & \frac{\partial f}{\partial x_2} & \cdots & \frac{\partial f}{\partial x_n} \end{bmatrix}^T \quad (25)$$

$$H_{i,j} = \frac{\partial^2 f}{\partial x_i \partial x_j} = \frac{\partial}{\partial x_i}\left(\frac{\partial f}{\partial x_j}\right) = \frac{\partial}{\partial x_j}\left(\frac{\partial f}{\partial x_i}\right) \quad (26)$$

This equality (26) is true by using the theorem of Schwarm on the partial differential. The matrix representation of H will be:

$$H = \begin{bmatrix} \frac{\partial^2 f}{\partial x_1^2} & \frac{\partial^2 f}{\partial x_1 \partial x_2} & \cdots & \frac{\partial^2 f}{\partial x_1 \partial x_n} \\ \frac{\partial^2 f}{\partial x_2 \partial x_1} & \frac{\partial^2 f}{\partial x_2^2} & \cdots & \frac{\partial^2 f}{\partial x_2 \partial x_n} \\ \vdots & \vdots & \ddots & \vdots \\ \frac{\partial^2 f}{\partial x_n \partial x_1} & \frac{\partial^2 f}{\partial x_n \partial x_2} & \cdots & \frac{\partial^2 f}{\partial x_n^2} \end{bmatrix} \quad (27)$$

The method of resolution could be classified by 2 by the function to optimize:

-If the function gradf and hessf is known

- If the function gradf and hessf is known

Our search is concerned only of the resolution's method without knowing the gradient and hessian like PSO or Particle Swarm Optimization, BADS or Bayesian Adaptive Direct Search, NM or Nelder Mead. The execution's time is not evaluation during our simulation.

*5.1 Algorithm NM ou Nelder Mead*

The algorithm could be shared to multiple functions: Order, centroid, reflection, expansions, contraction, shrinking. The algorithm Nelder Mead is highly inspired to the genetic algorithm.

Step 1: Ordering

$f(x_1) \le f(x_2) \le \cdots \le f(x_{n+1})$

Step 2 : Centroid $x_0 = \frac{1}{N}\sum x_i$

Step 3: Reflection

$x_r = x_0 + \alpha(x_0 - x_{n+1}); \alpha > 0$

Si $f(x_1) \le f(x_r) < f(x_n)$

$worst(x_{n+1}) = x_r$ go to step 1

Step 4: Expansion

If $f(x_r) < f(x_1)$

$x_e = x_0 + \gamma(x_r - x_0); \gamma > 1$

If $f(x_e) < f(x_r)$

$worst(x_{n+1}) = x_e$ go to step 1

Else

$worst(x_{n+1}) = x_r$ go to 1

Step 5: Contraction

$x_c = x_0 + \rho(x_{n+1} - x_0); 0 < \rho \le 0.5$

If $f(x_c) < f(x_{n+1})$

$worst(x_{n+1}) = x_c$ go to step 1

Step 6: Shrinking

Replace all points instead of $Best(x_1)$ by

$x_i = x_1 + \sigma(x_i - x_1)$

Note that: $\alpha, \gamma, \rho, \sigma$ is respectively the coefficients of reflection, expansions, contraction and shrinking. The standard value of this are:

$$\alpha = 1, \gamma = 2, \rho = \frac{1}{2}, \sigma = \frac{1}{2} \tag{28}$$

### 5.2. Algorithm PSO or Particle Swarm Optimization

The base of this algorithm is proposed by Kennedy and Eberhart on 1995:

$x_k^i$ Particle position

$v_k^i$ Particle velocity

$p_k^i$ Best remembered individual particle position

$p_k^g$ Best remembered swarm position

$c_1, c_2$ Cognitive and social parameters

$r_1, r_2$ Random number between 0 and 1

Updating the particle is defined by:

$$x_{k+1}^i = x_k^i + v_{k+1}^i \tag{29}$$

Updating velocity is defined by:

$$v_{k+1}^i = w_k v_k^i + c_1 r_1 \left(p_k^i - x_k^i\right) + c_2 r_2 (p_k^g - x_k^i) \tag{30}$$

| |
|---|
| Step 1: Initialization |
| Initialize $c_1, c_2, w_1$ |
| Initialize randomly $x_0^i \in D\ in\ \mathbb{R}^n$ for $i = 1, \dots, p$ |
| Initialize the velocity $0 \le v_0^i \le v_0^{max}$ for $i = 1, \dots, p$ |
| Step 2: Optimization |
| If $f_k^i < f_{best}^i$ then$f_{best}^i = f_k^i, p_k^i = x_k^i$ else $c_1 = 0$ |
| If $f_k^g < f_{best}^{best}$ then$f_{best}^g = f_k^g, p_k^g = x_k^g$ |
| If condition stopped go to step 3 |
| Updating velocity $v_k^i$ |
| Updating particle $x_k^i$ |
| Increment k |
| Go to step 2 |
| Step 3: Termination |

**5.3. Algorithm BADS or Bayesian Adaptive Direct Search**

The algorithm BADS or Bayesian Adaptive Direct Search is a combination with the combination with the algorithm MADS or Mesh Adaptive Direct Search and the BO or Bayesian Optimization found on 2017 par L. Acerbi, WJ. Ma [11]

| |
|---|
| Step 1: Evaluate f in his initial state |
| Step 2: Poll – Train - Search |
| Repeat until convergence or EvalMax(f) |
| Step 3: Evaluate up to 2D points around x, update x (Poll step) |
| Step 4: Train Gaussian Process on neighborhood of x |
| Step 5: Search of solution by using B0 (Bayesian Optimization) |

## 6. SELECTION OF OPTIMIZATION

The algorithm with high surface of propagation is the best. The author proposed to create 2 criteria which are the surface interior of the occupied angular noted IOB or In Of The Beam Width and the surface exterior of the occupied angular noted OOB or Out Of Beam Width [10] [11].

$$IOB = \int_{\theta_k \epsilon \theta_{BW}} P(W(\psi), \theta_k) d\theta_k \quad (31.a)$$

$$OOB = \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} P(W(\psi), \theta_k) d\theta_k - \int_{\theta_k \epsilon \theta_{BW}} P(W(\psi), \theta_k) d\theta_k$$

$$OOB = \left[\int_{-90}^{90} P(W(\psi), \theta_k) d\theta_k - \int_{\theta_k \epsilon \theta_{BW}} P(W(\psi), \theta_k) d\theta_k\right] \quad (31.b)$$

Knowing that:

$$\int_{-90}^{90} P(W(\psi), \theta_k) d\theta_k = \sum_i BW_i * max(\emptyset(\theta_k))$$

The author also creates algorithm for selection the best method of resolution by selection the high value of IOB. This algorithm takes the good decision for the proposed resolution.

If max($IOB_{BADS}, IOB_{PSO}, IOB_{NM}$) = $IOB_{NM}$

Use NM-optimization

If max($IOB_{BADS}, IOB_{PSO}, IOB_{NM}$) = $IOB_{PSO}$

Use PSO-optimization

If max($IOB_{BADS}, IOB_{PSO}, IOB_{NM}$) = $IOB_{BADS}$

Use BADS optimization

## 7. BPSK SIGNAL GENERATOR

With covariance matrix R, choosing the waveform is defined by [9]:

$$X = [x_1 \quad x_2 \quad \dots \quad x_{N_s}] \quad (32)$$

This waveform could be generated by following the equation:

$$X = N.\wedge^{1/2}.S^H \quad (33)$$

X is the vector of the transmitted signal $[S \quad \wedge \quad D] = SVD(R)$

N is random number following GRV

The synthesis of the covariance matrix using the GRV is proposed by Ahmet and al. This method consists to transmit numeric signal with the form [9]:

$$z_m = sign(x_m), m = \{1,2, \dots, n_T\}$$

The BPSK signal to be transmitted is defined by:

$$Z = sign(X) \quad (34)$$

For using GRV on two variable $x_p, x_q$ and the following BPSK variable $z_p$ and $z_q$.

$$E\{sign(x_p)sign(x_q)\} = \frac{2}{\pi} sin^{-1}(E\{x_p x_q\})$$

In this fact, we deduce that the relation between the real covariance R and the Gaussian covariance $R_g$ is defined by:

$$R = \frac{2}{\pi} sin^{-1}(R_g) \tag{35}$$

$R_g$ will be defined:

$$R_g = W(\psi).W^H(\psi) \tag{36}$$

For having FACE envelope with the angular direction$\theta_k$ , the minimized function will be:

$$\min J_{BPSK}(\psi) = \frac{1}{K}\sum_{k=1}^{K}(P(\psi) - \emptyset(\theta_k))^2 \tag{37}$$

The schema bloc resuming the formation of the waveform FACE-BPSK is represented by the Fig-1:

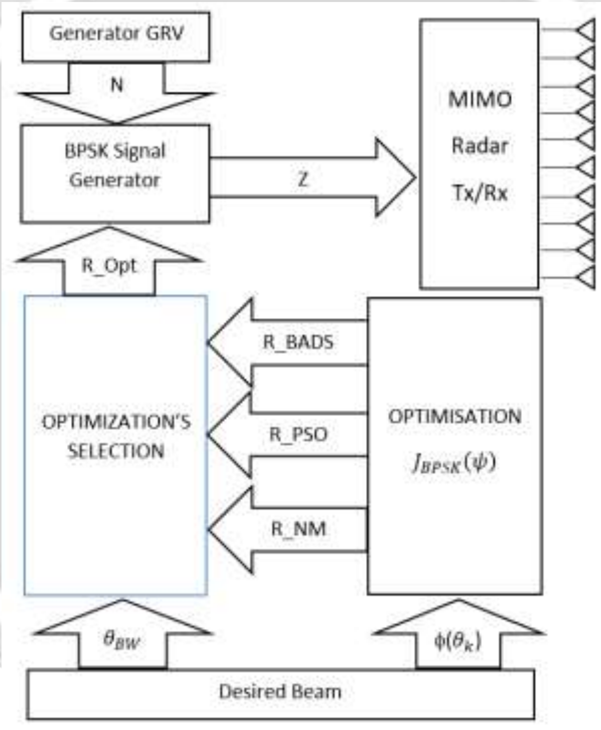



**Fig-1:** Schema Bloc to form waveform FACE-BPSK

All parameters used for our simulation are:

- Number antenna of the MIMO Radar= 10

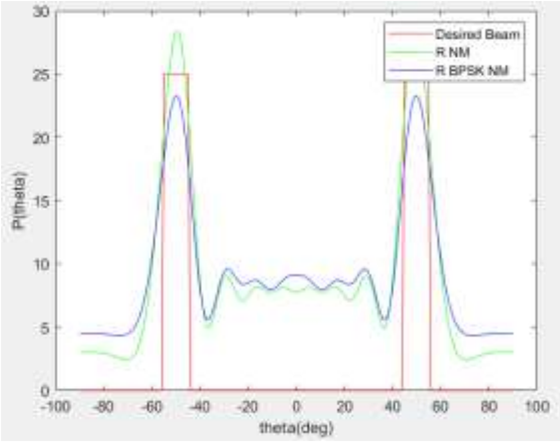
- Number antenna of the base station $\leq 10$

- Desired angular: $\theta_{BW} = \{[-55; -45], [45; 55]\}$

- The number of sampling = 100

The algorithm calculates the surface propagation and select following this the value maximal.

If NM has a best result:



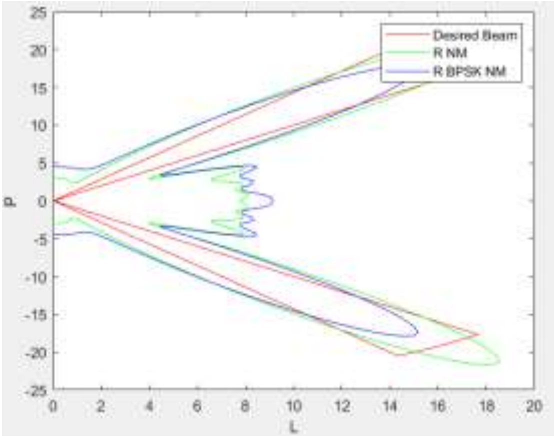



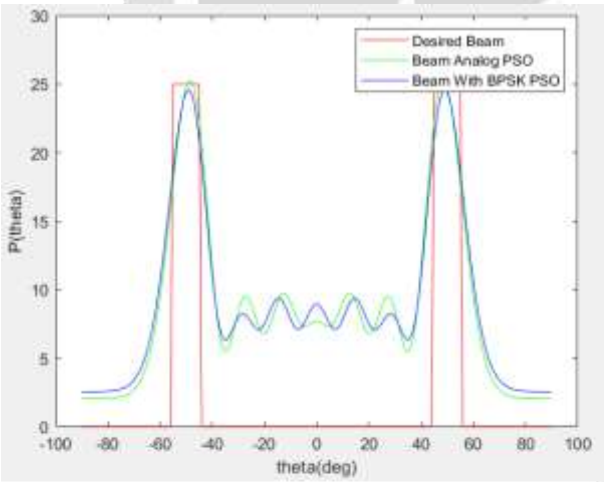

**Chart-1 :** BPSK-NM high IOB **Chart-2:** BPSK-NM high IOB

**Table-1:** Effect of IOB and OOB on NM, PSO and BADS

| | NM | PSO | BADS |
|---|---|---|---|
| IOB | 429.8543 | 406.0223 | 417.7756 |
| OOB | 1.2396e+03 | 1.1693e+03 | 1.1986e+03 |

By using the transform of the Cartesian co-ordinate to the polar coordinate, The Chart-1 is transformed to the Chart -2 The propagation of the desired function is the desired R. By using the algorithm FACE, the reference propagation will be R NM. The surfaces of the propagation BPSK-FACE are: 429.8543 with NM, 406, 0233 with PSO and 417,7756 with BADS. In this fact, our algorithm chooses NM for the waveform. The analyze of the OOB is not doing in this article, the MIMO Radar is concerning only of the optimization of the angular choose.

If PSO has a best result:



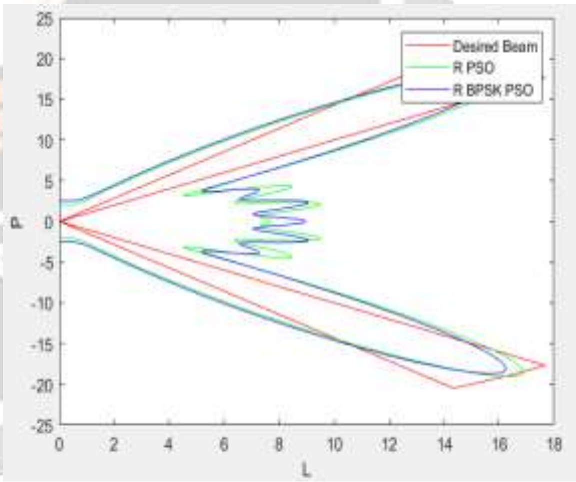




**Chart-3:** BPSK PSO high IOB **Chart-4:** BPSK PSO high IOB

**Table-2:** Effect of IOB and OOB on NM, PSO and BADS

| | NM | PSO | BADS |
|---|---|---|---|
| IOB | 402.0134 | 455.9346 | 411.6329 |
| OOB | 1.2540e+03 | 1.1678e+03 | 1.1527e+03 |

The surface of propagation BPSK-FACE are: 402.0134 with NM, 455.9346 with PSO and 417, 411.6329 with BADS. In this fact our algorithm chooses PSO for the waveform.

If BADS has a best result:

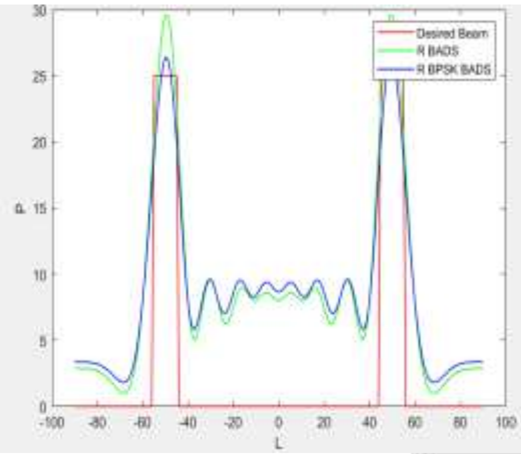



**Chart-5:** BPSK-BADS high IOB

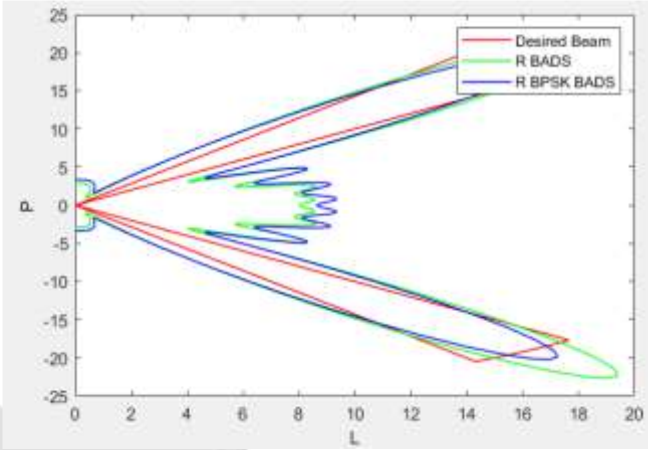



**Chart-6:** BPSK-BADS high IOB

**Table-3:** Effect of IOB and OOB on NM, PSO and BADS

| | NM | PSO | BADS |
|---|---|---|---|
| IOB | 410.6608 | 444.1823 | 478.0069 |
| OOB | 1.2445e+03 | 1.1988e+03 | 1.1224e+03 |

The propagation BPSK-FACE are: 410.6608 with NM, 444.1823 with PSO and 478.0069 with BADS. In this fact, our algorithm chooses BADS for the waveform resolution. We could deduce that our algorithm selects the best resolution method with high IOB.

## 8. QPSK SIGNAL GENERATOR

The matrix covariance complex GRV will be defined by [12] [13]:

$$\widetilde{R_g} = \Re(R_g) + j\Im(R_g) \tag{38}$$

$$E\{\widetilde{z_p}\,\widetilde{z_q^*}\} = \frac{2}{\pi}\left(sin^{-1}\left(E\{\widetilde{x_p}\widetilde{x_q}\}\right) + j sin^{-1}\left(E\{\widetilde{y_p}\widetilde{x_q}\}\right)\right) \tag{39}$$

The relation R and $R_g$ is also defined by:

$$\tilde{R} = \frac{2}{\pi}\left[sin^{-1}\left(\Re(R_g)\right) + j sin^{-1}\left(\Im(R_g)\right)\right]$$

By analogy with BPSK, The construction of the matrix $\widetilde{R_g}$ is defined by:

$$\widetilde{R_g} = \widetilde{W}(\psi)^H.\widetilde{W}(\psi)$$

$$\widetilde{W}(\psi) = \Re(W) + j\Im(W)$$

The covariance matrix QPSK is written to complex form and defined by:

$$\widetilde{R_g} = \left(\Re(W)^H\Re(W) + \Im(W)^H\Im(W)\right) + j\left(\Re(W)^H\Im(W) - \Im(W)^H\Re(W)\right) \tag{40}$$

The matrix spherical form do not use only the vector $\psi_{m,n}$ to form the waveform QPSK. It adds other criteria, which is:

$$\psi' = [\psi_1' \quad \psi_2' \quad \dots \quad \psi_{n_T}'] \tag{41}$$

$$card\{\psi'\} = n_T \tag{42}$$

$$W(\psi,\psi')=\begin{bmatrix} e^{j\psi'_1} & e^{j\psi'_2}\sin(\psi_{21}) & e^{j\psi'_3}\sin(\psi_{31})\sin(\psi_{32}) & \cdots & e^{j\psi'_{n_T}}\prod_{m=1}^{n_T-1}\sin(\psi_{n_T m}) \\ 0 & e^{j\psi'_2}\cos(\psi_{21}) & e^{j\psi'_3}\sin(\psi_{31})\cos(\psi_{32}) & \cdots & e^{j\psi'_{n_T}}\prod_{m=1}^{n_T-2}\sin(\psi_{n_T m})\cos(\psi_{n_T,n_T-1}) \\ \vdots & 0 & e^{j\psi'_3}\cos(\psi_{31}) & \ddots & \vdots \\ \vdots & \vdots & \ddots & \cdots & e^{j\psi'_{n_T}}\sin(\psi_{n_T 1})\cos(\psi_{n_T 2}) \\ 0 & 0 & \cdots & \cdots & e^{j\psi'_{n_T}}\cos(\psi_{n_T 1}) \end{bmatrix} \quad (43)$$

$$\mathrm{R}\left[W(\psi,\psi')\right]=\begin{bmatrix} \cos(\psi'_1) & \cos(\psi'_2)\sin(\psi_{21}) & \cos(\psi'_3)\sin(\psi_{31})\sin(\psi_{32}) & \cdots & \cos(\psi'_{n_T})\prod_{m=1}^{n_T-1}\sin(\psi_{n_T m}) \\ 0 & \cos(\psi'_2)\cos(\psi_{21}) & \cos(\psi'_3)\sin(\psi_{31})\cos(\psi_{32}) & \cdots & \cos(\psi'_{n_T})\prod_{m=1}^{n_T-2}\sin(\psi_{n_T m})\cos(\psi_{n_T,n_T-1}) \\ \vdots & 0 & \cos(\psi'_3)\cos(\psi_{31}) & \ddots & \vdots \\ \vdots & \vdots & \ddots & \cdots & \cos(\psi'_{n_T})\sin(\psi_{n_T 1})\cos(\psi_{n_T 2}) \\ 0 & 0 & \cdots & \cdots & \cos(\psi'_{n_T})\cos(\psi_{n_T 1}) \end{bmatrix} \quad (44)$$

$$\mathrm{I}\left[W(\psi,\psi')\right]=\begin{bmatrix} \sin(\psi'_1) & \sin(\psi'_2)\sin(\psi_{21}) & \sin(\psi'_3)\sin(\psi_{31})\sin(\psi_{32}) & \cdots & \sin(\psi'_{n_T})\prod_{m=1}^{n_T-1}\sin(\psi_{n_T m}) \\ 0 & \sin(\psi'_2)\cos(\psi_{21}) & \sin(\psi'_3)\sin(\psi_{31})\cos(\psi_{32}) & \cdots & \sin(\psi'_{n_T})\prod_{m=1}^{n_T-2}\sin(\psi_{n_T m})\cos(\psi_{n_T,n_T-1}) \\ \vdots & 0 & \sin(\psi'_3)\cos(\psi_{31}) & \ddots & \vdots \\ \vdots & \vdots & \ddots & \cdots & \sin(\psi'_{n_T})\sin(\psi_{n_T 1})\cos(\psi_{n_T 2}) \\ 0 & 0 & \cdots & \cdots & \sin(\psi'_{n_T})\cos(\psi_{n_T 1}) \end{bmatrix} \quad (45)$$

$$J_{QPSK}(\psi,\psi') = \frac{1}{K}\sum_{k=1}^{K}\left[\left(P(\psi,\psi') - \emptyset(\theta_k)\right)^2\right] \quad (46)$$

The variable number to optimize with the waveform QPSK is defined by:

$$n_{var} = (n_T - 1) * \frac{n_T}{2} + n_T = \frac{(n_T + 1) * n_T}{2} \quad (47)$$

When the matrix covariance R is synthetized, the waveform selected will be:

$$\tilde{X} = \left[\widetilde{x_1} \quad \widetilde{x_2} \quad \ldots \quad \widetilde{x_{N_s}}\right] \quad (48)$$

The signal has a form complex I/Q data.

The signal to be transmitted $\tilde{Z}$ on QPSK will be:

$$\widetilde{Z} = \frac{1}{\sqrt{2}}\left[sign[\Re(X)] + sign[\Im(X)]\right] \quad (49)$$

This waveform could be generated following the equation:

$$\tilde{X} = \tilde{N}.\widetilde{\Lambda}^{1/2}.\tilde{S}^H \quad (50)$$

$\tilde{X}$ is the symbol vector to be transmitted, $\left[\tilde{S} \quad \widetilde{\Lambda} \quad \tilde{D}\right] = SVD(\tilde{R})$

The schema bloc summarize all the theories is represented by the Fig-2.

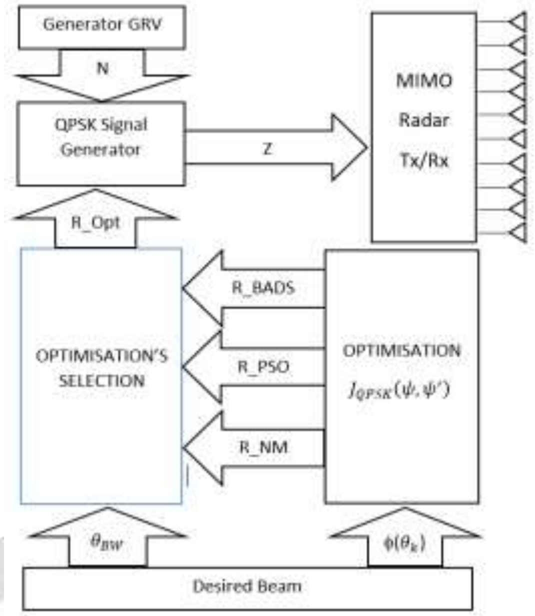



**Fig-2:** *schema bloc to form waveform FACE-QPSK*

Our simulation uses the following parameters:

- Number antenna of the MIMO Radar = 10

- Number antenna of the Base Station ≤ 10

- The desired angular : $\theta_{BW} = \{[-60; -40], [40; 60]\}$

- The number of sampling: 100

If NM has a best result:

Like the case with the BPSK, to result to be extracted is the data QPSK with the maximal of the IOB.

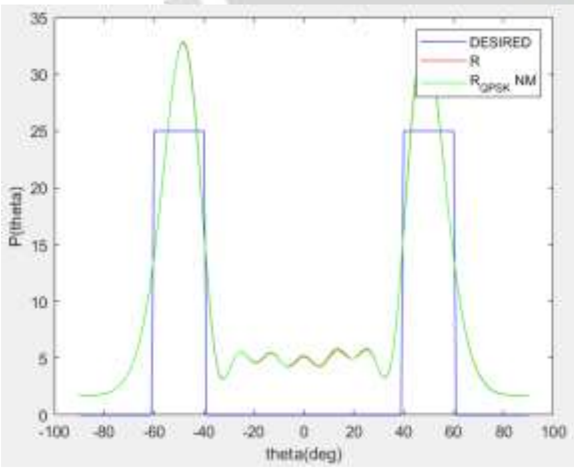



**Chart-7:** QPSK NM with high IOB

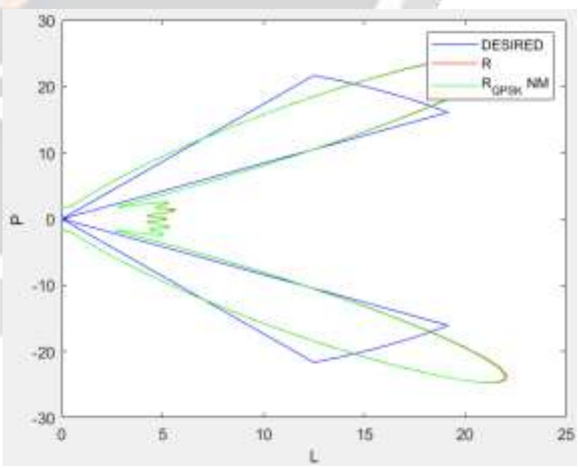



**Chart-8:** QPSK NM with high IOB

R_QPSK in the Chart-7 and Figure 6 (b) concerns the case with QPSK with number of sampling 100. When this sampling tends to the infinity, the propagation will be represented by the matrix covariance R.

**Table-4:** Effect of IOB and OOB on NM, PSO and BADS

| | NM | PSO | BADS |
|---|---|---|---|
| IOB | 1.0377e+03 | 773.1805 | 1.0275e+03 |
| OOB | 669.9335 | 967.6678 | 677.3554 |

The table shows that the maximal value of the surface of each optimization methods is the NM. In this fact, our algorithm chooses the method NM.

If PSO has the best result:

The algorithm PSO with FACE-QPSK will be presented at the Chart-9 and Chart-10. By transforming the propaged power to the angular at the Chart-9 to a polar co-ordinate, We could have he presentation of the space's propagation by the Chart-10. This surface will be calculated by making integration of the power on the concerned angular.

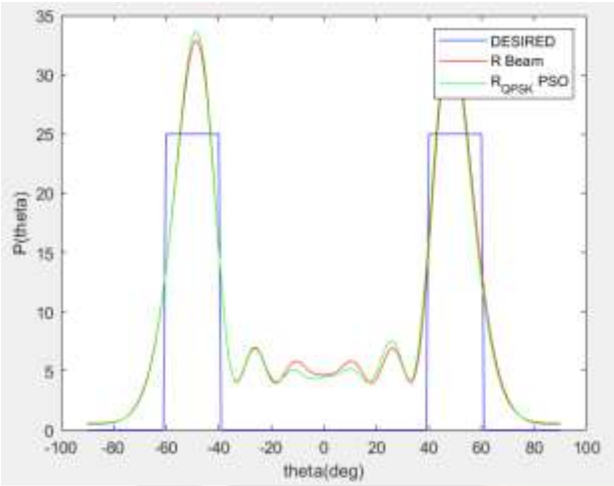



**Chart-9:** QPSK PSO with high IOB

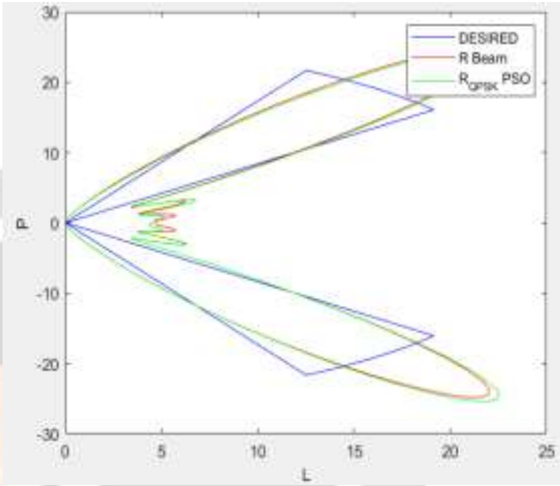



**Chart-10:** QPSK PSO with high IOB

**Table-5:** Effect of IOB and OOB on NM, PSO and BADS

| | NM | PSO | BADS |
|---|---|---|---|
| IOB | 1.0094e+03 | 1.0319e+03 | 1.0198e+03 |
| OOB | 680.4906 | 626.3871 | 660.8336 |

Following the propagation, the algorithm PSO has a maximal value of the surface of the propagation, which is$1.0319.10^3$.

If BADS has a best result :

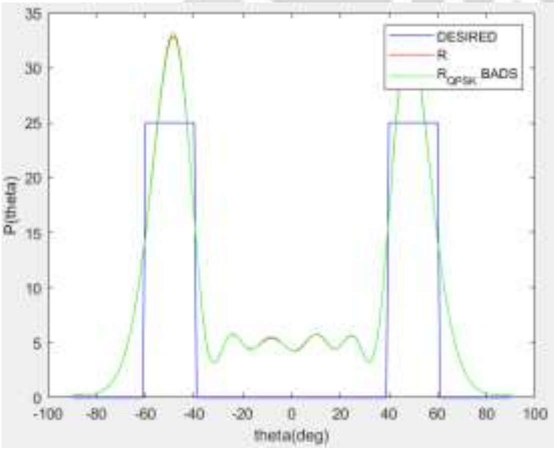



**Chart-11:** *QPSK BADS with high IOB*

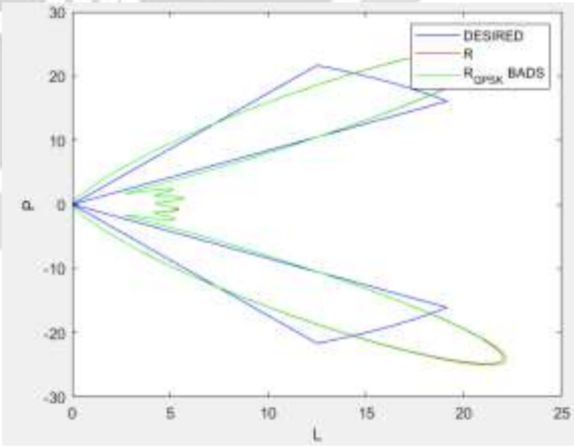



**Chart-12:** *QPSK BADS with high IOB*

Following the propagation, the algorithm BADS has a maximum value of the propagation, which is$1.0453.10^3$.

**Table-6:** Effect of IOB and OOB on NM, PSO and BADS

| | NM | PSO | BADS |
|---|---|---|---|
| IOB | 1.0342e+03 | 683.4214 | 1.0453e+03 |
| OOB | 672.0949 | 1.0227e+03 | 622.7082 |

## 9. NSP

### 9.1. Cellular network modelization

We consider that the cellular network with the canal MIMO, with $\kappa$ equipped with base station which has $N_{BS}$ antenna reception and transmission. Each i-th BS supports $L_i$ UE or User equipment. Each UE has a multi-antenna with $N_{UE}$ reception and transmission's antenna. If $S_j$ is the signal to transmit to the j-th UE for the i-th cellular, so the received signal with the i-th base station could be defined by [14] [15] :

$$y_i(n) = \sum_j H_{i,j} S_j(n) + \eta(n), \quad (51)$$

$$pou 1 \leq i \leq \kappa \text{ et } 1 \leq j \leq L_i$$

$H_{i,j}$ is the transfert matrix between the i-th BS and j-th UE

η (n) is the Gaussian noise.

### 9.2. Interference Model

We defined the interference matrix is:

$$H_i = \begin{bmatrix} h_i^{(1,1)} & \cdots & h_i^{(1,n_T)} \\ \vdots & \ddots & \vdots \\ h_i^{(N_{BS},1)} & \cdots & h_i^{(N_{BS},n_T)} \end{bmatrix} \quad (52)$$

With $i = 1,2, \ldots, \kappa$ and the element $h_i^{(l,k)}$ is the canal's coefficient part of the k-th antenna of the MIMO Radar, l-th antenna of the i-th base station

### 9.3 Spectrum sharing scenarios

By considering the coexistence with the $\kappa$ canal's interference, the signal of the i-th base station could be written by the form [14] [15]:

$$y_i(n) = H_i x(n) + \sum_j H_{i,j} S_j(n) + \eta(n) \quad (53)$$

Instead of avoiding the interference of the i-th BS, the radar forms this envelope like on null space of the $H_i$ by :

$$H_i . x(n) = 0 \quad (54)$$

It possible to use the SVD decomposition for calculating the null space projection:

$$H_i = U_i \Sigma_i V_i^H \quad (55)$$

$$\tilde{\Sigma}_i = diag(\tilde{\sigma}_{i,1}, \tilde{\sigma}_{i,2}, \ldots, \tilde{\sigma}_{i,p})$$

With $p = min(N_{BS}, n_T)$ and $\tilde{\sigma}_{i,1} > \tilde{\sigma}_{i,2} > \cdots > \tilde{\sigma}_{i,q} > \tilde{\sigma}_{i,q+1} = \tilde{\sigma}_{i,2} = \cdots = \tilde{\sigma}_{i,p} = 0$

$$\Sigma_i' = diag(\tilde{\sigma}_{i,1}', \tilde{\sigma}_{i,2}', \ldots, \tilde{\sigma}_{i,p}')$$

with

$$\sigma_{i,u}' = \begin{cases} 0 \ pour \ u \leq q \\ 1 \ pour \ u > q \end{cases}$$

In this fact, $$P_i = V_i \Sigma_i' V_i^H \quad (56)$$

### 9.4. Propriety of the projection

- $P_i$ is a null space projection if only: $P_i = P_i^H = P_i^2$

$$P_i^H = (V_i \Sigma_i' V_i^H)^H = P_i$$

$$P_i^2 = V_i\, \Sigma_i' V_i^H \times V_i\, \Sigma_i' V_i^H = P_i$$

- $P_i$ is an orthogonal projection with the null space $H_i$

**9.5. Algorithm NSP**

| |
|---|
| Calculate of SVD: $U_i \Sigma_i V_i^H = H_i$<br>Construction: $\widetilde{\Sigma}_i = diag(\tilde{\sigma}_{i,1}, \tilde{\sigma}_{i,2}, \ldots, \tilde{\sigma}_{i,p})$<br>Consturction: $\Sigma_i' = diag(\tilde{\sigma}_{i,1}', \tilde{\sigma}_{i,2}', \ldots, \tilde{\sigma}_{i,p}')$<br>Calculate of the projection: $P_i = V_i\, \Sigma_i' V_i^H$ |

## 10. OPTIMIZED SIGNAL GENERATOR

After capturing the interference and determine the signal to be transmitted Z. The optimized signal NSP to be transmitted will be defined [9]:

$$Z_{NSP} = Z.P_i^H \tag{57}$$

## 11. BPSK AND NSP

The diagram summarizes the schema bloc of the BPSK NSP will be presented by (9) [9] [14] [15]. The function to be optimized needs the null space projection by:

$$J_{BPSK-NSP}(\psi) = \frac{1}{K}\sum_{k=1}^{K}\left(P_i P(\psi) P_i{}^H - \emptyset(\theta_k)\right)^2 \tag{58}$$

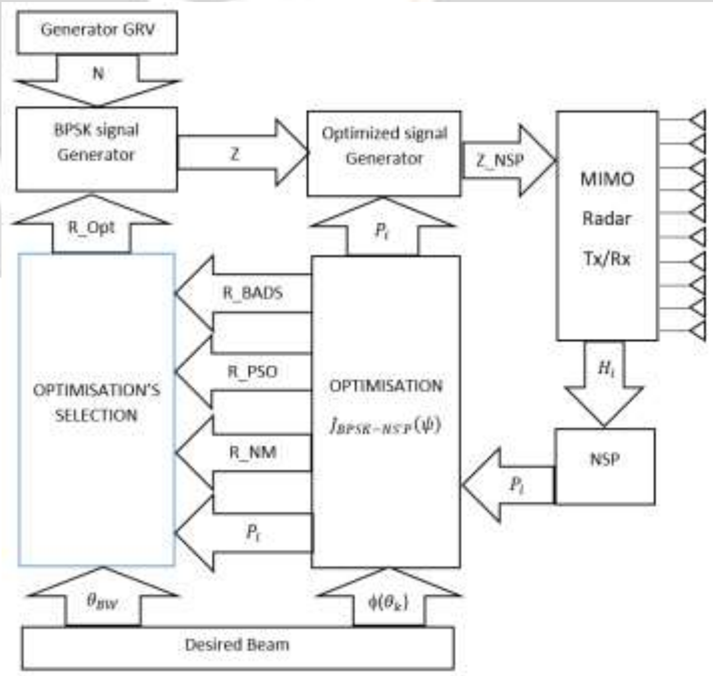



**Fig-3:** *Schema bloc to form the waveform FACE-BPSK with NSP*

If PSO has a best result:

**Table-7:** Effect of IOB and OOB on NM, PSO and BADS

| No NSP | NM | PSO | BADS |
|---|---|---|---|
| IOB | 50.4343 | 60.1173 | 37.5579 |
| OOB | 341.4592 | 239.0999 | 380.8874 |

**Table-8:** Effect of IOB and OOB on NM, PSO and BADS

| Avec NSP | NM | PSO | BADS |
|---|---|---|---|
| IOB | 101.6536 | 145.1768 | 140.0213 |
| OOB | 415.9135 | 537.1671 | 508.5189 |

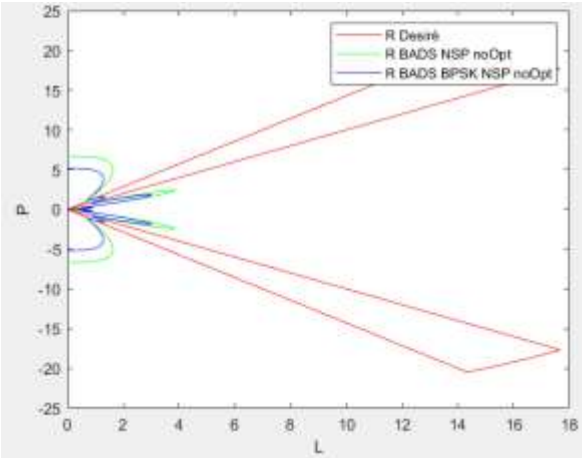



**Chart-13:** *BADS BPS without NSP*

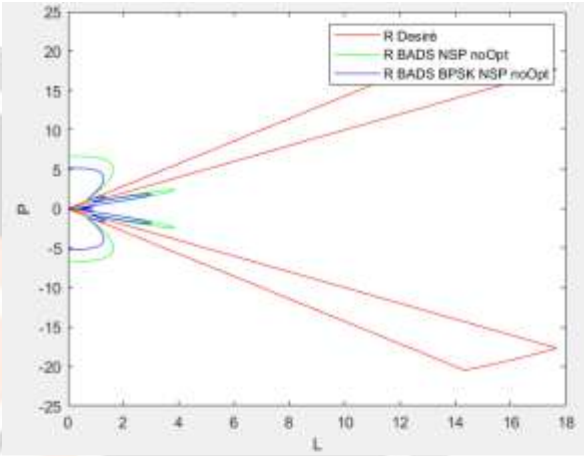



**Chart-14:** *BADS BPS with NSP*

The propagation with maximum IOB with NSP compared to NM, PSO, and BADS is the algorithm PSO. Our algorithm chooses also PSO.To more finding the deterioration of the envelope; we study in particular the propagation with worst IOP without NSP, which is the BADS methods.

We could see that the surface occupied by the propagation increase when the algorithm use the NSP. Without the NSP, the envelope does not follow the desired beam.

If BADS has a best result:

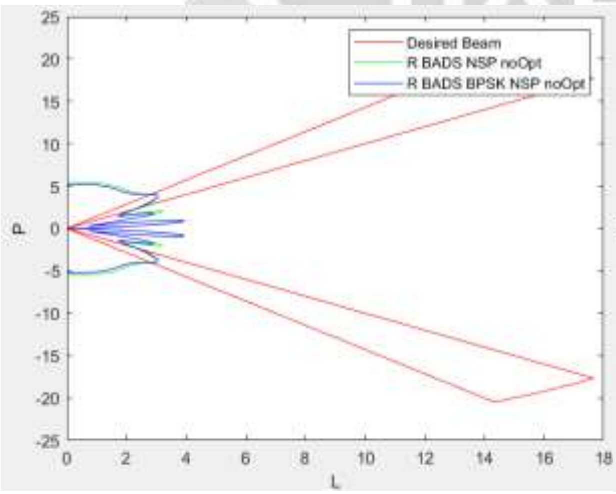



**Chart-15:** *BADS BPSK without NSP*

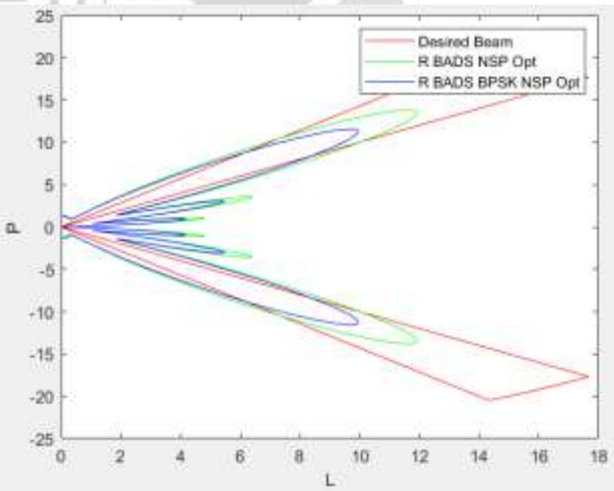



**Chart-16:** *BADS BPSK with NSP*

**Table-9:** Effect of IOB and OOB on NM, PSO and BADS

| No NSP | NM | PSO | BADS |
|---|---|---|---|
| IOB | 183.1048 | 262.0497 | 90.2642 |
| OOB | 584.9542 | 570.7490 | 558.5109 |

**Table-10:** Effect of IOB and OOB on NM, PSO and BADS

| With NSP | NM | PSO | BADS |
|---|---|---|---|
| IOB | 261.5825 | 267.5848 | 275.4209 |
| OOB | 538.9625 | 488.9956 | 484.8907 |

Our algorithm chooses BADS, which has a high IOB. For studying in particular the deterioration of the envelope without NSP, we choose to show the Figure with worst IOB without NSP, which is the BADS methods. We could see that the surface occupied by the propagation increase when it use NSP. Without NS, the width of the envelope decrease.

## 12. QPSK AND NSP

The function to be optimized needs also knowing the projection to null space.

$$J_{QPSK-NSP}(\psi,\psi') = \frac{1}{K}\sum_{k=1}^{K}\left(P_i P(\psi,\psi')P_i^{H} - \emptyset(\theta_k)\right)^2 \quad (59)$$

The Fig-4 summarizes the schema bloc to form waveform FACE-QPSK with NSP.

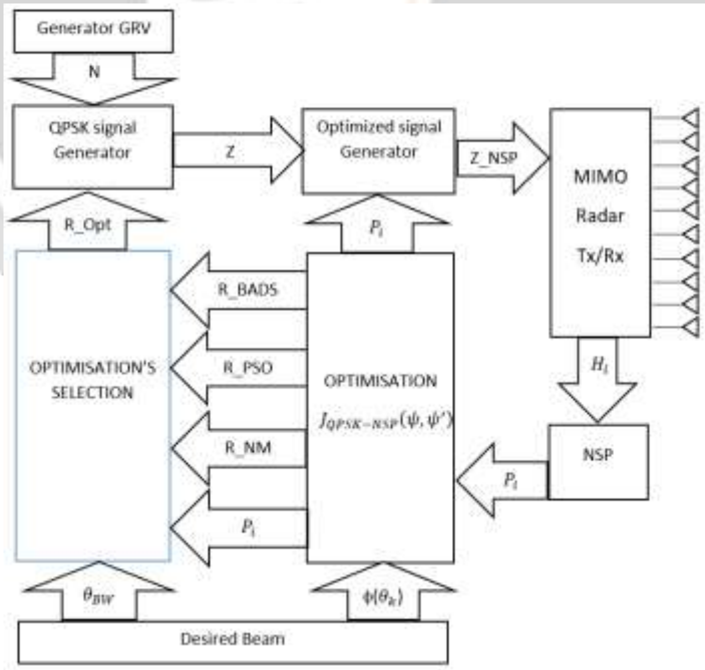



**Fig-4:** *Schema bloc to form waveform FACE-QPSK with NSP*

If NM has a best result:

**Table-11:** Effect of IOB and OOB on NM, PSO and BADS

| No NSP | NM | PSO | BADS |
|---|---|---|---|
| IOB | 806.2703 | 749.3184 | 701.9524 |
| OOB | 636.2181 | 728.0744 | 784.0097 |

With NSP, the propagation with the surface maximal is the algorithm NM with the value928.1608. For analyzing the efficacy with the NSP, We will studied in particular the algorithm with the worst IOB without NSP, which is the algorithm BADS

**Table-12:** Effect of IOB and OOB on NM, PSO and BADS

| With NSP | NM | PSO | BADS |
|---|---|---|---|
| IOB | 928.1608 | 857.8483 | 922.0548 |
| OOB | 535.8034 | 519.9619 | 411.5190 |

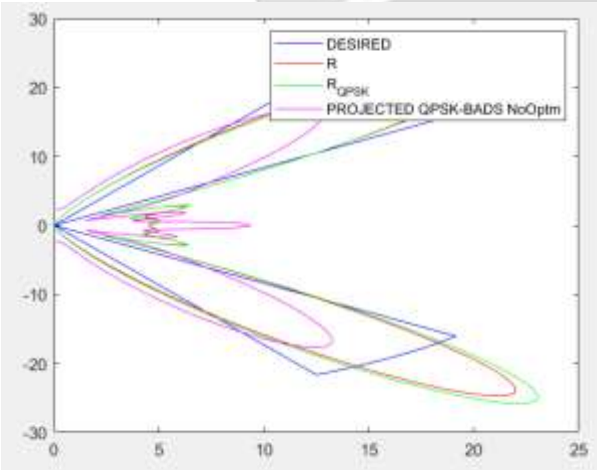



**Chart-17:** *QPSK BADS without NSP*

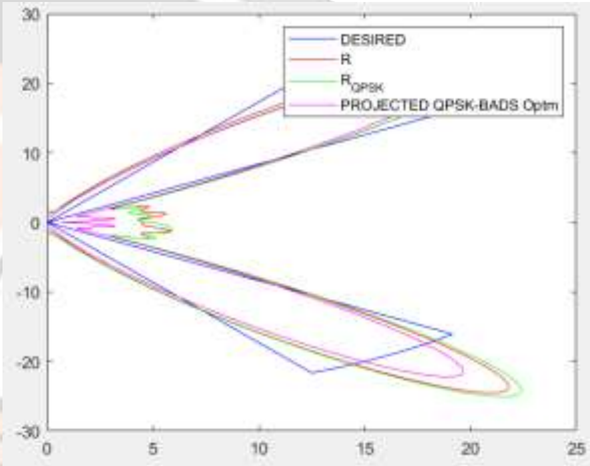



**Chart-18:** *QPSK BADS with NSP*

The NSP reduces the surface of the propagation of the Radar MIMO. By using algorithm NSP, this surface increase 701 to 922.

If PSO has a best result:

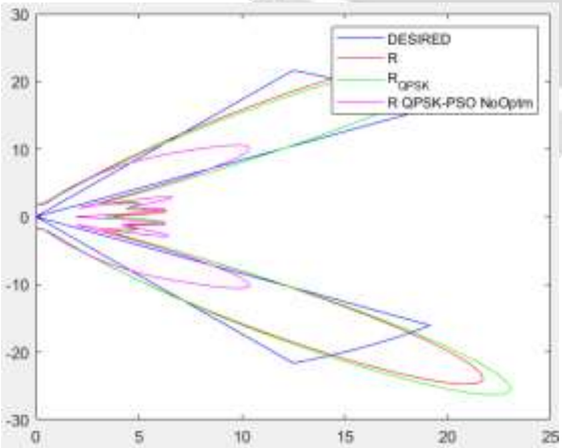



**Chart-19:** *QPSK PSO without NSP*

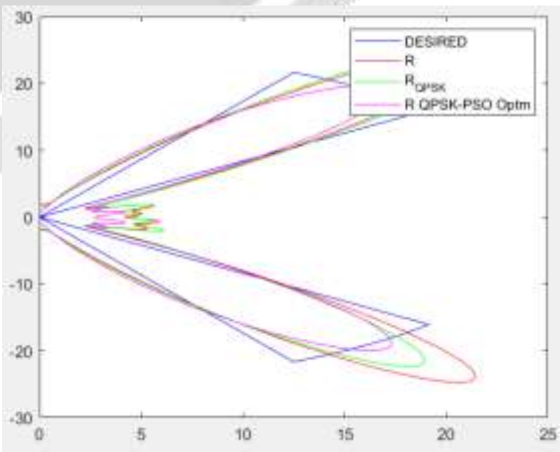



**Chart-20:** *QPSK PSO with NSP*

**Table-13:** Effect of IOB and OOB on NM, PSO and BADS

| No NSP | NM | PSO | BADS |
|---|---|---|---|
| IOB | 539.7685 | 503.7266 | 555.7099 |
| OOB | 685.2593 | 631.2911 | 695.3669 |

With NSP, the propagation with the surface maximal is the algorithm PSO, which has the value 883.4497. To analyze efficacy of the NSP, we study in particular the algorithm with the worst IOB without NSP, which is the algorithm PSO.

**Table-14:** Effect of IOB and OOB on NM, PSO and BADS

| With NSP | NM | PSO | BADS |
|---|---|---|---|
| IOB | 744.9044 | 883.4497 | 842.5344 |
| OOB | 678.8082 | 600.9229 | 459.5174 |

The NSP reduces the surface of the propagation of the Radar MIMO. By using algorithm NSP, this surface increase 503 to 883.

## 13. CONCLUSION

Our study permit us to know and observe the interest of the algorithm null space in the surface of the propagation of the waveform. The surface IOB or In Of the Bandwidth of the algorithm without NSP is not better than the algorithm with this. The signal to be transmitted with the waveform BPSK or QPSK will be calculated with this projection to the null space with this optimization. To have any envelope with the direction of the given angular, the MSE or Minimum Square Error between the power of the signal and the desired function should be minimal as possible. Our experimentation consist to create any algorithm to select the best algorithm to solve this problem of the minimization. Our choice is based on the algorithm without knowing the function gradient and hessian of the problem like the BADSn the NM and the PSO. The extraction of the best result of the methods of the resolution is classified on the tables for analyzing the IOB and OOB. Our study shows us that all algorithm has a probability to be selected and to have a best surface propagation. By knowing the interference channel, certain solver is most performing than other. The result final of our simulation conduct to this conclusion: the result of the waveform of the QPSK is better than QPSK. Explanation of this result could be interpreted by the matrix dimension to be resolved with the optimization on QPSK increase than the BPSK. The last result, is that the surface occupied with the IOB with NSP increase than without NSP.

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